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A Traffic-Jam Theory of Growth

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Abstract

We investigate the growth-finance nexus in an endogenous growth model with search frictions and congestion effects in credit and innovation markets. The interplay between these two frictions generates a nonlinear relationship between finance and growth. Financial development eases the financing of innovation but can exacerbate bottlenecks in R&D. In a calibration close to the U.S. economy, finance has a negative impact on growth. This effect is quantitatively small—consistent with the observation that, in the last century, most developed economies have experienced an expansion of the financial sector and almost constant growth rates of GDP.

JEL codes: E51, G24, O40.

Keywords: Growth, Finance, Search frictions, Technology, Innovation.

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Some academic researchers, pharma executives, and other experts have decried this explosion of [clinical] trials as a counterproductive glut motivated more by the race for money than good science and warned *that many of these efforts may not finish because of a lack of participants*.¹ (Kaiser, 2018)

1 Introduction

A long tradition in economics, exemplified by Solow (1956) and anchored in the Age of Enlightenment, views science and innovation as the main driving forces of long-run growth. The role played by financial development in this process is not trivial. To be true, research is costly so that, if inventions require investment in R&D beyond the means of innovators, financial development affects growth positively. However, the link between finance and growth is not one-way as financial institutions are costly to develop and maintain, and their profitability is, by essence, affected by growth prospects. Furthermore, the finance-growth nexus cannot be linear as, over the last century, per-capita GDP in the United States has been growing, except for historical accidents, at an average 2% annual rate while the development of the financial sector has accelerated. Something is thus hindering, and maybe even reversing, the contribution of finance to growth: our paper suggest that the interplay of congestion externalities in finance and in R&D might be the culprit.

Our formalization relies on the interaction between two markets, credit and R&D, both plagued by frictions. In a world where innovation itself entails no friction, removing the sole hurdle (finance) standing in the way of innovators necessarily enhances growth. This is the traditional mechanism underlying many policy recommendations for financial liberalization.² However, by contrast with the Solowian utopia in which innovations are instantly provided for free to researchers by a *deus ex machina*, R&D takes time and effort and its success rate is unpredictable. In addition, there is no presumption, contrary to common assertions, that more science and more research are always better for growth: there might be other bottlenecks and congestion effects that might be exacerbated by financial liber-

¹Emphasis added.

²They are akin to labor-market reform policies advocated on the basis of the Diamond-Mortensen-Pissarides model: if search and matching frictions on the labor market as the sole hindrance to unemployment, a "better" functioning labor market is all it takes to reduce unemployment.

alization. For instance, proliferation of clinical trials for checkpoint inhibitors in cancer immunotherapy might be "too much of a good thing" (Kaiser, 2018). Similarly, seemingly further removed from growth theory but in fact anchored in Jevons' (1866) paradox, Braess (1968) warns that traffic could be impeded by the addition of a new road — a remark at the core of the "Lewis-Mogridge position" that postulates that punctual improvements in a road network often shift congestion to another traffic node, thereby negating the original effort and possibly exacerbating overall road congestion. Our starting point is thus that models that ignore bottlenecks in research and innovation adopt, without nuance, a Renaissance-inspired belief in the unlimited scope for progress. Is it reasonable to assume that the more resources we pour into innovation the faster we will grow on average? Shall we not eventually lack individuals with the ability to do research?; or run out of the less skilled workers who provide infrastructure complementary to research (e.g., building or maintenance)? Or patients to enroll into clinical trials? Our answer is: possibly.

We investigate this conundrum in an endogenous growth model with search frictions in both credit and innovation markets. In our world, all growth is innovation-led and, partly in line with the Solowian view of the world, there is a fixed number of ideas ready to be "fetched" by innovators. Entrepreneurs do not have the wealth (or ability) to self-finance innovation and need to look for financiers. We show that, all else equal, there is a negative relationship between growth and tightness in both innovation and credit markets. But once all feedback effects are taken into account, financial deepening has a non-monotonic effect on long-run growth: after a certain threshold, more finance (which entails less tightness in credit markets) actually increases congestion in the ideas market, so that growth might fall.³

The finance-growth nexus has been the subject of an extensive empirical literature. Recent studies (Popov, 2018, Arcand et al., 2015, Aghion et al., 2019) suggest that, beyond a certain threshold, financial development has no effect or could even be detrimental for growth. In theory (see Levine, 2005 and Aghion et al., 2018 and references therein), well functioning financial systems can promote growth by improving resource allocation, fostering innova-

³We look at this issue also from a normative perspective and show that entry in both markets is efficient once innovators and financiers are compensated for their contribution to growth and that the social planner internalizes the interactions between the two congestion frictions. These results are available upon request.

tion or by facilitating monitoring and pooling of risky projects. At the same time, "too much finance" could lead to a misallocation of talents to less productive sectors of the economy (Tobin, 1984) or an increase in financial fragility (Minsky, 1974 and Rajan, 2005). Aghion et al. (2019) document an inverted-U relationship between credit constraints and productivity growth at a sectoral level in France and propose a theory according to which better access to credit allows less efficient incumbent firms to remain longer on the market. In the same spirit, Malamud and Zucchi (2019) propose a theoretical model where financing frictions affects differently entrant and incumbent firms and hence and the composition of growth. Our paper proposes a different, and not necessarily exclusive, explanation for the non-monotonic relationship between finance and productivity growth.

In our model, financiers provide funds to entrepreneurs to invest in R&D. All else equal, through this channel finance has a positive effect on growth. In this respect, our work contributes to the literature on innovation-led growth (see e.g. Aghion et al. 2005, Laeven et al., 2015, Chiu et al., 2017, Aghion et al., 2018 and Kogan et al., 2017). However, we depart from that literature in two important respects: i) we model bottlenecks in R&D by introducing search frictions in innovation markets, in the spirit as Silveira and Wright (2010)⁴ ii) we incorporate search frictions in financial markets. Our modeling of finance, borrowed from Wasmer and Weil (2004), embraces the view of Jaffee and Stiglitz (1990) according to which credit markets are better described as customer markets where borrowers have a single relationship with lenders. Cipollone and Giordani (2019a) and Cipollone and Giordani (2019b) provide some recent empirical support to a search and matching view of financial markets. Our stylized innovation market captures also external technology acquisitions, i.e. the strategic acquisitions of innovative firms by large firms *de facto* outsourcing R&D (Phillips and Zhdanov, 2013).

Our work is is part of a literature, stemming from Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2015) and Chiu et al. (2017), which studies the interactions between multiple trading frictions. The latter paper also deals with innovation-led growth in the presence of financial frictions though it restricts liquidity through a collateral constraint. In con-

⁴Along similar lines, in Frydman and Papanikolaou (2018), managers can improve technology by searching for new investment opportunities.

trast in our set-up, R&D is not pledgeable.

Empirically, Bloom et al. (2020) document a sharp decline in research productivity and a substantial rise in research effort in many sectors. Gordon (2016) suggests that inequality, education, demographic and fiscal factors are fours possible forces holding back productivity growth. In our model, search frictions in the innovation market aim at broadly capturing hurdles in the production function of ideas, i.e. success in R&D requires both resources and time. Our paper provides a theoretical explanation behind the empirical observation that "ideas are getting harder to find."

The remainder of the paper is organized as follows. Section 2 illustrates the links between growth and finance in a simple accounting framework. Section 3 models these links in an equilibrium model with search-and-matching frictions in both innovation and financial markets. Section 4 extends our benchmark model along many dimensions. Finally, Section 5 concludes.

2 Growth: an accounting framework

To describe the impact on growth of the possible interplay finance and R&D, we start from a simple, firm-level, accounting framework.

Imagine two inputs are required, sequentially, to improve the productivity of a firm: financing and R&D. Suppose these inputs cannot be found instantly: it first takes τ_1 units of time to find a banker/financier, and then τ_2 units of time to innovate. When innovation occurs, it upgrades previous productivity by a factor γ . As a result, the average growth rate of productivity per unit of time is

$$g = \frac{\gamma}{1 + \tau_1 + \tau_2}.\tag{1}$$

Trivially, the more time it takes to find either factor, given the time to find the other factor, the slower growth.

Now what happens when the two waiting times are related—as they are, for instance, on a highway or river where upstream traffic impacts downstream activity? To model the

interdependence between τ_1 and τ_2 , write, without loss of generality, that

$$\tau_1 + \tau_2 = T(\tau_1) \tag{2}$$

where *T* is the total time required to find both factors.

Three scenarios are possible:

- If T' > 0 everywhere, financial liberalization (lower τ_1) always *raises* growth. This will occur either if τ_2 falls at the same time as τ_1 (general traffic improvement), or if τ_2 increases a bit, but not too much, when τ_1 falls. Upstream traffic improvement causes moderate downstream congestion.
- If T' < 0 everywhere, financial liberalization (lower τ_1) always *lowers* growth because τ_2 *rises* by more than τ_1 falls. Upstream traffic improvement causes so much downstream congestion that total travel time rises.
- If T is U-shaped with a minimum at τ_1^* , then the growth rate g is hump-shaped with a maximum at τ_1^* . Financial liberalization (a lower τ_1) only raises the growth rate if finance is very hard to find ($\tau_1 > \tau_1^*$) but lowers if when it is readily available $\tau_1 < \tau_1^*$. Fut differently, the maximum growth rate is reached when $T'(\tau_1) = 0$, that is, by the definition of T(.), when

$$\frac{d\tau_2}{d\tau_1} = -1. ag{3}$$

Then, at the margin, a reduction in τ_1 is met with an equal rise in τ_2 and the total time required to procure both inputs is unaffected by a change in τ_1 .

We investigate, in the next section, which of these scenarios prevails once we turn this firm-level accounting framework into an aggregative model. To that effect, we derive the times required to find each input, as well as their interaction, as the outcome of the profit-maximizing decisions of firms confronted with search costs and frictions.

⁵We do not spell out, for brevity, the opposite scenario in which *T* is hump-shaped.

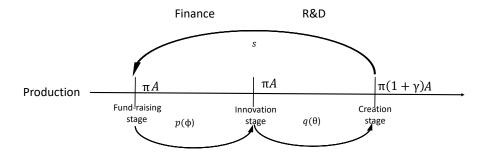


Figure 1: Markets and transitions through different stages.

3 Growth with credit and innovation frictions

We now turn the foregoing accounting framework into a *bona-fide* growth model by introducing search-and-matching frictions affecting the two inputs we assume are required for growth: credit and innovation. The matching probabilities on each market, and thus the average time and resources spent by profit-maximizing firms searching for each input, reflect endogenous market tensions and determine the equilibrium growth rate. In the spirit of the accounting framework presented above, we depict how these two equilibrium probabilities interact, as a function of search costs, matching functions and optimizing behavior. These interactions generate the non-trivial relation between finance and growth at the heart of this paper.

3.1 The life cycle of firms

Assume that firms go through four stages of life, depicted in Figure 1:

• *Stage 0*. First, a newly-created firm immediately produces flow output π (without the need for workers) and suffers a concomitant flow production cost we set to π for simplicity.⁶ It might be convenient to think of a firm as a robot or an automated produc-

⁶All costs and benefits below are to be understood as deflated by average aggregate productivity, whose endogenous growth rate will be determined below.

tion line.

- Stage 1. Second, a newly-created firm needs to find an intermediary (e.g., a bank or
 a venture capitalist) before it can look for an upgraded blueprint for their production
 line. We call p the instantaneous probability a firm meets a banker.
- *Stage 2*. Third, after the firm has met its banker, it looks for an innovator who knows how to upgrade the productivity of its robot by a factor $1 + \gamma$, so that output net of production cost is $\pi \gamma$ after the upgrade. We call q the probability that the firms finds an innovator.⁷
- *Stage 3*. Lastly, the upgraded firm is destroyed with an exogenous instantaneous separation probability *s*.

According to the timeline specified above, the evolution of the long-run values of a firm in the four different stages is described by:

$$(r-g)\hat{F}_0 = -c + p[\hat{F}_1 - \hat{F}_0]$$
 (4)

$$(r-g)\hat{F}_1 = q[\hat{F}_2 - \hat{F}_1]$$
 (5)

$$(r-g)\hat{F}_2 = \pi\gamma - \rho + s[\hat{F}_3 - \hat{F}_2]$$
(6)

$$\hat{F}_3 = \hat{F}_0. \tag{7}$$

where $\hat{F}_i \equiv \frac{F_i}{A}$ is the value of a firm deflated by average productivity and c and ρ denote flow search costs borne by firms in the fund-raising stage and the contracted repayment to the financier, respectively. The firm thus spends, in expectation, 1/p units of time looking for a bank, then 1/q units of time looking for an innovator, and finally 1/s units of time producing at the upgraded profit level until it is destroyed. The fraction of the firm's expected lifetime spent at high productivity is thus

$$\frac{1/s}{1/p + 1/q + 1/s} = \frac{1}{1 + s/p + s/q}.$$
 (8)

⁷Thus all meeting probabilities are computed from the perspective of the firm.

This fraction equals 1 if there is no destruction (s = 0), or if meeting a bank *and* innovator is instantaneous ($p = q = \infty$). As shown in the appendix, this fraction also measures the steady-state proportion of firms who have met an innovator. The next proposition links this ratio to the growth rate of productivity in this economy.

Proposition 1 Along a balance growth path, the growth rate of average productivity is the fraction of upgraded firms times the magnitude of the productivity jump γ stemming from each innovation, namely:

$$g = \frac{1}{1 + s/p + s/q}\gamma. \tag{9}$$

Proof: See the Appendix. •

If credit and innovation are found instantly (i.e., if $p=q=\infty$), the growth rate reaches the growth rate of innovation γ , as in the Solow (1956) model. We will refer to γ as the *potential growth rate*. If either credit or innovation is found with delay (p or q below infinity), the growth rate falls short of its potential γ . Obviously, the growth rate is zero and the economy stagnates if it is impossible to meet the bank required to find innovators (p=0) or the innovators themselves (q=0).

Assume that both finance and innovation are subject to search-and-matching frictions. Suppose the probability p that a firm meets a banker depends negatively on the credit market tension⁸ ϕ defined, *from the firm's standpoint*, as the ratio of the number of firms searching for banks to the number of banks searching for firms:

$$p = p(\phi), \qquad p'(\cdot) < 0. \tag{10}$$

with $p(0) = \infty$ and $p(\infty) = 0$. The reciprocal probability of a bank finding a firm, $\phi p(\phi)$, is increasing in credit market tension ϕ .

Assume, furthermore, that the probability a firm meets an innovator depends negatively on the innovation market tension θ defined, *again from the firm's standpoint*, as the ratio of

⁸Throughout the paper we will use the terms "tension" and "tightness" interchangeably.

⁹Those properties follow from assuming a constant-returns-to-scale matching function. See the appendix for all mathematical details.

the number of firms searching for innovators to the number of innovators looking for firms:

$$q = q(\theta), \qquad q'(\cdot) < 0, \tag{11}$$

with $q(0) = \infty$ and $q(\infty) = 0$. All else equal, the tighter the credit or innovation market, the fewer the firms with an upgraded productivity, and the smaller the aggregate average rate of growth of productivity. Nevertheless, as shown below, in equilibrium, tightness in the two markets interact with each others thereby creating a non-monotonic relation between finance and growth. We next turn to the equilibrium ϕ and θ to elucidate this point.

3.2 Equilibrium credit market tension under free entry

To close the model and determine the equilibrium credit tension we need to specify the financial contract between firms and financiers. We assume that in the fund-raising stage, liquidity-constrained firms can borrow from financiers the amount n (per unit of time) to cover their cost of searching for an innovator. The contract is then settled in the next-stage when the firm promises to repay the amount ρ as long as it operates. This implies that, along the balance growth path, the value of a bank follows

$$(r-g)\hat{B}_0 = -k + \phi p(\phi)[\hat{B}_1 - \hat{B}_0]$$
(12)

$$(r-g)\hat{B}_1 = -n + q(\theta)[\hat{B}_2 - \hat{B}_1]$$
 (13)

$$(r-g)\hat{B}_2 = \rho + s[\hat{B}_3 - \hat{B}_2],$$
 (14)

where everything is discounted by aggregate technology, i.e. $\hat{B}_i = \frac{B_i}{A}$ and we assume that $B_0 = B_3$, i.e. once a match is dissolved, banks and firms return to the fundraising stage. The stipulated loan repayment is decided by Nash bargaining according to the rule:

$$\rho = \arg\max(\hat{B}_1 - \hat{B}_0)^{(1-\omega)} (\hat{F}_1 - \hat{F}_0)^{\omega}, \tag{15}$$

where ω measures the firm's bargaining power. By combining the equilibrium loan repayment with the free-entry conditions for firms and financiers it pins down credit market tight-

ness.

Equilibrium credit tension depends on the attractiveness of entry into the market and thus, for bank and firm, on the balance of costs and benefits of operation. The costs flow from expensive and time-consuming searches incurred while seeking a match, while the benefits stem from the output upgrade afforded by the eventual match between firm and innovator. Namely, the total surplus value of a firm-bank match can be expressed as:

$$\frac{q(\theta)}{r - g + q(\theta)} \left(\frac{\pi \gamma}{r - g + s} - \frac{n}{q(\theta)} \right) := S[q(\theta), g; \gamma], \tag{16}$$

where r is the (subjective) interest rate of risk neutral agents. ¹⁰ The term in parenthesis on the left-hand side is the expected present discount value of the output upgrade enjoyed until the destruction of the firm, net of search of the cost of searching for an innovator, and measured at the time bank and firm meet. It is discounted by a factor q/(r-g+q), which measures the expected value, at the time of the meeting with the banker, of one unit of good at the random time of the meeting with an innovator. The higher $q(\theta)$, the shorter and thus cheaper the search for an innovator, and hence the higher the the expected discounted profits $(S_q > 0)$. Similarly, the faster the economy grows, or the larger the innovation, the larger the profits $(S_g > 0)$ and $(S_q > 0)$. Note that $(S_q(\theta), S_q)$ depends on $(S_$

Under Nash-bargaining between firm and bank, parties split the surplus of their match according to their exogenous bargaining weights $(\omega, 1 - \omega)$. If entry in the credit market is unfettered for both banks and firms, ¹¹ profits are driven to zero in equilibrium so that the

¹⁰We omit for simplicity from the arguments of the $S(\cdot)$ function variables for which we will not perform comparative statics experiments.

¹¹In order to fix the maximum scale of output without frictions, we keep the number of innovators constant, in the same way as the number of workers is kept fixed, for instance, in Mortensen and Pissarides (1994) or in Wasmer and Weil (2004).

costs each party incurs to find a match must equal its share of the surplus of the the match: 12

$$\frac{c}{p(\phi)} = \omega S[q(\theta), g; \gamma], \tag{17}$$

$$\frac{k}{\phi p(\phi)} = (1 - \omega)S[q(\theta), g; \gamma], \tag{18}$$

where *c* denotes the flow search cost incurred by the firm to search for a bank, and *k* the flow search cost of the bank.

These two free-entry condition immediately imply that, under Nash-bargaining, equilibrium credit market tension equals

$$\phi^* = \frac{\omega}{1 - \omega} \frac{k}{c},\tag{19}$$

so that the equilibrium probability a firm finds a bank is

$$p^* = p \left[\frac{\omega}{1 - \omega} \frac{k}{c} \right],\tag{20}$$

which defines a vertical line PP at $p = p^*$ in (p,g) space. The credit market is tight and the matching probability correspondingly low when firms drive a hard bargain with banks (ω high), if their flow search cost c is low, or the banks' search cost k is high — since all three factors attract more firms and/or fewer banks to the credit market.

Credit market tightness, ϕ^* , and thus p^* , is independent of θ and g. Note that we will introduce below fixed search costs for banks which break this simplicity and make equilibrium credit market tension dependent on the endogenous growth rate.

3.3 Equilibrium growth rate

Now that we have computed the equilibrium credit-matching probability, we need to characterize the equilibrium innovation-matching probability - since the growth rate depends on both p and q.

¹²If bargaining fails, banks and firms remain in their unmatched status with zero value (their outside option) because of free entry. As a result, the surplus is simply by the expected present discounted value of the output upgrade stemming from a match, given in expression (16).

To that effect, notice that the free-entry condition (17) can be inverted to yield a relationship between the innovation-matching probability q, and the credit-matching probability p and the growth rate g:¹³

$$q = \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi \gamma}{r-g+s} - \frac{c}{\omega p}} := Q(p,g). \tag{21}$$

The *spillover function* $Q(\cdot,\cdot)$ summarizes the crucial interaction between credit and innovation markets described in the accounting framework of section 2.¹⁴ For each credit matching probability and growth rate, it provides the innovation matching probability that is consistent with zero firm profits under free-entry. Unsurprisingly, $Q_p < 0$ and $Q_g < 0$: if the firm finds a bank faster or the growth rate rises, the expected profitability of the firms rises so that the probability of finding an innovator must fall concomitantly — else equilibrium profits would not be zero as free entry requires. For the same reason, lower credit flow search costs c or an increase in the magnitude γ of the innovation (which both raise the profitability of the firm) shift the spillover function Q(p,g) down, and thus lower q, for given p and g.

Using the information provided by the spillover function (21) into the definition of the growth rate (9), we obtain a relationship between the growth rate and the credit matching probability under free entry:

$$g = \frac{\gamma}{1 + s/p + s/Q(p, g)}.$$
(22)

This equation defines in (p, g) space a GG curve whose shape we will characterize shortly. Its intersection with the PP curve, which is vertical at $p = p^*$ as derived in equation (20), provides the equilibrium value g^* of the growth rate and thus also, using equation (21), the equilibrium innovation matching probability.

Before we proceed, it is useful to introduce two restrictions on parameters that guarantee the existence of the equilibrium and will be imposed throughout the paper:

Condition 1 $r > \gamma$,

Condition 2
$$\Psi \leq \omega \frac{\pi \gamma}{r - g^0 + s}$$
,

¹³We could as well use the other free-entry condition (18) as it is implied by the combination of equations (17) and (19).

¹⁴For simplicity, the dependence of *Q* on exogenous variables not shifted in the paper not spelled out.

where $g^0 \equiv \frac{\gamma}{\frac{S}{p}+1}$ is the growth rate that would prevail in an economy with frictionless innovation markets and $\Psi \equiv \frac{c}{n}$.

Proposition 2 *The equilibrium exists under the two parameter assumptions above.*

Proof: The first assumption guarantees, by equation (9), that r > g in equilibrium, so that all discounted sums in the paper are finite. The second assumption ensures, by equation (21), that Q is positive since it implies that $\omega((\pi\gamma)/(r-g^0+s)) < \omega((\pi\gamma)/(r-g+s))$. The condition states that at, the lowest possible value of tightness in the innovation market ($\theta = 0$), a firm can enter and make more profits than the expected vacancy costs of entering in the financial market (Ψ).

Implicitly, the second condition defines a minimum p for which the equilibrium exists, we denote this as p_{min} . To characterize the shape of the GG curve, it is useful to establish what happens for extreme values of p:

Lemma 1 The GG curve goes through the origin. It has an horizontal asymptote at $g_{\infty} > 0$, with $0 < g_{\infty} < \gamma$ when $p \to \infty$.

When p=0, it is impossible to meet a bank and there is no growth since a firm cannot by assumption finance the search for innovators on its own. When the match with a bank occurs instantly $(p=\infty)$, the difficulty of finding an innovator is the only brake to growth so that, from the free entry condition (21), $q=n(r-g+s)/(\pi\gamma)$ while, from (22), $g=\gamma/(1+s/q)<\gamma$. The asymptotical growth rate g_{∞} is the positive root of the quadratic equation obtained by combining these two conditions, and the (positive) limit value of q follows immediately. The following proposition enables us to gauge when the maximum growth rate is achieved:

Proposition 3 (Maximum growth rate) Let $\mu = -Q_p p/p > 0$ denote the elasticity of q with respect to p along the spillover function. Then g is maximum when the ratio of the expected time spent looking for a bank over the the expected time spent looking for a firm equals to μ , i.e., $(1/p)/(1/q) = \mu$.

Proof: From equation (14), and the fact that $Q_g < 0$, g is maximal when 1/p + 1/Q(p,..)

is minimal. Since there is, because of free-entry in the credit market, a negative spillover

between congestion in credit and innovation markets $(Q_p < 0)$, a one-percent increase in

1/p is associated with a one-percent fall in 1/q. These two conflicting effects balance out

in levels, and the growth rate reaches a maximum, when the condition of the proposition is

satisfied. It can be verified that this condition characterizes a global maximum.

Proposition 3 is consistent with two possibilities. Either the GG curve is rising monoton-

ically from 0 to g^{∞} — in which case the maximum growth rate described by the proposition

is reached at the asymptote g^{∞} when $p = \infty$, i.e., when credit matching is instantaneous.

Or the GG curve is hump-shaped, first rising with p, then reaching above g^{∞} the maximum

described by 3, and finally declining towards the horizontal asymptote at g^{∞} .

In the benchmark symmetrical case when the flow cost of searching for a bank, c, equals

the flow cost ωn of searching for an innovator borne by the firm it is straightforward to es-

tablish that the GG curve is hump-shaped, i.e. the relation between finance and growth is

indeed non-monotonic as shown in the following proposition:¹⁵

Proposition 4 (hump-shaped GG curve) Suppose $c = \omega n$. Then the GG curve is hump-shaped.

The growth rate is maximal and the total expected search time is minimal when expected

credit and innovation search times are equal 1/p = 1/q.

Proof: See the Appendix.•

The proof, provided in the Appendix, exploits the fact that, if the GG curve has a hump,

the growth rate, which depends negatively on the *total* expected search time 1/p+1/q, must

be insensitive to a first order to a change in p. For that to be the case, an infinitesimal in-

crease (decrease) in the expected credit search time 1/q must be met by an exactly offsetting

¹⁵The remainder $(1-\omega)n$ of the cost of finding an innovator is borne, after Nash-bargaining, by the bank

the firm has met.

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decrease (increase) in the expected innovation search time that leaves, therefore, 1/p + 1/q constant. In the symmetric case $c = \omega n$, this occurs when 1/p = 1/q, i.e., when credit and innovation expected search times are equal. This equality determines, through the spillover function (21), the point on the GG curve at which the growth rate is maximal and total expected search time is minimal.

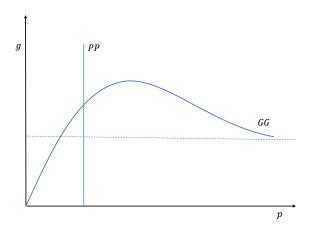


Figure 2: Hump-shaped GG curve

To understand the intuition behind this result, illustrated in Figure 2, think of the problem of driving from the Italian mainland to Catania in Sicily. This involves confronting congestion twice: first to cross the straight of Messina (currently by ferry) and second on Sicilian roads to Catania. Suppose the cost of time waiting for a ferry is the same as the cost of time driving on congested Sicilian roads (this hypothesis is analogous to our symmetrical cost assumption). Will an increase in the number of ferries or the construction of bridge across the straight reduce total travel time to Catania? It all depends on relative congestion. If the main bottleneck is on the mainland it will. If it is on Sicilian roads, it won't. Total expected travel time will be minimized when expected time spent on the continent and on the island are equalized.

The GG curve is hump-shaped because a rise in p has two conflicting effects on growth: on the one hand, by making financing easier, it increases the proportion of firms that can search for innovators — which contributes to raising the growth rate. On the other hand,

as evidenced by the negative derivative Q_p of the spillover function, a rise in p is associated with a fall in q that contributes to lowering the growth rate — because easier credit attracts more firms relative to banks, thereby raising the innovation market tension and lowering p up to the point where zero profit are reestablished. Which of these two effect dominates depend on the magnitude of p. When it is close to zero, the first effect dominates. When p is large, the second effect dominates. This simple result can be generalized to the case when $c \neq \omega n$, as shown in the Appendix.

A simple approximation. To clarify the role played by the search costs in the two markets and better grasp the gist of these results, some back-of-the envelope calculations are useful. For values of interest rates close to the growth rate of the economy ($r \approx g$), the free entry condition becomes, approximately,

$$1/q + 1/p \approx \frac{\pi \gamma}{sn} + (1 - \frac{c}{\omega n})(1/p).$$

The left-hand side of this expression is the expected total waiting time in the two markets, a quantity that has a direct impact on growth. The equilibrium growth rate is then, approximately,

$$g \approx \frac{\gamma}{1 + s(1/q + 1/p)} = \frac{\gamma}{1 + s(\frac{\pi \gamma}{sn} + (1 - \frac{c}{\omega n})\frac{1}{n})} := g(p).$$

The two expressions above make clear that the link between finance, i.e. $\frac{1}{p}$, and growth g is directly related to the relative magnitude of search costs in the two markets via its impact on total waiting time. Specifically, $g_p > 0$ if and only if $c/\omega < n$. Before we proceed, it is useful to note that the non-negativity requirement on q imposes that

$$p \ge \frac{c/\omega}{\pi \gamma/s} := p_{min},$$

so that the equilibrium growth rate evaluated at the minimum p_{min} is

$$g(p_{min}) = \gamma/(1 + \pi \gamma \omega/c)$$

while, for $p \to \infty$,

$$g(\infty) = \gamma/(1 + \pi \gamma/n)$$
.

We can then distinguish between three cases:

- 1. If $c/\omega < n$, the growth rate is increasing in p. Its minimum value is $g(p_{min})$, and its maximum value is $g(\infty)$.
- 2. If $c/\omega > n$, g is decreasing in p. Its minimum value is $g(\infty)$, and its maximum value is $g(p_{min})$.
- 3. If $c/\omega = n$, g is independent of p, and it is equal to the following expression

$$g(p_{min}) = \gamma/(1 + \frac{\pi \gamma}{c/\omega}) = \gamma/(1 + \frac{\pi \gamma}{n}) = g(\infty).$$

The intuition behind these results is the following. Increasing the probability of meeting a financier always leads to a fall in q because of the free-entry condition: if a firm that makes zero profit spends less time looking for credit, it must be spending more time looking for an innovation. When the cost of looking for credit, corrected by the firm's bargaining power, is lower than the cost of looking for an innovation (case 1), the profit of the firms would fall if the total search time were to stay the same or, a fortiori, increase. But that's inconsistent with zero profits and some firms will exit the market. As a result, the total search time falls and growth must rise. The same explanation holds, $mutatis\ mutandis$, in case 2 and explains why, in case 3, the total search time remains constant when p varies. In terms of our road traffic analogy, suppose that bridge tolls are less expensive than road tolls on Sicily (case 1). Reducing traffic jams on the first (cheaper) bridge intensifies traffic jams on the second (expensive) road, thereby increasing the total cost of traveling. In the long-run, higher expenses will push some trucks off the road and eventually ease traffic congestion on the whole route.

3.3.1 Comparative statics

Let us look at some qualitative comparative statics. In the following we will evaluate the equilibrium effects of lower search costs for financiers, of higher search costs for firms and of an increase in the size of the productivity jump.

More finance. A lower search cost *k* for banks reduces equilibrium credit market tension ϕ by encouraging bank entry. This raises the equilibrium credit matching probability p^* , thereby shifting the PP curve to the right with an unchanged GG curve. The effect on the equilibrium growth rate is positive, left of the hump of the GG curve, if p is initially small and the increase in k is small enough. Right of the hump, however, if p is initially small, a rise in *p* lowers the growth rate.

Higher search costs for firms, c, have too a benign effect on equilibrium credit market tension. However in this case, the probability of a match in the innovation market increases. In equilibrium, higher costs discourage firms entry in both markets and innovation tension heta decreases . 16 Graphically, the GG curve shifts upward and the PP curve to the right. The final effect on growth is positive.

Larger innovation. What happens if the size of the productivity jump γ stemming from innovation increases? This increase has two counteracting effects: a direct positive effect on g and an indirect negative effect due to the tightening in the innovation market. Higher benefits from the output upgrade encourage entry in the financial markets by both firms and banks—thereby leaving the equilibrium financial market tightness unchanged. At the same time, more firms will find it profitable to search for innovators—thereby creating congestion in that market and impinging on growth. Graphically, the GG curve could shift either upward or downward, depending on parameters values. ¹⁷ Thus, the final effect on the equilibrium growth rate is ambiguous.

3.3.2 A simple numerical exercise

The purpose of this subsection is to evaluate the equilibrium properties of our model with a simple calibration exercise. Table 1 summarizes our parametrization.

Our calibration is based on annual data. The risk-free rate, r, is 3.5% and the separation rate, s, is set to 4%. We assume a symmetric bargaining power for banks and firms, $\omega = .5$.

¹⁶Here we assumed that $p=p_0\phi^{-\eta}$, so that $\frac{c}{p}=\frac{c^{1-\eta}}{p}\left(\frac{\omega k}{1-\omega}\right)^{\eta}$, and hence q and p are increasing in c. ¹⁷For the calibration outlined in the subsection below, an increase in the productivity jump has positive

effect on equilibrium growth.

Table 1: Parametrization

Parameter	Value	Parameter	Value
r	3.5%	$1/\phi$	0.06
S	4%	γ	0.023
ω	0.5	c	0.166
π	2.75	n	0.331
		k	2.758

The target duration in credit markets (for creditors) and innovation markets (for firms) are, respectively, slightly below 1 months and 2 years. The first number together with our target for credit market tightness, ϕ , implies a duration in credit markets for firms slightly above 1 year, as in Wasmer and Weil (2004). The second number is in line with the average time for patent approvals for 2020 according to data published by the USPTO. The productivity jump, γ , is set to target an annual growth rate, g, of 2%. For simplicity, we assume that the flow costs of searching for banks, c equals the flow cost of searching for innovators borne by the firms, ωn , while k is set to target a fraction of employed in the financial sector over total employment of 6% according to the Bureau of Labor Statistics data for 2020. The value for π is chosen to normalize total discounted output net of production costs to 1. Finally, tightness in innovation market, θ , is set to match the fraction of employed in scientific research and development services over total employed of 0.5% in 2020 (BLS)

Table 2 reports the equilibrium growth rate expected waiting time in the innovation market that are predicted by our model in four different cases: our current calibration, low credit market frictions, low innovation market frictions and low frictions in both markets. To decrease the level of credit market frictions we let $p_0 \to \infty$ while to reduce search time in the innovation markets, we double the search costs for innovators, n.¹⁸

Before commenting on the results of Table 2, it is good to recall that given our chosen calibration ($c = \omega n$), as shown in section 3.3, the maximum growth rate would be achieved for p = q. In our case, with p > q, the economy is on the right of the hump in a rather flat region. Moreover, as exemplified by our back of the envelope calculations at the end of the same section, when the real rate is close to the growth rate of the economy, the latter is

¹⁸An increase in the search costs for innovators reduce a firm profitability. To keep profit at zero, the probability of meeting innovators must increase (equation 21).

Table 2: Lower frictions in credit and innovation markets

	Benchmark	Low credit	Low innovation	Low frictions
		frictions	frictions	in both markets
g	2.000%	1.997%	2.071%	2.122%
$\frac{1}{q}$	2 yr	3.4 yr	1.03 yr	1.75 yr

relatively insensitive to access to credit. The main take away from this simple exercise is that, for a calibration close to US data, only a combined reduction of frictions in both markets would result in a higher growth rate, as illustrated by the last column in the table. Conversely, changes in credit markets have only a marginal (negative) effect on growth as they mainly exacerbate bottlenecks in the innovation market. Lower frictions in the innovation market have also (positive) marginal effects on growth. Our results thus substantiate the OECD view that innovation is the outcome of a "system" favorable to growth rather than the result of isolated pro-growth measures. ¹⁹ All in all, the results above show that for a calibration chosen to mimic the current US economy, financial factors play only a moderate role for growth. In light of what shown in the previous section, this should perhaps not come as a surprise when the real and growth rate are close to each other and search costs in both markets are symmetric. More generally, potential growth, as captured by the productivity jump γ plays an important role. Specifically, is it possible to show²⁰ that the elasticity of the growth rate with respect to finance is a multiple of the factor $\frac{\gamma-g}{\gamma}$. That is, as long as potential growth is close to the actual growth rate, growth is relatively insensitive to changes in financial factors.

4 Extensions

In this section, we evaluate the robustness of our findings to changes in some model assumptions. First, we modify the original model to allow for a direct feedback, absent heretofore, from growth onto credit market tension. To that effect, we assume that entering in

¹⁹For example, the OECD's 2015 innovation strategy argues that policy makers can promote innovations by focusing on five areas of action among which education and training systems as well as a business environment supporting investment in knowledge based capital.

²⁰These results are available upon request.

financial markets entails a fixed licensing cost for banks. Second, we introduce a remuneration for innovators and show under which conditions frictions in the financial and innovation markets might negatively interact with each other and hamper growth. Finally, we show that the relationship between finance and growth is monotonic if the market for ideas is frictionless.

Fixed entry cost for banks 4.1

Assume that a bank incurs a fixed entry cost *K*, say a licensing cost, at the instant it enters the credit market to offer its services to firms. The annuity value of that cost in a growing economy is (r-g)K, and it adds up to the flow search cost k paid by the bank. Since the growth rate is endogenous in our model, the effect of the fixed entry cost *K* depends on its impact on the equilibrium growth rate—an effect absent when growth is exogenous. This introduces into our model a direct feedback from growth into finance that we now investigate.

Assume, in addition, that the fixed market-entry cost K is paid by the bank each and every time it starts searching for a firm—i.e. that, upon exogenous separation from the firm or in case of negotiation failure, the firm reverts to its zero pre-entry value.²¹ As a result, the fixed cost K does not affect the surplus of the bank-firm match, and the free entry conditions (17) and (18) become

$$\frac{c}{p(\phi)} = \omega S[q(\theta), g; \gamma], \tag{23}$$

$$\frac{c}{p(\phi)} = \omega S[q(\theta), g; \gamma], \tag{23}$$

$$\frac{k + (r - g)K}{\phi p(\phi)} = (1 - \omega)S[q(\theta), g; \gamma]. \tag{24}$$

Retracing the steps taken in section 3, tightness in credit market is given by

$$\phi = \frac{\omega}{1 - \omega} \frac{k + (r - g)K}{c}.$$
 (25)

The novel term (r - g)K on the right hand-side of the equation captures the annuity value,

²¹This approach is similar to that of Pissarides (2009) in a labor setting where fixed training costs are incurred each time a match occurs.

in an economy growing at the endogenous rate g, of the fixed entry cost K. The faster the economy grows, the smaller the fixed cost looms in the bank's cost computations. The equilibrium impact of the fixed cost thus depends on whether it slows down or raises growth. In the first case, lower growth amplifies the impact of barriers to credit entry. In the latter, it mitigates them. Interestingly, there is now a direct positive effect of growth on financial deepening. Higher growth reduces the annuity value of the licensing cost, thereby inducing entry of new banks and reducing the tightness of credit markets.

4.1.1 Equilibrium

The equilibrium credit matching probability obeys

$$p = p \left[\frac{\omega}{1 - \omega} \frac{k + (r - g)K}{c} \right]. \tag{26}$$

Since $p'(\cdot) < 0$, and provided r > g (a condition always satisfied in equilibrium), this equation defines, given K, an upward sloping PP curve in (p,g) space—as shown in Figure 3.²² When K = 0, the PP curve is vertical. Raising K shifts the PP curve to the left and flattens it. Furthermore, since the surplus S does not depend on K, neither does the spillover function, i.e. $q = Q(p,g;c,\gamma)$ — so that the equation and position of the GG curve defined in (22) are unaffected by the introduction of the fixed entry cost K.

These results are represented in Figure 3. ²³

As in our benchmark model, a policy intervention which lowers costs for financial intermediaries stimulates entry and decreases credit market tightness. At the margin, lower licensing costs have a stronger impact on market tightness the lower the initial growth rate and for higher level of interest rates. As before, the interaction between credit and innovation frictions makes the policy have a negative effect on congestion in the innovation market, and the equilibrium effect on growth is ambiguous. Graphically, a reduction in search costs shifts the PP curve to the right while lowering licensing costs steepens the curve and

²²When $p \to \infty$, the PP curve has a horizontal asymptote at $(k + rK)/K := g_p^{\infty}$. The minimal value of p occurs when g = 0.

²³The feedback from growth to credit tightness, captured by the new PP curve could potentially result in multiplicity of equilibria. This possibility is ruled out under standard assumptions on the matching probabilities. The proof is available in the online appendix.

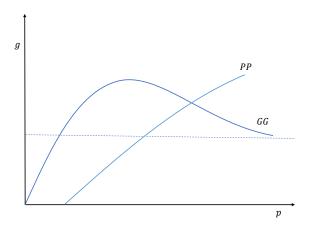


Figure 3: Equilibrium growth and credit matching probability (K > 0) moves it to the right.

4.2 Innovators share the surplus

We have assumed so far, for ease of exposition, that innovators are not compensated for their work. 24 In this section, we depart from that simplifying assumption. Suppose that, after a successful match, innovators receive a wage w that is *negotiated through Nash bargaining*. This implies that the value of innovator along the balance growth path evolves according to:

$$(r-g)\hat{I}_1 = \theta q(\theta) [\hat{I}_2 - \hat{I}_1]$$
(27)

$$(r-g)\hat{I}_2 = w + s[\hat{I}_3 - \hat{I}_2]$$
 (28)

$$\hat{I}_3 = \hat{I}_1. \tag{29}$$

where, as before, all values are discounted is discounted by average productivity, i.e., A. Assume that bargaining in the financial and innovation markets occurs independently. Specifically, firms and banks do not take into account the effect of credit repayment ρ on wages.²⁵ In the innovation market, the newly formed bank-firm pair bargains with a innovator. If we

²⁴Our main results can be easily generalized to a set-up where innovators receive an exogenously given compensation.

 $^{^{25}}$ This assumption keeps the solution for credit repayment unchanged from what we found in the previous sections.

define the joint bank-firm value \hat{J}_i as:

$$\hat{J}_i = \hat{B}_i + \hat{F}_i$$

the wage negotiated under Nash bargaining for the innovator solves the following maximization problem:

$$w = \operatorname{arg\,max} (\hat{I}_2 - \hat{I}_1)^{1-\alpha} (\hat{I}_2 - \hat{I}_1)^{\alpha},$$

in which $\alpha \in (0,1)$ denotes the bargaining weight of innovators. As shown in the appendix, the solution of this problem is a standard surplus-sharing rule:

$$w = \alpha \left[\pi \gamma + \theta \, n - \left((1 - \theta)(r - g) + s \right) K(\phi) \right],\tag{30}$$

where $K(\phi) \equiv \frac{c}{p} + \frac{k}{p\phi}$ measures total discounted search costs in the financial market. The equilibrium wage is increasing in innovators productivity $(\pi\gamma)$, the innovation market tightness (θn) and the growth rate, while decreasing in total search costs in financial markets. All else equal, less frictions in credit markets (a higher p_0), have a positive effect on innovators' compensation. Trivially, if innovators had no bargaining power, $\alpha = 0$, Eq. 30 encompasses the case w = 0, considered so far.

Proceeding as in Section 3.3 and taking into account the wage rule above, we can express the new spillover function as:

$$Q(p,g) = \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi \gamma - w(p,g)}{r-g+s} - \frac{c}{\omega p}}.$$
(31)

As in our original model, the spillover function summarizes the interaction between the growth rate and frictions in credit and the innovation markets, but it becomes more complex in its expression. A full analytical characterization of the equilibrium properties of this extended model is beyond the scope of this paper, but we can outiline some properties of Eq.31. Specifically, it remains true, that the growth rate affects negatively the spillover function, i.e. $Q_g < 0$. At the same time, finance has a layered role. With endogenous wages, if a firm finds a financier faster, both its expected revenues and its labor costs rise. For α suf-

ficiently small, the first effect dominates, and the probability of finding an innovator must fall to satisfy the free-entry condition. If this is case, finance can create bottlenecks in the innovation market as in our the benchmark model, i.e. $Q_p < 0$. In the Appendix, we show that indeed this is always the case for $\alpha < 1$.

4.3 Search frictions only in one market

In the foregoing analysis, the interaction between search frictions in financial and innovation markets generates a non-monotonic relationship between growth and finance. We now show that this non-monotonicity vanishes when there is only one friction.

Specifically, assume that a firm only needs a financial intermediary to find an upgraded blueprint to boost its productivity. Finding a financiers requires effort and time but, once the firm has met a bank, an innovator is found costlessly and instantly. As shown in the appendix, the equilibrium of this simple model can be summarized by two equations

$$BB : \frac{k}{\phi p(\phi)} = \frac{(1-\omega)}{(r-g+s)} \left[\pi \gamma + c - \frac{\omega}{1-\omega} \frac{k}{\phi} \right],$$

$$GG : g = \frac{\gamma}{\frac{s}{p(\phi)} + 1}.$$

The first equation represents a free-entry condition in the financial sector, the second captures the average growth rate of the economy when firms meet innovators instantaneously after having secured financing. Both curves are downward sloping in the (ϕ, g) plane, as shown in figure 4 26 .

In this one-friction set-up, a more efficient matching between firms and financiers modeled as an increase in p_0 makes the first curve shifts to the left, i.e. for given g the discounted search cost for banks decreases thereby inducing more banks to enter and thus reducing credit tightness. The GG curve moves upward, since finance has a positive direct effect on the share of innovating firms. An improvement in credit markets therefore results unequivocally into higher growth and less tightness in financial markets. Finance is always good for growth because it is the only hindrance to innovation. This is the traditional mechanism

²⁶The existence of an equilibrium is ensured by the condition $r > \gamma$.

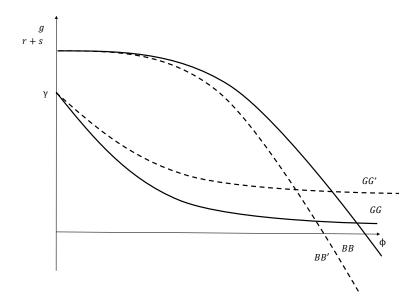


Figure 4: More efficient credit markets when innovation markets are frictionless underlying simplistic policy recommendations for financial liberalization.²⁷

5 Conclusions

In the last century, most developed economies have experienced a widespread expansion of the financial sector yet almost constant growth rates of GDP (save for wars and financial or health crises). In this paper, we build a parsimonious endogenous growth model with search frictions in credit and innovation markets to shed more light on this empirical observation. In our model, higher growth induces firms and banks to enter in the market. All else equal, tight credit and innovation markets have a negative effect on growth. However, we show that once all the matching feedback effects are taken into account, financial deepening beyond a certain threshold is harmful for growth. Finance has a non-monotonic effect on long-run growth since there are other bottlenecks than money hindering innovation, i.e. a too big financial sector can create congestion in the innovation markets. For a calibration chosen to mimic the actual US economy, with relatively well functioning credit and innovation markets, the effects of finance on growth are indeed negative but only marginal, in line with empirical evidence. Our parsimonious theory of growth builds on a representative

²⁷Similarly, if only innovation markets were frictional while access to credit were free, more developed R&D markets would always be beneficial for growth. This last case is shown in the Appendix of the working paper version of this article.

agent framework. In truth, firms, innovators or financial intermediaries exhibit significant heterogeneity which could enhance or diminish the effect of finance on growth. For example, young firms could be more exposed to financial frictions relative to larger firms, while for the latter innovation could be more cumbersome. Furthermore, in a set-up with various level of innovation quality, more finance and congestion in R&D could tilt the balance between radical versus incremental innovation. Finally, some type financial intermediaries (e.g., venture capitalists) could be more be more effective in influencing the outcomes of R&D; this channel could emphasize the impact of finance-type on growth.²⁸ We leave the exploration of these important extensions for future research.

All in all, our results corroborate the view that growth-stimulating policies should be designed as a system taking into account different headwinds slowing down technological progress.

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Appendix A The model

This section describes in more detail our model.

A.1 Matching functions and probabilities

We assume the following functional forms:

$$\begin{split} q(\theta) &\equiv \frac{\mu(\mathcal{F}_1, \mathcal{I}_1)}{\mathcal{F}_1} = \mu\left(1, \theta^{-1}\right) = q_0 \theta^{-\varepsilon} \\ p\left(\phi\right) &\equiv \frac{m\left(\mathcal{F}_0, \mathcal{B}_0\right)}{\mathcal{F}_0} = m\left(1, \theta^{-1}\right) = p_0 \phi^{-\eta} \end{split}$$

where $\mu(\cdot)$ and $m(\cdot)$ are constant return to scale technologies producing matches in the innovation and the credit markets, respectively.

A.2 Value of a firm

The values of a firm along the balanced path in the four stages are described by

$$rF_{0} = \pi A - \pi A - cA + p(\phi)[F_{1} - F_{0}] + \dot{F}_{0}$$

$$rF_{1} = \pi A - \pi A + q(\theta)[F_{2} - F_{1}] + \dot{F}_{1}$$

$$rF_{2} = \pi (1 + \gamma) A - \pi A - \rho A + s[F_{3} - F_{2}] + \dot{F}_{2}$$

$$F_{3} = F_{0}$$

In the fund-raising stage, firms produce πA , pay a flow cost cA to search for banks and production costs πA . Once the match is created, in stage 1, they will still produce πA (and sustain production costs πA) and with probability $q(\theta)$, they will meet an innovator and have access to a better technology, where $\theta = \frac{\mathscr{F}_1}{\mathscr{F}_1}$, i.e. the ratio between entrepreneurs and available innovators, represents tightness in the market of idea and $q'(\theta) < 0$. In the next stage, 2, they will produce $\pi (1 + \gamma) A$ and repay the contracted amount ρA to the bank, and sustain costs πA . Matches are exogenously destroyed with probability s at the end of that

stage. By dividing both left and right-hand sides by A, we can rewrite the equations as:

$$r\hat{F}_{0} = -c + p(\phi)[\hat{F}_{1} - \hat{F}_{0}] + \frac{\dot{F}_{0}}{A}$$

$$r\hat{F}_{1} = q(\theta)[\hat{F}_{2} - \hat{F}_{1}] + \frac{\dot{F}_{1}}{A}$$

$$r\hat{F}_{2} = \pi\gamma - \rho + s[\hat{F}_{3} - \hat{F}_{2}] + \frac{\dot{F}_{2}}{A}$$

$$\hat{F}_{3} = \hat{F}_{0}.$$

where $\hat{F}_i \equiv \frac{F_i}{A}$. Finally recalling that $\frac{\dot{F}_i}{A} = \dot{F}_i + g\hat{F}_i$ and that along the BGP $\dot{F}_i = 0$, we can rewrite the equations above as they appear in the main text:

$$(r-g) \hat{F}_{0} = -c + p(\phi) [\hat{F}_{1} - \hat{F}_{0}]$$

$$(r-g) \hat{F}_{1} = q(\theta) [\hat{F}_{2} - \hat{F}_{1}]$$

$$(r-g) \hat{F}_{2} = \pi \gamma - \rho + s [\hat{F}_{3} - \hat{F}_{2}]$$

$$\hat{F}_{3} = \hat{F}_{0}.$$

With free entry $\hat{F}_0 = 0$, so that:

$$\hat{F}_{1} = \frac{c}{p(\phi)}$$

$$\hat{F}_{1} = \frac{q(\theta)}{r - g + q(\theta)} \frac{\pi \gamma - \rho}{r - g + s}$$

$$\frac{c}{p(\phi)} = \frac{q(\theta)}{r - g + q(\theta)} \frac{\pi \gamma - \rho}{r - g + s}.$$

A.3 Value of a bank

Similarly, we can express the Bellman equations describing the evolution of the bank values along the balanced growth path as:

$$(r-g)\hat{B}_0 = -k + \phi p(\phi)[\hat{B}_1 - \hat{B}_0]$$

 $(r-g)\hat{B}_1 = -n + q(\theta)[\hat{B}_2 - \hat{B}_1]$
 $(r-g)\hat{B}_2 = \rho + s[\hat{B}_3 - \hat{B}_2]$

where k and n are search costs in the financial and innovation markets, respectively.

With free-entry:

$$\hat{B}_{1} = \frac{k}{\phi p(\phi)}$$

$$\hat{B}_{1} = -\frac{n}{r - g + q(\theta)} + \frac{q(\theta) \rho}{(r - g + q(\theta))(r - g + s)}$$

$$\frac{k}{\phi p(\phi)} = -\frac{n}{r - g + q(\theta)} + \frac{q(\theta) \rho}{(r - g + q(\theta))(r - g + s)}.$$

If we instead assume that banks need to pay a fix cost K upon entry, $\hat{B}_0 = \hat{B}_3 = K$,

$$\hat{B}_{1} = \frac{(r-g)K + k}{\phi p(\phi)}$$

$$\hat{B}_{2} = \frac{\rho + sK}{r - g + s}$$

$$\hat{B}_{1} = \frac{-n + q(\theta)\hat{B}_{2}}{r - g + q(\theta)}$$

$$= \frac{-n + q(\theta)\hat{B}_{2}}{r - g + q(\theta)} + \frac{q(\theta)}{r - g + q(\theta)} \frac{\rho + sK}{r - g + s},$$

by equating forward and backward Bellman equations, we get:

$$\frac{\left(r-g\right)K+k}{\phi p\left(\phi\right)} = \frac{-n}{r-g+q\left(\theta\right)} + \frac{q\left(\theta\right)}{r-g+q\left(\theta\right)} \frac{\rho}{r-g+s}.$$

A.4 Value of an innovator

The Bellman equations describing the steady-state values of the innovator over the four stages are

$$(r-g)\hat{I}_1 = \theta q(\theta) [\hat{I}_2 - \hat{I}_1]$$
$$(r-g)\hat{I}_2 = s[\hat{I}_3 - \hat{I}_2]$$
$$\hat{I}_3 = \hat{I}_1$$

There is no free-entry in this market and the total number of inventors is normalized to one

$$\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2 = 1$$
.

Then, the dynamics of the flow of inventors searching for entrepreneurs can be expressed as

$$\mathcal{I}_1 = s(1 - \mathcal{I}_1) - \theta q(\theta) \mathcal{I}_1.$$

Abstracting from the very short-run $\mathcal{I}_1 = 0$:

$$\mathscr{I}_1 = \frac{s}{s + \theta \, q(\theta)}.$$

A.5 Bargaining

No entry-costs

Banks and firms share the surplus

$$S = \frac{q(\theta)}{r - g + q(\theta)} \frac{\pi \gamma}{r - g + s} - \frac{n}{r - g + q(\theta)}$$

according to the rule:

$$\rho = \arg\max(\hat{B}_1 - \hat{B}_0)^{(1-\omega)} (\hat{F}_1 - \hat{F}_0)^{\omega}.$$

Then we have

$$(1-\omega)\hat{F}_{1} = \omega\hat{B}_{1}$$

$$(1-\omega)\left[\frac{q(\theta)}{r-g+q(\theta)}\frac{\pi\gamma-\rho}{r-g+s}\right] = \omega\left(\frac{q(\theta)\rho}{\left(r-g+q(\theta)\right)\left(r-g+s\right)} - \frac{n}{r-g+q(\theta)}\right)$$

$$\rho = (1-\omega)\pi\gamma + \omega\frac{n\left(r-g+s\right)}{q(\theta)}.$$

Furthermore, using the backward definition of \hat{F}_1 and \hat{B}_1 , it follows that

$$(1 - \omega) \frac{c}{p(\phi)} = \omega \frac{k}{\phi p(\phi)}$$
$$\phi = \frac{\omega}{1 - \omega} \frac{k}{c}.$$

Entry costs

When there are fixed entry costs in the banking sector, we assume that they do not affect the outside option of the bank

$$\rho = \operatorname{argmax}(\hat{B}_1)^{(1-\omega)}(\hat{F}_1 - \hat{F}_0)^{\omega}.$$

As a result, the surplus is unaffected:

$$S = \frac{q(\theta)}{r - g + q(\theta)} \frac{\pi \gamma}{r - g + s} - \frac{n}{r - g + q(\theta)}.$$

The equilibrium tightness is determined using the backward definition of \hat{F}_1 and \hat{B}_1 :

$$\phi = \frac{\omega}{1 - \omega} \frac{(r - g)K + k}{c}.$$

A.6 Flow equations and technology evolution

The number of entrepreneurs in the four stages evolves according to the following equations:

$$\mathcal{F}_{2} = -s\mathcal{F}_{2} + q(\theta)\mathcal{F}_{1}$$

$$\mathcal{F}_{1} = -q(\theta)\mathcal{F}_{1} + p(\phi)\mathcal{F}_{0}$$

$$\mathcal{F}_{0} = s\mathcal{F}_{2} - p(\phi)\mathcal{F}_{0}.$$

Thus, abstracting from the very short run $\mathcal{F}_i = 0$, we have

$$\mathcal{F}_1 = \frac{s}{q(\theta)} \mathcal{F}_2$$
$$\mathcal{F}_0 = \frac{s}{p(\phi)} \mathcal{F}_2.$$

Since

$$(\mathscr{F}_1 + \mathscr{F}_0 + \mathscr{F}_2) = \mathscr{F},$$

it follows that

$$\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)\mathcal{F}_2 = \mathcal{F}$$

and thus that

$$\frac{\mathcal{F}_2}{\mathcal{F}} = \frac{1}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.$$

Proof of Proposition 1: Equilibrium growth rate

Proof: The law of motion of average technology in discrete time would be

$$\begin{split} A_{t+1} &= \frac{(\mathcal{F}_1 + \mathcal{F}_0)}{\mathcal{F}} A_t + \frac{\mathcal{F}_2}{\mathcal{F}} A_t \left(1 + \gamma \right) \\ \frac{A_{t+\Delta} - A_t}{\Delta} &= \frac{\left(\mathcal{F}_1 + \mathcal{F}_0 + \mathcal{F}_2 \left(1 + \gamma \right) - \mathcal{F} \right)}{\mathcal{F}} A_t \\ \frac{A_{t+\Delta} - A_t}{\Delta} &= \frac{\mathcal{F}_2}{\mathcal{F}} \gamma A_t, \end{split}$$

so that, in continuous time, letting $\Delta \rightarrow 0$, we have:

$$g \equiv \frac{\dot{A}}{A} = \frac{\gamma}{\left(\frac{s}{q(\theta)} + \frac{s}{p(\phi)} + 1\right)}.$$

•

Appendix B Growth and finance

As a preliminary to the proof of the next two propositions, it is useful to define P = 1/p and Q = 1/q. From the definition of the growth rate, we have

$$P + Q = (\gamma/g - 1)/s := M(g).$$
 (B.1)

If GG has a hump, it must be that (locally) dP = -dQ so that dg = 0.

From the free-entry condition:

$$cP = \frac{\omega}{1 + Q(r - g)} \left[\frac{\pi \gamma}{r - g + s} - nQ \right]$$
 (B.2)

or, in logs,

$$\log c + \log P = \log \omega - \log[1 + Q(r - g)] + \log[\frac{\pi \gamma}{r - g + s} - nQ]$$
 (B.3)

We can now derive the condition under which dP = -dQ and dg = 0 according to the (log) free entry condition. To do so, let's keep g constant and totally differentiate so that:

$$\frac{dP}{P} = -\frac{(r-g)dQ}{1+Q(r-g)} - \frac{ndQ}{\frac{\pi\gamma}{r-g+s} - nQ}.$$
(B.4)

Hence dP = -dQ and dg = 0 if and only if

$$\frac{1}{P} = \frac{(r-g)}{1 + Q(r-g)} + \frac{n}{\frac{\pi\gamma}{r-g+s} - nQ}$$
 (B.5)

$$= \frac{(r-g)}{1+Q(r-g)} + \frac{n\omega/cP}{1+Q(r-g)}$$
 (B.6)

or

$$1 = \frac{P(r-g) + n\omega/c}{Q(r-g) + 1}$$
 (B.7)

or

$$P - Q = \frac{1 - n\omega/c}{r - g} := N(g).$$
 (B.8)

The hump thus occurs at (P, Q, g) that are defined by the three equations (1), (2) and (8). Now (1) and (8) can be solved, given g, to provide:

$$P = \frac{M(g) + N(g)}{2} \tag{B.9}$$

$$Q = \frac{M(g) - N(g)}{2}. (B.10)$$

Substituting these values in B.2 provides a single equation in g, and thus the value of the g at hump. Existence condition for the hump boils down to checking under which condition this equation has a solution below r.

More specifically, the free-entry condition is:

$$cP[1 + Q(r - g)] = \omega \Pi - n\omega Q \tag{B.11}$$

or

$$[cP + n\omega Q] + cPQ(r - g) - \omega \Pi = 0.$$
(B.12)

Now, at the hump,

$$Q = P - N(g) = P + \frac{n\omega/c - 1}{r - g},$$
 (B.13)

so that

$$PQ = P^2 + \frac{n\omega/c - 1}{r - g}P.$$
(B.14)

Using the last two equations to eliminate *Q* and *PQ* from the free-entry condition yields:

$$\left[cP + n\omega\left[P + \frac{n\omega c - 1}{r - g}\right]\right] + c\left[P^2 + \frac{n\omega/c - 1}{r - g}P\right](r - g) - \omega\Pi = 0.$$
 (B.15)

Collecting terms, we get another quadratic equation in P, with coefficients depending on r - g:

$$c(r-g)P^{2} + 2n\omega P + n\omega \frac{n\omega/c - 1}{r - g} - \omega \frac{\pi\gamma}{r - g + s} = 0.$$
(B.16)

Its discriminant is

$$\Delta = 4n^2\omega^2 - 4c(r-g)\left(n\omega\frac{n\omega/c - 1}{r - g} - \omega\frac{\pi\gamma}{r - g + s}\right)$$

$$= 4n^2\omega^2 - 4cn\omega(n\omega/c - 1) + 4c(r - g)\omega\frac{\pi\gamma}{r - g + s}$$

$$= 4cn\omega + 4c(r - g)\omega\frac{\pi\gamma}{r - g + s} > 0$$

Proof of Proposition 4: Hump-shaped GG curve

Proof: In the special case $n\omega/c = 1$, N(g) = 0 and P = Q = M(g)/2 which can be inserted into the free-entry condition B.2 to provide the value of g at the hump.

$$cP(1+P(r-g)) = \omega \left(\frac{\pi \gamma}{r-g+s} - nP\right)$$
$$cP(1+P(r-g)) = \omega \Pi - cP$$

For a given g, the equation above provides a quadratic expression in P with the following

two roots

$$P_{1} = \frac{-\sqrt{c^{2} + c\Pi\omega(r - g)} - c}{c(r - g)}$$
$$P_{2} = \frac{\sqrt{c^{2} + c\Pi\omega(r - g)} - c}{c(r - g)}$$

It's clear that if (r-g) > 0, $P_1 < 0$ and $P_2 > 0$. Using the fact that $P = \frac{M(g)}{2}$, we have

$$P_2 = \frac{-c + \sqrt{c^2 + c\frac{\pi\gamma\omega(r-g)}{r-g+s}}}{c(r-g)} = \frac{M(g)}{2}.$$

The left-hand side of this equation evaluated at 0 and r is

$$\frac{M(g)}{2} = \frac{\frac{\gamma}{g} - 1}{s}$$

$$M(r) = \frac{\frac{\gamma}{r} - 1}{2s} < 0$$

$$M(0) = \infty.$$

Similarly, the right-hand side is

$$\begin{split} P_{1}\left(0\right) &= \frac{-c + \sqrt{c^{2} + cr\omega\frac{\pi\gamma}{r+s}}}{cr} > 0\\ P_{1}\left(r\right) &= \lim_{g \to r} \frac{\sqrt{c^{2} + c\frac{\pi\gamma}{r-g+s}\omega\left(r-g\right) - c}}{c\left(r-g\right)}\\ &= \lim_{g \to r} \frac{\left(-\frac{1}{2}\right)\left(c^{2} + c\frac{\pi\gamma}{r-g+s}\omega\left(r-g\right)\right)^{-\frac{1}{2}}\left(c\frac{\pi\gamma}{r-g+s}\omega\right)}{-cg} = \frac{\left(-\frac{1}{2}\right)c\frac{\pi\gamma}{s}\omega}{-c^{2}r} = \frac{\pi\gamma\omega}{2scr}. \end{split}$$

So there is always a solution between 0 and r, i.e. the left-hand and right-hand sides cross at least once in that interval since:

$$P_1(0) < M(0)$$

 $P_1(r) = \frac{\pi \gamma \omega}{2scr} > 0 > \frac{\gamma - r}{rs} = M(r)$.

Proposition 5 (Hump-shaped GG curve general case) The GG curve is hump-shaped if

$$n^2 < \frac{c}{\omega} \left[(r - g_{\infty}) \Pi_{\infty} + n \right]. \tag{B.17}$$

Proof: For ease of exposition, it is useful to divide the proof into few steps.

Step 1: Positive *P* The quadratic equation in *P* (derived above)

$$c(r-g)P^{2} + 2n\omega P + n\omega \frac{n\omega/c - 1}{r - g} - \omega \frac{\pi\gamma}{r - g + s} = 0$$

has two roots P_1 and P_2 whose sum and product are

$$\begin{split} P_1 + P_2 &= -\frac{2n\omega}{c(r-g)} < 0 \\ P_1 P_2 &= \frac{\omega \left(n \frac{n\omega/c - 1}{r-g} - \frac{\pi \gamma}{r-g + s} \right)}{c(r-g)}. \end{split}$$

If

$$n\frac{n\omega/c-1}{r-g} - \frac{\pi\gamma}{r-g+s} < 0 \Rightarrow$$

$$n\frac{n\omega/c-1}{r-g} - \Pi < 0,$$

then one of the two roots is positive.

Step 2: Sufficient condition The condition above can be simplified as

$$n\omega - c < \frac{c}{n} (r - g) \Pi$$

$$n^2 < \frac{c}{\omega} [(r - g)\Pi + n].$$

This condition involves an endogenous variable (g), but a sufficient condition for it to

hold is

$$n^2 < \frac{c}{\omega} \left[(r - g_{\infty}) \Pi_{\infty} + n \right]$$

where g_{∞} is the horizontal asymptote of the GG curve when $p \to \infty$.

Step 3: r - g > 0 *Finally, we need to check that* r - g > 0. *Let's denote with* P_1 *the positive root, recall that*

$$P_1(g) = \frac{M(g) + N(g)}{2}.$$

The right-hand side of this equation is

$$\frac{M(g)+N(g)}{2}=\frac{1-n\omega/c}{r-g}+\frac{\frac{\gamma}{g}-1}{s}$$

$$M(r) + N(r) = \infty$$

$$M(0) + N(0) = \infty.$$

The left-hand side is:

$$P_{1}(0) = \frac{-n\omega + \sqrt{cn\omega + cr\omega \frac{\pi\gamma}{r+s}}}{cr} > 0$$

$$P_{1}(r) = \lim_{g \to r} \frac{-n\omega + \sqrt{cn\omega}}{c(r-g)} = \infty.$$

Hence there must be at least a solution in between 0 and r.

Proposition 6 (Uniqueness of the hump) *If the GG curve has an hump, it's unique.*

Proof: Recall the equation describing the GG curve:

$$g = \frac{\gamma}{1 + s/p + s/Q(p, g)},\tag{B.18}$$

where the function Q(p, g) is derived implicitly from the free-entry condition:

$$q = \frac{(r-g)\frac{c}{wp} + n}{\frac{\pi\gamma}{r-g+s} - \frac{c}{wp}} := Q(p,g)$$

with $Q_p < 0$ and $Q_g < 0$. This implies after straightforward differentiation that the sign of the slope of the GG curve (dg/dp) is the same as the sign of

$$A := 1/p^2 + Q_p/q^2 \tag{B.19}$$

since

$$G_{p} = \frac{dg}{dp} = \frac{\gamma s}{\left(\frac{s}{q} + \frac{s}{p} + 1\right)^{2}} \left(\frac{q_{p}}{q^{2}} + \frac{1}{p^{2}}\right) = A(p) \frac{\gamma s}{\left(\frac{s}{q} + \frac{s}{p} + 1\right)^{2}}$$

$$G_{pp} = \frac{dg}{dpdp} = 2 \frac{\gamma s^{2}}{\left(\frac{s}{q} + \frac{s}{p} + 1\right)^{3}} A(p)^{2} + \frac{\gamma s}{\left(\frac{s}{q} + \frac{s}{p} + 1\right)^{2}} A_{p}$$

where $A_p = \frac{q_{pp}}{q^2} - \frac{2q_p^2}{q^3} - \frac{2}{p^3}$. At the hump, p^* , $A(p^*) = 0$ so that

$$G_{pp} = \frac{\gamma s}{\left(\frac{s}{q} + \frac{s}{p^*} + 1\right)^2} A_p(p^*).$$

With some algebra, it is possible to show that at the hump the GG function is concave, i.e.

 $A_p < 0$

$$A_{p} = \frac{q_{pp}}{q^{2}} - \frac{2q_{p}^{2}}{q^{3}} - \frac{2}{p^{3}}$$

$$= -\frac{2q_{p}}{q^{2}} \left(\frac{1}{p} - \frac{\Psi_{p}}{(\omega\Pi - \Psi)} \right) - \frac{2q_{p}^{2}}{q^{3}} - \frac{2}{p^{3}}$$

$$= -\frac{2q_{p}}{q^{2}} \left(\frac{1}{p} - \frac{\Psi_{p}}{(\omega\Pi - \Psi)} + \frac{q_{p}}{q} \right) - \frac{2}{p^{3}}$$

$$= \frac{2}{p^{2}} \left(\frac{1}{p} - \frac{\Psi_{p}}{(\omega\Pi - \Psi)} + \frac{q_{p}}{q} - \frac{1}{p} \right)$$

$$= \frac{2}{p^{2}} \left(\frac{q_{p}}{q} - \frac{\Psi_{p}}{(\omega\Pi - \Psi)} \right)$$

$$= \frac{2}{p^{2}} \left(\omega\Psi_{p} \frac{(r - g)\Pi + n}{(\omega\Pi - \Psi)(\omega n + (r - g)\Psi)} - \frac{\Psi_{p}}{(\omega\Pi - \Psi)} \right)$$

$$= \underbrace{\left(\frac{2}{p^{2}}\right)}_{+} \underbrace{\Psi_{p}}_{-} \underbrace{\left(\frac{\omega n + \omega(r - g)\Pi}{\omega n + (r - g)\Psi} - 1\right)}_{+} < 0$$

We can show that the last term is positive using the no free-entry condition:

$$\Psi = \omega \beta \left(\Pi - \frac{n}{q} \right) < \omega \beta \Pi < \omega \Pi$$

where

$$\beta = \frac{q}{r - g + q} < 1$$

$$\Pi = \frac{\pi \gamma}{r - g + s}$$

$$\Psi = \frac{c}{p}.$$

We used the following expressions:

$$q = \frac{\omega n + (r - g)\Psi}{(\omega \Pi - \Psi)}$$

$$\Psi_{p} = -\frac{c}{p^{2}} < 0, \ \Psi_{pp} = 2\frac{c}{p^{3}} > 0, \ \frac{\Psi_{pp}}{\Psi_{p}} = -\frac{2}{p}$$

$$q_{p} = \Psi_{p} \frac{(r - g)(\omega \Pi - \Psi) + \omega n + (r - g)\Psi}{(\omega \Pi - \Psi)^{2}} = \omega \Psi_{p} \frac{(r - g)\Pi + n}{(\omega \Pi - \Psi)^{2}} < 0$$

$$\frac{q_{p}}{q} = \omega \Psi_{p} \frac{(r - g)\Pi + n}{(\omega \Pi - \Psi)^{2}} \frac{(\omega \Pi - \Psi)}{\omega n + (r - g)\Psi} = \omega \Psi_{p} \frac{(r - g)\Pi + n}{(\omega \Pi - \Psi)(\omega n + (r - g)\Psi)}$$

$$q_{pp} = \omega \Psi_{pp} \frac{(r - g)\Pi + n}{(\omega \Pi - \Psi)^{2}} + 2\omega \Psi_{p}^{2} \frac{(r - g)\Pi + n}{(\omega \Pi - \Psi)^{3}} =$$

$$= q_{p} \frac{\Psi_{pp}}{\Psi_{p}} + \frac{2q_{p}\Psi_{p}}{(\omega \Pi - \Psi)} = \frac{-2q_{p}}{p} + \frac{2q_{p}\Psi_{p}}{(\omega \Pi - \Psi)} = -2q_{p} \left(\frac{1}{p} - \frac{\Psi_{p}}{(\omega \Pi - \Psi)}\right) > 0.$$

Appendix C Block Bargaining

Let's assume that the joint bank-firm pair bargains with a innovator. If we define:

$$J_i = B_i + F_i$$
.

Then, in different stages:

$$(r-g)\hat{J}_2 = \pi\gamma - w - s\hat{J}_2$$
$$(r-g)\hat{J}_1 = -n + q(\theta)[\hat{J}_2 - \hat{J}_1]$$
$$K(\phi) \equiv \frac{k}{\phi p(\phi)} + \frac{c}{p(\phi)} = \hat{J}_1.$$

In the innovation market, the negotiated wage for the innovator is the solution to

$$w = \arg\max(\hat{J}_2 - \hat{J}_1)^{1-\alpha} (\hat{I}_2 - \hat{I}_1)^{\alpha}$$

$$\Rightarrow \alpha (\hat{J}_2 - \hat{J}_1) = (1 - \alpha) (\hat{I}_2 - \hat{I}_1)$$

$$\alpha (\hat{J}_2 - \hat{J}_1) = (1 - \alpha) (\hat{I}_2 - \hat{I}_1)$$

$$(1 - \alpha) (\hat{I}_2 - \hat{I}_1) = \alpha (\hat{J}_2 - \hat{J}_1).$$

It follows that

$$(1 - \alpha) (\hat{I}_{2} - \hat{I}_{1}) = \alpha (\hat{J}_{2} - \hat{J}_{1})$$

$$(1 - \alpha) (r - g) (\hat{I}_{2} - \hat{I}_{1}) = \alpha (r - g) (\hat{J}_{2} - \hat{J}_{1})$$

$$(1 - \alpha) [w - s(\hat{I}_{2} - \hat{I}_{1}) - (r - g) \hat{I}_{1}] = \alpha [\pi \gamma - w - s(\hat{J}_{2} - \hat{J}_{1}) - (r - g + s) \hat{J}_{1}]$$

$$(1 - \alpha) [w - (r - g) \hat{I}_{1}] = \alpha [\pi \gamma - w - (r - g + s) \hat{J}_{1}]$$

$$w = \alpha (\pi \gamma - (r - g + s) \hat{J}_{1}) + (1 - \alpha) (r - g) \hat{I}_{1}$$

$$(r-g)\hat{I}_{1} = \theta q(\theta) \left[\hat{I}_{2} - \hat{I}_{1}\right] =$$

$$= \theta q(\theta) \frac{\alpha}{(1-\alpha)} \left[\hat{J}_{2} - \hat{J}_{1}\right]$$

$$= \theta q(\theta) \frac{\alpha}{(1-\alpha)} \left[\frac{(r-g)\hat{J}_{1} + n}{q(\theta)}\right]$$

$$= \theta q(\theta) \frac{\alpha}{(1-\alpha)} \left[\frac{(r-g)K(\phi) + n}{q(\theta)}\right]$$

$$= \theta \frac{\alpha}{(1-\alpha)} \left[(r-g)K(\phi) + n\right]$$

We can use the expression above in the equilibrium wage:

$$w = \alpha \left(\pi \gamma - (r - g + s)\hat{J}_{1}\right) + (1 - \alpha)(r - g)\hat{I}_{1}$$

$$= \alpha \pi \gamma + (1 - \alpha)(r - g)\hat{I}_{1} - \alpha(r - g + s)K(\phi)$$

$$= \alpha \pi \gamma + (1 - \alpha)\theta \frac{\alpha}{(1 - \alpha)}((r - g)K(\phi) + n) - \alpha(r - g + s)K(\phi)$$

$$= \alpha(\pi \gamma + \theta(r - g)K(\phi) + \theta n - (r - g + s)K(\phi))$$

$$= \alpha(\pi \gamma + \theta n + (\theta(r - g) - (r - g + s))K(\phi))$$

$$= \alpha(\pi \gamma + \theta n - ((1 - \theta)(r - g) + s)K(\phi))$$

For what follows, It is useful to slightly rewrite the equilibrium wage by noting that in equilibrium, substituting $\phi = \frac{\omega}{1-\omega} \frac{k}{c}$ in $K(\phi)$:

$$K(\phi) = \frac{c}{p} + \frac{k}{p\phi} = \frac{c}{\omega p}$$

so that

$$w = \alpha \left(\pi \gamma + \theta \, n - \left((1 - \theta) \left(r - g \right) + s \right) \frac{c}{\omega p} \right)$$

It follows that we can rewrite the new spillover function, i.e. Eq 31 in the text, as

$$Q(p,g) = \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi \gamma - \omega(p,g)}{r-g+s} - \frac{c}{\omega p}}$$

$$= \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi \gamma(1-\alpha) - \alpha\theta n + \alpha((1-\theta)(r-g)+s)\frac{c}{\omega p}}{r-g+s} - \frac{c}{\omega p}}$$

$$= \frac{(r-g)\frac{c}{\omega p} + n}{\frac{\pi \gamma(1-\alpha) - \alpha\theta n + \alpha((1-\theta)(r-g)+s)\frac{c}{\omega p}}{r-g+s} - \frac{c}{\omega p}}$$

$$= \frac{(r-g)\frac{c}{\omega p} + n}{\frac{r-g+s}{r-g+s} - \frac{c}{\omega p}}$$

Given that $\alpha < 1$, the denominator of the expression above increases with p, hence, as before, the spillover function decreases with p.

Appendix D Only financial frictions

The value of a bank

The Bellman equations describing the steady-state values of the bank over the two stages are:

$$(r-g)\hat{B}_0 = -k + \phi p(\phi)[\hat{B}_1 - \hat{B}_0]$$
$$(r-g)\hat{B}_1 = \rho + s[\hat{B}_0 - \hat{B}_1]$$

where *k* is a search cost in the financial market. With free-entry:

$$\hat{B}_{1} = \frac{k}{\phi p(\phi)}$$

$$\hat{B}_{1} = \frac{\rho}{(r-g+s)}$$

$$\frac{k}{\phi p(\phi)} = \frac{\rho}{(r-g+s)}.$$

The value of an Innovator/Firm

Similarly, the value of a firm on the balanced growth path is:

$$(r-g)\hat{F}_0 = -c + p(\phi)[\hat{F}_1 - \hat{F}_0]$$
$$(r-g)\hat{F}_1 = \pi\gamma - \rho + s[\hat{F}_0 - \hat{F}_1]$$

or

$$(r-g)[\hat{F}_1-\hat{F}_0] = \pi\gamma - \rho + c - (s+p)[\hat{F}_1-\hat{F}_0].$$

There is no free-entry in this market and the total number of Firms is normalized to 1:

$$\mathscr{F} = \mathscr{F}_1 + \mathscr{F}_0 = 1.$$

Then, the dynamics of the flow of firms searching for banks can be expressed as

$$\mathscr{F}_0 = s(1-\mathscr{F}_0) - p\mathscr{F}_0.$$

Abstracting from the very short-run so that $\mathcal{F} = 0$:

$$\mathcal{F}_0 = \frac{s}{s+p}$$

$$\mathcal{F}_1 = \frac{p}{s+p}.$$

Nash Bargaining

Under Nash bargaining, the surplus is shared according to the bargaining weights

$$(1-\omega)\left[\hat{F}_1-\hat{F}_0\right]=\omega\hat{B}_1.$$

After some algebra, we have:

$$(1-\omega)\left(r-g\right)\left[\hat{F}_{1}-\hat{F}_{0}\right]=\omega\left(r-g\right)\hat{B}_{1}$$

$$(1-\omega)\left[\pi\gamma-\rho+c-\left(s+p\right)\left(\hat{F}_{1}-\hat{F}_{0}\right)\right]=\omega\left[\rho-s\hat{B}_{1}\right]$$

$$(1-\omega)\left[\pi\gamma-\rho+c-p\left(\hat{F}_{1}-\hat{F}_{0}\right)\right]-s\left(1-\omega\right)\left(\hat{F}_{1}-\hat{F}_{0}\right)=\omega\rho-s\omega\hat{B}_{1}$$

$$(1-\omega)\left[\pi\gamma-\rho+c-p\left(\hat{F}_{1}-\hat{F}_{0}\right)\right]=\omega\rho$$

$$(1-\omega)\left[\pi\gamma-\rho-\left(r-g\right)\hat{F}_{0}\right]=\omega\rho$$

$$\rho=(1-\omega)\left[\pi\gamma-\left(r-g\right)\hat{F}_{0}\right].$$

Moreover

$$(r-g)\hat{F}_{0} = -c + p(\phi)[\hat{F}_{1} - \hat{F}_{0}] = -c + p(\phi)\frac{\omega\hat{B}_{1}}{1-\omega}$$

$$(r-g)\hat{F}_{0} = -c + p\frac{\omega}{1-\omega}\frac{k}{\phi p}$$

$$(r-g)\hat{F}_{0} = -c + \frac{\omega}{1-\omega}\frac{k}{\phi}.$$

Plugging the expression above in the equilibrium ρ :

$$\rho = (1 - \omega) \left[\pi \gamma + c - \frac{\omega}{1 - \omega} \frac{k}{\phi} \right].$$

Therefore, the free entry condition in the banking sector can be rewritten as

$$\begin{split} \frac{k}{\phi p\left(\phi\right)} &= \frac{\rho}{\left(r-g+s\right)} \\ \frac{k}{\phi p\left(\phi\right)} &= \frac{\left(1-\omega\right)}{\left(r-g+s\right)} \left[\pi \gamma + c - \frac{\omega}{1-\omega} \frac{k}{\phi}\right]. \end{split}$$

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