Staff memo

Monetary policy transmission and the cash-flow channel via nominal and real interest rates

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Summary

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The authors work at the Riksbank's Monetary Policy Department.[1]

In the standard New Keynesian framework, which has been used to guide monetary policy during the last 25 years, monetary policy affects household consumption via an intertemporal substitution channel. According to this channel, forward-looking households adjust their consumption and savings decisions when actual and expected real interest rates change. The empirical support for this transmission channel is, however, mixed and other potential channels have been proposed. One such channel is the cash-flow channel which suggests that changes in interest rates can have a direct effect on household consumption via interest expenses.

We incorporate the cash-flow channel into a standard New Keynesian framework and analyse how adding this channel affects monetary policy transmission under different modelling assumptions. Notably, we discuss under which assumptions real versus nominal interest rates matter for household consumption. We focus on the intuition behind the mechanisms and the analysis is intended for illustrative purposes, not to quantify the effects of monetary policy.

Based on our theoretical framework, the key findings are as follows. First, the cash-flow channel amplifies the macroeconomic effects of monetary policy shocks and the effects are stronger when household debt-ratios are higher. Second, different specifications of borrowing constraints lead to different conclusions on whether it is nominal or real interest rates that matter for household consumption. Third, depending on assumptions about the borrowing constraints, monetary policy can have an immediate or a lagged impact on household consumption. Fourth, according to the cash-flow channel, it is realised, or ex-post, rather than ex-ante interest rates that matter for consumption. Finally, we find that in the case of a demand shock, monetary policy can more easily stabilise household consumption if borrowing constraints are designed in nominal

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terms, while in the case of a supply shock, monetary policy can more easily stabilise household consumption if borrowing constraints are in real terms.

1 Introduction

In the standard New Keynesian (NK) framework, which has been used to guide monetary policy during the last 25 years, monetary policy affects household consumption via an intertemporal substitution channel. According to this channel, forward-looking households adjust their consumption and savings decisions when the real interest rate changes.[2] The empirical support for the intertemporal-substitution channel is mixed and other channels have been suggested. Interest rates can, for example, have a direct effect on household consumption via the so called cash-flow channel. Specifically, when households hold debt with adjustable interest rates, a change in the policy rate can affect household consumption through a change in household interest expenses and, consequently, disposable income. If households behave as in the standard NK framework, households may not alter consumption much when there are changes in interest rates. However, if households face liquidity or credit constraints, or are unwilling to use up their savings, changes in interest rates will have a larger effect on consumption than in the standard NK framework. Hughson *et al.* (2016), Di Maggio *et al.* (2017), Flodén *et al.* (2021), Holm *et al.* (2021) provide empirical evidence for cash-flow effects and their results suggest that the cash-flow channel can play an important role in monetary policy transmission.[3,4]

The purpose of this paper is to illustrate the transmission of monetary policy via the cash-flow channel. We study the mechanisms through which monetary policy affects household consumption under different specifications of financial contracts (borrowing constraints). We assume that there is a limit to how much a household is allowed to borrow. This limit can be designed in different ways. For example, the maximum amount of borrowing can be measured in terms of consumption goods (real) or in terms of money (nominal).

Empirically, borrowing constraints may be neither purely real nor purely nominal. For example, in Sweden, a mortgage is generally restricted to maximum 85 percent of the purchase value of the house, i.e., it is relative to the house price. In addition, the loan-to-value ratio (in combination with loan-to-income) determines how much a household needs to amortize. This amount is in turn used as input for the (nominal) discretionary income, the so-called KALP ("kvar att leva på"), that needs to be positive for a mortgage to be approved and that banks compute to establish the borrowing limit for the household. The KALP is based on the household current income and on some standard measures of expenses. However, the standard expenses (including the nominal interest rate) used as input for this constraint are infrequently updated and there

[2] Real interest rates are affected by unexpected changes to inflation via the Fisher relation between nominal and real interest rates.

[3] Cloyne *et al*. (2020) study both direct (cash-flow) and indirect (general equilibrium) effects of monetary policy.

[4] Flodén *et al.* (2021) also show theoretically that, under some circumstances, a change in the nominal interest rate has a direct effect on the consumption of credit constrained households via the cash-flow channel.

are restrictions on how often the loan-to-value ratio can be updated when computing amortization rates.

In this paper, a particular focus is on whether it is the nominal or real interest rate that matters for consumption dynamics and we show that this depends on the design of the borrowing constraints. To demonstrate a cash-flow channel, we first compare the macroeconomic effects of monetary policy shocks under different, stylized, assumptions of borrowing arrangements. Second, we analyse the effects of demand and supply shocks and their implications for systematic monetary policy given the different assumptions on the design of borrowing constraints.

In terms of modelling aspects, we extend the standard NK framework by including two types of agents: financially unconstrained households (savers) and financially constrained households (borrowers).[5] Borrowers face a borrowing constraint and we assume that they always borrow the maximum amount allowed.[6] The main difference between the two types of households is that savers behave as the standard NK household and can smooth consumption over time, whereas borrowers face financial frictions which inhibit them from smoothing consumption. Accordingly, monetary policy affects the savers' consumption via the intertemporal substitution channel, whereas it affects the borrowers' consumption via the cash-flow channel. Moreover, we model different specifications of loan contracts by altering the design of borrowing constraints. It should be noted that this paper focuses on illustrating the intuition behind the cash-flow channel under different model specifications. Thus, our quantitative implications are based on simple models and intended for illustrative purposes only.

The main findings of this paper are as follows. First, by including the cash-flow channel, macroeconomic variables, such as GDP and inflation, are more sensitive to monetary policy shocks compared to the standard NK model. The primary reason is that borrowers are more sensitive to changes in interest rates than savers, consequently consumption and GDP change more in the extended NK framework.[7] Second, we analyse the implications of different specifications of borrowing constraints. Particularly, we show that if the borrowing limit is in real terms, then it is the real interest rate that affects financially constrained household consumption through the cash-flow channel. If on the other hand, the borrowing limit is stipulated in nominal terms, then it is the nominal interest rate that affects financially constrained household consumption. This is also the case if the borrowing arrangement is defined as a function of the current nominal interest rate. Depending on the timing of the nominal interest rate in the loan contracts, monetary policy can have an immediate or a lagged impact on household consumption. Third, with the cash-flow channel, it is actual (ex-post) rather than ex-ante interest rates that matter for the borrowers' consumption.

[5] Throughout the paper we use the terms "savers (borrowers)" and "financially unconstrained households (financially constrained households)" interchangeable when referring to different types of households.

[6] We assume that borrowers are less patient than savers. As a result the borrowers always borrow the maximum amount and this implies that borrowing constraints are always binding.

[7] The models are kept simpler than, for example, Finocchiaro *et al.* (2016) and Di Casola and Iversen (2019) that include more financial and real frictions.

Finally, we examine the implications of different assumptions of borrowing constraints for systematic monetary policy and for the effects of demand and supply shocks. We show how different specifications of financial arrangements can affect the ability of monetary policy to stabilise the economy. In the case of a demand shock, monetary policy can more easily stabilise household consumption if the nominal interest rate matters, while in the case of a supply shock, monetary policy can more easily stabilise consumption when the real interest matters. The derived results depend on if the relevant interest expense is determined by nominal or real interest rates. In addition to the effect of changes in the policy rate, the debt-inflation channel may affect household consumption as well.

In this paper we contribute to the existing literature related to the New Keynesian framework with credit constrained households. Based on the framework of Iacoviello (2005), previous studies such as Walentin (2014), Finocchiaro *et al.* (2016), Di Casola and Iversen (2019) and Bekiros *et al.* (2020) formulate household borrowing constraint in real terms. The corollary of this borrowing constraint specification is that the real interest rate is what matters for consumption. We demonstrate that depending on how the borrowing constraints are specified, the nominal interest rate can also matter for household consumption.

Section 2 presents the theoretical framework that consists of an outline for the financially unconstrained and constrained households, using three different borrowing constraint specifications for the latter. Primarily, we focus on whether the nominal or real interest rate matters for household consumption. Section 3 analyses the cash-flow channel of monetary policy transmission based on the impulse responses of macroeconomic variables to a monetary policy shock. Section 4 examines systematic monetary policy in response to macroeconomic shocks in the different models. In Section 5 we discuss other types of borrowing constraints: debt-service-to-income and loan-to-value constraints. Section 6 concludes.

2 Theoretical framework

In this section, we extend the standard NK framework by allowing for the presence of cash-flow effects. First, we present savers' consumption behaviour. In particular, we describe how monetary policy transmits via the intertemporal substitution channel. Second, we present the borrower's consumption problem and discuss how monetary policy transmits via the cash-flow channel. Particularly, we illustrate how different ways of specifying borrowing constraints can lead to different conclusions on whether nominal or real interest rates matter for borrowers' consumption.

In the standard NK framework, the household sector can be represented by one household type, the representative household, and this household can smooth consumption over time. Our framework shares many basic features with the standard NK DSGE (dynamic stochastic general equilibrium) framework. Thus, we only briefly state the basic features but focus mainly on the distinctive features. In our extended framework, the economy is populated by two types of households: financially unconstrained (savers) and financially constrained (borrowers). Both types of households consume and work. The financially unconstrained households are forward-looking and can save in financial markets to facilitate consumption smoothing over time. The financially constrained households, on the other hand, are credit constrained and behave as hand-to-mouth consumers.[8] Savers lend directly to borrowers. Finally, our theoretical framework consists of three different specifications of borrowing constraints. We refer to the Appendix for more details on model features and for derivations of key equations.

The basic features of the standard NK framework are as follows. Labour is the only input used for production and there are no labour market frictions so the nominal wage is determined in equilibrium (by equating the demand for and supply of labour). The goods market is characterized by monopolistic competition and prices of goods are sticky. The central bank follows a standard monetary policy rule, described in more detail in Section 3.

2.1 Financially unconstrained household

A financially unconstrained household (saver) chooses her consumption and labour supply. The utility function is expressed as:

$$1. \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(lnC_t - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right),$$

where C_t denotes consumption, N_t is hours worked/labour supply, β is the discount factor and φ is the Frisch labour-supply elasticity. The saver maximises her expected utility subject to a budget constraint. The household can smooth her consumption through saving in the form of private bonds (denoted by B_t). For the financially unconstrained household, expenditures consist of consumption and new purchases of bonds, whereas income comes from the return from holding bonds, wage income and the lump-sum profit. The budget constraint in nominal terms is written as:

$$2. \quad P_t C_t + i_{t-1} B_{t-1} = B_t + W_t N_t + T_t.$$

P_t is the consumer price level and W_t is the nominal wage. i_t is the (gross) nominal policy rate that is set by the central bank in period t and $\pi_t = P_t / P_{t-1}$ is the current (gross) inflation rate. Nominal lump-sum profits from firms and lump-sum taxes as

[8] The financially constrained households are also forward-looking, but less patient than the unconstrained households and want to borrow to finance more consumption today. We assume that the household faces a borrowing constraint and always borrows up to the borrowing limit, see more details in Section 2.2.

well as transfers are denoted by T_t.[9] Equation (2) can be written in real terms by dividing with P_t. The household therefore faces the following real budget constraint:

$$3.\quad C_t + \frac{i_{t-1}}{\pi_t} b_{t-1} = b_t + w_t N_t + \tau_t.$$

The amount of one-period lending in real terms from the saver to the borrower is given by $-b_t$, where $b_t = B_t/P_t$. The ex-post real interest rate is i_{t-1}/π_t. The real return from issuing private bonds is $-(i_{t-1}/\pi_t)b_{t-1}$, so changes in the current inflation affect the ex-post real interest rate. The real wage is $w_t = W_t/P_t$. Finally, τ_t denotes the real lump-sum profits, taxes and transfers. The financially unconstrained household maximizes the utility function (1) given the budget constraint (3). The first-order conditions with respect to C_t and b_t can be summarised by the standard Euler equation:

$$4.\quad \frac{1}{C_t} = E_t\left(\frac{i_t}{\pi_{t+1}} \frac{1}{C_{t+1}}\right).$$

The consumption-Euler equation in terms of log-linearization is written as:

$$5.\quad \hat{C}_t = E_\text{t}\left(\hat{C}_{t+1}\right) - E_\text{t}(\hat{\imath}_t - \hat{\pi}_{t+1})\ .$$

Eq. (5) relates expected changes in the consumption path to the ex-ante real interest rate, $E_\text{t}(\hat{\imath}_t - \hat{\pi}_{t+1})$.[10] For instance, a rise in the ex-ante real interest rate increases savings and decreases current consumption, all else equal. Since the saver is a forward-looking consumer and makes decisions about both current and future consumption, the household is affected by changes in the ex-ante real interest rate.

2.2 Financially constrained household: borrowing constraint in real terms (Model A)

We incorporate two features into the standard NK model. First, we allow for heterogeneity among households by including two types of households: savers and borrowers. Both types of households have the same utility function, but they are different in terms of how much they discount the future. Specifically, we assume borrowers are less patient than savers, thus the borrowers prefer debt to internal funds, i.e., to saving. Second, we introduce a simple version of incomplete markets (financial frictions) by imposing an exogenous borrowing constraint on borrowers. In reality, there is of course heterogeneity also among borrowers (and savers). In our models, we make the simplifying assumption that borrowers are always at the borrowing constraint and are more impatient than savers. These two features, impatience and financial frictions, hence imply financially constrained households (borrowers) face a binding borrowing

[9] Since the financially unconstrained household behaves as the Ricardian household, changes in lump-sum taxes have no effects on her consumption. In this paper, monetary policy is not affected by the assumption of fiscal policy. For macroeconomic effects of fiscal policies on monetary policy, see the SELMA DSGE model of National Institute of Economic Research by Akkaya *et. al.* (2021).

[10] The hatted variables denote percent changes from the steady state, and those without time subscript denotes steady state.

constraint and always borrow up to the borrowing limit.[11] As a result, the borrowers are not able to smooth her consumption and do not save sufficiently to make the borrowing constraint irrelevant.[12]

A financially constrained household (the superscript F stands for a financially constrained household) finances her consumption and repayment of existing debt with labour income and new borrowing. Borrowing is one period and is always repaid in full in the next period. Nominal repayment is defined as the principal plus the interest rate at origination times the borrowed amount. In this section we consider the case when the borrowing constraint is specified in real terms. The utility function is:

$$6. \quad E_0 \sum_{t=0}^{\infty} \beta_t^F \left(lnC_t^F - \frac{\left(N_t^F\right)^{1+\varphi}}{1+\varphi} \right).^{13}$$

We assume that the borrower's discount factor β^F is lower than that of the saver. The amount of borrowing (new loans) in nominal terms is denoted by B_t^F. The debt contract is set in nominal terms, i.e., the debt repayment is given by $i_{t-1}B_{t-1}^F$. The borrower faces the following nominal budget constraint:

$$7. \quad P_t C_t^F + i_{t-1} B_{t-1}^F = B_t^F + W_t N_t^F.$$

As in the previous section, the above constraint can be written in real terms.[14] The borrower faces the following real budget constraint:

$$8. \quad C_t + \frac{i_{t-1}}{\pi_t} b_{t-1}^F = b_t^F + w_t N_t^F.$$

We denote b_t^F as the amount of new loans in real consumption unit terms. In other words, the household borrows the amount of kronor that corresponds to a certain multiple of consumption (basket) units. The household faces the following borrowing constraint in real terms:

$$9. \quad b_t^F \leq \overline{b}.^{15}$$

We assume that the borrower always borrows up to the borrowing limit, i.e., the household behaves as a rule-of-thumb consumer and $b_t^F = \overline{b}$. Since the borrower is at her borrowing limit, the current consumption is determined by the budget constraint:

$$10. \quad C_t^F = \left[1 - \frac{i_{t-1}}{\pi_t}\right] \overline{b} + w_t N_t^F.$$

[11] For models that feature occasionally binding constraints, see Guerrieri and Iacoviello (2015), Grodecka (2020) and Swarbrick (2021).

[12] For more discussions on the assumptions of household heterogeneity and financial frictions, see, e.g., Quadrini 2011.

[13] In principle the labour supply term is not needed in the utility function of the borrower as we assume that the consumer does not optimize with respect to that variable. We include it for expositional purposes.

[14] Since the lump-sum taxes do not enter the financially constrained household budget constraint, changes in lump-sum taxes have no effects on the rule-of-thumb household consumption.

[15] The exogenous borrowing limit in real terms implies that if there is any change in the price level, the nominal value of the borrowing limit will adjust such that the amount of borrowing in real terms is fixed at a certain value.

Thus, household consumption equals a cash-flow term $\left[1-\frac{i_{t-1}}{\pi_t}\right]\overline{b}$ for repayment of debt and the labour income term $w_t N_t^F$.[16] The borrower's consumption equation in terms of log-linearization can then be written as:

$$11.\ \frac{C^F}{Y}\hat{C}_t^F = -\frac{1}{\beta}\frac{\overline{b}}{Y}(\hat{\imath}_{t-1}-\hat{\pi}_t)+\frac{wN}{Y}(\widehat{w}_t+\widehat{N}_t).$$ [17]

For this household, all else equal, a higher level of the previous period's nominal interest rate, $\hat{\imath}_{t-1}$, has a negative effect on current consumption. On the other hand, an increase in current inflation, without an increase in the nominal interest rate, contributes positively to consumption. As inflation rises, the real value of the nominal debt repayment declines. This has a positive effect on current consumption, all else equal. An increase in the ex-post real interest rate, $\hat{\imath}_{t-1}-\hat{\pi}_t$, i.e., if the nominal interest rate rises more than inflation, this increases the interest expense (for a given debt ratio), which in turn has a negative impact on the borrower's consumption.[18]

If the debt to income ratio, $\overline{b}/Y$, increases, the effects of changes in the ex-post real interest rate on the interest expense, and thus on consumption, are amplified. Another channel of monetary policy transmission comes from the labour income channel. If the real labour income decreases via a decline in the real wage and/or employment, the financially constrained household reduces her consumption.

We can compare the borrower's and the saver's consumption (Eq. (11) versus Eq. (5)). For the borrower, it is the ex-post interest rate that matters for consumption, while the ex-ante interest rate matters for the saver's consumption. This difference stems from the fact that the financially unconstrained household can smooth her consumption. In contrast, the financially constrained household faces financial frictions in the form of borrowing constraints that impede consumption smoothing.

2.3 Financially constrained household: borrowing constraint in nominal terms (Model B)

A financially constrained household faces the same budget constraint as in Model A. However, this section considers a nominal loan contract instead of a loan contract in real terms. The second specification of the borrowing constraint is:

$$12.\ B_t^F \leq \overline{B}.$$

[16] Throughout models A to C we assume that interest expenses on the previous period's debt are given by: $i_{t-1}B_{t-1}$. This means that today, in period t, the borrower pays a predefined nominal interest rate on her existing debt (from $t-1$). Because debt in this model is one period we do not distinguish between loan-contracts with adjustable or fixed interest rates. This assumption is without loss of generality as long as bond markets open every period.

[17] The above equation is divided by Y (GDP) to facilitate the calibration of the steady state level of debt-to-GDP ratio.

[18] The derivation of the borrower's consumption based on Model A can be found in Appendix A2.

The maximal amount of new loans is defined in nominal terms as B_t^F. In simple words, the constraint relates to the amount of kronor that the borrower can borrow and it cannot be greater than $\overline{B}$. As in Model A, we assume that the borrower always borrows up to the borrowing limit, i.e., the household behaves as a rule-of-thumb consumer and thus $B_t^F = \overline{B}$.[19] Because the borrower is at its borrowing limit, consumption is determined by the budget constraint:

$$13.\ C_t^F = [1 - i_{t-1}]\frac{\bar{B}}{P_t} + w_t N_t^F.$$

As before, consumption equals the cash-flow channel $[1 - i_{t-1}]\frac{\bar{B}}{P_t}$ plus the labour income component $w_t N_t^F$. The cash-flow channel in Model B is primarily captured by changes in the nominal interest rate, while changes in the real interest rate have a direct effect on consumption in Model A. Given Eq. (13), the consumption equation in terms of log-linearization can be written as:

$$14.\ \frac{C^F}{Y}\hat{C}_t^F = -\frac{1}{\beta}\frac{\bar{b}}{Y}\hat{\imath}_{t-1} + \frac{wN}{Y}\left(\widehat{w}_t + \widehat{N}_t\right).$$

From Eq. (14), it is clear that the previous period's nominal interest rate matters for consumption, in addition to changes in real labour income. A higher level of the nominal interest rate has a negative effect on consumption, in line with the intuition from the cash-flow channel.[20] This is in line with the results from a partial equilibrium model in the Online Appendix of Flodén *et al.* (2021). Furthermore, an increase in the debt ratio amplifies the effect of a rise in the nominal interest rate on consumption.

2.4 Financially constrained household: borrowing constraint with the current nominal interest rate (Model C)

A financially constrained household faces the same specification of the budget constraint as in Models A and B. The loan contract is similar to Model B. However, here we imagine the simplest form of a type of debt service constraint, where the borrowing constraint varies with the nominal interest rate in the following manner:

$$15.\ i_t B_t^F \leq \bar{B}.$$

Here we just assume that debt service cannot exceed a nominal amount, $\overline{B}$, without specifying how this amount is set. In reality, a macroprudential authority can use the design of borrowing restrictions as a policy tool. In Section 5.1 we discuss a specific example of this constraint that directly links household income to the amount of new

[19] In contrast to Model A, the borrowing limit is fixed in nominal terms. Thus, the nominal value of the borrowing limit does not respond to changes in the price level, so implicitly the borrowing limit in real terms can change with the price level.

[20] Note that inflation targeting monetary policy implies that the price level is a non-stationary process. Since we log-linearize around a non-stochastic steady state we make the following assumption to be able to conduct model simulations: $[1/\beta - 1]\left(\overline{b}/Y\right)\hat{P}_t \approx 0$ in Eq. (14). With our calibration of β, $[1/\beta - 1] \approx 0$.

debt: interest expenses cannot exceed a fraction of the household's wage income and this fraction is set by the macroprudential authority.

As in the above models, we assume that the borrowing constraint is binding, i.e., $i_t B_t^F = \bar{B}$. Given this setup of the borrowing constraint, household consumption can be expressed as:

$$16.\ C_t^F = [\frac{1}{i_t} - 1]\frac{\bar{B}}{P_t} + w_t N_t^F.$$

Based on Eq. (16), the cash-flow channel is captured by the term $[\frac{1}{i_t} - 1]\frac{\bar{B}}{P_t}$ and the labour income channel is captured by the term $w_t N_t^F$. In contrast to Model B (Eq.13), the current nominal rate rather than previous period's nominal rate has a direct effect on consumption. In Model B, the interest rate that the household pays on the existing debt is key for the monetary policy transmission. Because we specify that the household repayment of debt is given by $i_{t-1}\bar{B}$, the lagged nominal rate matters for household consumption in the previous model.

For Model C, we assume that the borrowing constraint itself is affected by the current interest rate, hence the interest rate on new loans is key for the monetary policy transmission. As a result, current consumption is directly affected by the current nominal rate. Thus, monetary policy can have a lagged or immediate impact on household consumption depending on the timing of the nominal interest rate specified in loan contracts.

Finally, the financially constrained household's consumption equation in terms of log-linearization can be written as:

$$17.\ \frac{C^F}{Y}\hat{C}_t^F = -\beta\frac{\bar{b}}{Y}\hat{\imath}_t + \frac{wN}{Y}(\widehat{w}_t + \widehat{N}_t).$$

Similar to Model B (Eq.14), the nominal rate matters for consumption, but Eq. (17) implies that it is the current nominal rather than last period's interest rate that matters. Given this specification of the borrowing constraint, monetary policy thus has an immediate impact on consumption.[21]

3 Cash-flow channel of monetary policy transmission

This section illustrates, using model simulations, the mechanisms of monetary policy transmission in the standard NK model compared to the three different models from Section 2. We compare impulse responses of

[21] Note that inflation targeting monetary policy implies that the price level is a non-stationary process. To be able to conduct model simulations, we assume the following term $[1-\beta](\bar{b}/Y)\hat{P}_t \approx 0$ in Eq. (16). With our calibration of β, $[1-\beta] \approx 0$.

key macroeconomic variables to a monetary policy shock. We focus on illustrating the intuition behind the cash-flow channel under the different model assumptions and the results from this section should not be interpreted as quantitatively realistic.

In order to study the effects of a monetary policy shock, we assume that the central bank follows a standard monetary policy rule that reacts to deviations in inflation from the target, $\hat{\pi}_t$, and deviations in output from the steady state, $\hat{Y}_t$:

$$18.\ \hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\left(\rho_\pi \hat{\pi}_t + \rho_y \hat{Y}_t\right) + \varepsilon_t^i,$$

where ε_t^i captures an unexpected monetary policy shock. We often assume hump-shaped responses in output and inflation in empirical DSGE and VAR studies. Thus, we allow for a habit-formation in the saver's consumption function in our model simulations to help generate a hump-shaped response in output. We also allow for an inertia in inflation dynamics to generate a hump-shaped response in inflation. Including the habit formation and inertia in inflation do not alter our main results or conclusions qualitatively.[22] Table 1 in Appendix presents the baseline calibration.

3.1 Monetary policy shock

We compare the results from the two-agent models (Models A to C) to a standard, representative agent, NK model. In our standard NK model, the share of financially constrained households (borrowers) is set to zero. The other models consist of both types of households. The baseline debt-to-annual GDP ratio is set at 1.9 and the share of borrowers is set to 0.35.[23] Note that GDP in our simple models equals total household consumption which comprises financially unconstrained and constrained household consumption. To facilitate the comparisons, the impulse responses of the policy rate from all four models are based on a 1 percentage point increase in the nominal policy rate and the same policy rate path.

Figure 1 displays impulse responses of macroeconomic variables to a contractionary monetary policy shock in the standard NK model and the three cash-flow models (Models A to C). All variables are measured in percent deviation from the steady state. The responses of inflation as well as nominal and real interest rate are expressed in annual terms. In general, the responses of GDP, inflation, consumption and the real wage to a contractionary monetary policy shock are negative. As we can see, the models with cash-flow effects yield a larger decline in GDP and inflation than the standard NK model. The purpose of these theoretical simulations is to highlight the intuition of how monetary policy is transmitted when we allow for a cash-flow channel.

[22] For more details on habit formation and inertia in inflation, see Christiano *et al.* (2005) and Akkaya *et. al.* (2021). In the Appendix, we show the saver's consumption equation with habit formation and the Phillips curve with inertia in inflation.

[23] Swedish household debt as percentage of annual disposable income was 190 percent in 2019.

All four models are thought of a being conceptual and are therefore not calibrated to match Swedish data. One can thus only interpret the qualitative difference in responses as an illustration of how the mechanisms differ and not as quantitative estimates. We show that by adding the constrained borrower, the cash-flow channel matters for the transmission of monetary policy.

A rise in the policy rate leads to an increase in the ex-ante real interest rate which in turn has a negative effect on savers' consumption via the intertemporal substitution channel. Unlike the standard NK model, the NK cash-flow models include both the intertemporal substitution as well as the cash-flow channel.

In Model A, a rise in the nominal interest rate triggers an increase in the interest expense for borrowers, all else equal. This has a negative effect on the borrowers' consumption. Additionally, the decrease in inflation raises the real value of debt payments, which also has a negative effect on the borrowers' consumption. The combination of the increase in the nominal rate and the decrease in inflation results in an increase in the ex-post real interest rate, thereby increasing the cost of debt service.[24]

Similar to Model A, an increase in the nominal interest rate has a negative effect on financially constrained household consumption in Model B. However, when we compare Model A and Model B, the response of borrowers' consumption in Model B is smaller than in Model A. The main reasons are as follows. A change in inflation has a limited impact on the interest expense in Model B. In Model A, a decrease in inflation reinforces an increase in the interest expense. This channel is called the debt-deflation channel which is discussed in Iacoviello (2005), whereas in Model B it is (mainly) a movement in the nominal rate that affects the interest expense.[25]

In Model C, the response of financially constrained household consumption is similar to that of Model B, with a somewhat stronger immediate response of consumption. The main reason is that the current rather than the lagged nominal interest rate matters for borrowers' consumption in Model C.

In the considered models, the decline in aggregate demand triggers a fall in the real wage and as a consequence real labour income drops. This also has a negative effect on the financially constrained household consumption.

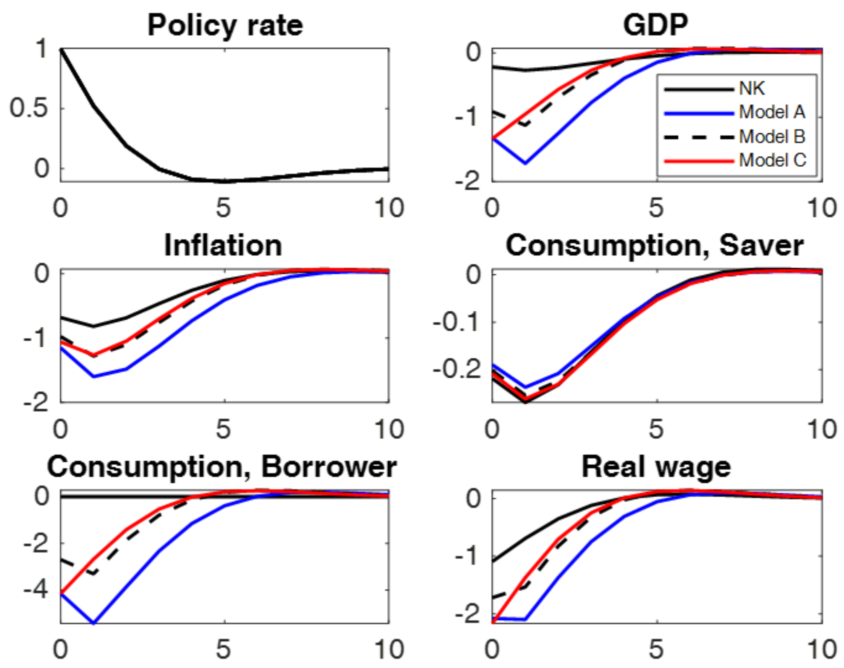
To summarize, different specifications of borrowing constraints lead to different conclusions regarding whether it is nominal or real interest rates that matter for household consumption. Moreover, depending on stipulations about the timing of nominal interest rate in loan contracts, monetary policy can have an immediate or a lagged impact on household consumption. Lastly, in our simple cash-flow framework, it is

[24] Note that if we instead make assumptions such that the nominal rate matters for both constrained and unconstrained agents, by assuming that the debt contract is indexed to inflation, the response of GDP to an unexpected monetary policy shock will be more muted than that of Model A.

[25] Figure 1D in the appendix shows impulse responses of additional variables such as the ex-ante and ex-post real interest rates as well as interest expenses.

mainly the ex-post, rather than ex-ante real interest rate that matters for the consumption of constrained households.

Figure 1. Impulse responses to a monetary policy shock



Note: The figure shows the effects of an unexpected increase in the policy rate in different models.
Source: Own calculations

3.2 Monetary policy shock with different levels of debt ratios

A higher debt increases the sensitivity of household consumption to a monetary policy shock. It is intuitive, and it is also clear from Section 2, that an increase in the debt-to-GDP ratio amplifies the consumption response to a policy rate change in all three cases (Models A to C). In this section we compare, quantitatively, the effect of changing the debt ratio in the different models.

In Figures 1E−3E in the appendix we show impulse responses to a monetary policy shock given different levels of the debt-to-GDP ratio. We consider debt-to-GDP ratios of 0.9, 1.9 (baseline) and 2.9. As the debt ratio rises, the transmission of monetary policy shock is amplified in all three cases. Particularly, borrowers become more sensitive to interest rate changes as the debt ratio increases. This result indicates that a higher debt-ratio amplifies the response of interest expense, which in turn magnifies the responses of GDP and inflation. The sensitivity of interest expense is higher in

Model A than in the other two models since Model A takes into account the debt-deflation channel.

To conclude, monetary policy, all else equal, becomes more powerful in an economy with a more highly-leveraged household sector. This conclusion is in line with the findings from Finocchiaro *et al.* (2016) and Di Casola and Iversen (2019). Stockhammar *et al.* (2022) also show empirically that the effects of monetary policy on household consumption has increased as household debt has increased (as the share of loans with a short interest-rate fixation period is large).

4 Macro-shocks and systematic monetary policy

This section analyses the systematic monetary policy response to macroeconomic shocks in the presence of the cash-flow channel. Specifically, we analyse the case of demand and supply shocks. The demand shock is modelled as an increase in the patience of financially unconstrained household and this leads to a drop in aggregate demand and inflation. The supply shock is instead modelled as a positive cost-push shock that leads to an increase in inflation while GDP decreases.[26]

The key findings are as follows. In the case of a demand shock, the response of the financially constrained households' consumption is largest in the model where the real, rather than the nominal, interest rate matters, i.e. when the borrowing limit is defined in real terms. In the case of a supply shock, the response of the financially constrained households' consumption is largest in the model where the borrowing limit is defined in nominal terms.

4.1 Demand shock

A demand shock is defined as a shock that generates a positive relationship between GDP and inflation. In this exercise, the demand shock is modelled as a positive discount factor shock to financially unconstrained households, i.e., an increase in the patience of financially unconstrained households. Monetary policy responds in a systematic manner through the monetary policy rule (see Eq. 18).

[26] Di Casola and Iversen (2019) analyse how monetary policy reacts in a systematic way to macro-exogenous shocks in the context of different debt-ratio levels. We instead focus on the implications of different assumptions about borrowing constraints.

Figure 1F in Appendix F shows the impulse responses of macroeconomic variables to the demand shock in Models A to C. In all three models, a positive shock to the financially unconstrained households' discount factor induces savers to reduce their consumption. In a second step, the drop in consumption has a negative impact on labour demand and thus on the real wage. The decrease in the real wage in turn has a negative effect on the borrowers' consumption via the labour income channel. Finally, the decrease in aggregate demand puts downward pressure on inflation. According to the Taylor rule, the central bank reacts to the decrease in both inflation and GDP by cutting the policy rate. All else equal, the decrease in the policy rate has a positive impact on the financially constrained households' consumption via the cash-flow channel. However the labour income channel dominates the cash-flow channel, thus borrowers reduce their consumption. Overall, total household consumption and GDP both decrease after the negative demand shock.

We now focus on how the consumption of the financially constrained households responds in Models A to C.[27] In Model A, the relevant interest rate expense is determined by the ex-post real interest. Particularly, a decline in the policy rate has a positive effect on household consumption, while a decline in inflation has a negative effect on household consumption. In Model A, the debt-deflation channel implies an increase in the real value of the debt repayment and this amplifies a demand-type shock (consistent with Iacoviello, 2005). In contrast, in Models B and C, the relevant interest expenses are instead based on the nominal interest rate. As a result, the borrowers' consumption, as well as total GDP, decline more in Model A than in Models B and C.

To sum up, when the economy is hit by a demand shock, the response of the financially constrained households' consumption is larger when the real interest rate matters for the cash-flow channel compared to when the nominal interest rate matters. This entails that household consumption is more sensitive if the borrowing limit is defined in real rather than in nominal terms. The policy ramification from these simple models is that monetary policy can more easily stabilise household consumption if borrowing constraints are stipulated in nominal terms.

4.2 Supply shock

A supply shock is defined as a shock that yields a negative relationship between inflation and GDP. This section considers the effects of a positive supply shock in the form of a so-called cost-push shock.

Figure 1G in Appendix G shows the impulse responses of macroeconomic variables to a cost-push shock in Models A to C. In all three models, a positive cost-push shock triggers an increase in inflation. The surge in inflation reduces the real wage and this

[27] Figure 2F in the appendix shows impulse responses of additional variables such as ex-ante and ex-post real interest rates as well as interest expenses.

has a negative effect on borrowers' consumption in all models. In addition, the central bank raises the policy rate in response to the increase in inflation. The positive cost-push shock therefore has a negative impact on borrowers' consumption in all models through both the cash-flow and the labour income channel. Overall, total consumption and GDP both decrease while inflation rises following the positive cost-push shock.

As for the case with the demand shock, we can now focus on how the consumption of the financially constrained households respond in Models A to C. In Models B and C, the relevant interest expenses that determine consumption are based on the nominal interest rate rather than the real rate. In both models, nominal interest expenses increase and as a result, the borrowers' consumption decline more in Models B and C than in Model A.[28] In fact, in contrast to Models B and C, the relevant interest expense in Model A initially declines since it is the real interest rate that matters for households. In Model A, the surge in inflation reduces the real debt payments via the debt-inflation channel. Especially, the ex-post real interest rate determines the interest expense in Model A, while the nominal interest rate determines the relevant interest expenses in Models B and C. As a result, the interest expense in Model A is lower than that of Models B and C. Thus, the response of financially constrained household consumption in Model A is less sensitive to the cost-push shock relative to Models B and C.

The dynamics of the cost-push shock implies a trade-off between returning inflation to the target and stabilising the real economy. Specifically, the central bank reacts to the surge in inflation by raising the policy rate and this action has a negative impact on GDP. This trade-off becomes more or less problematic depending on how financial contracts are designed. If borrowing constraints are specified in nominal terms, as in Models B and C, then the trade-off is amplified as the financially constrained households do not benefit from the debt-inflation channel.[29]

To conclude, the response of financially constrained households' consumption in response to a supply shock is larger when the borrowing limit is specified in nominal terms. The implication for monetary policy is that the central bank can more easily stabilise household consumption in an economy where financial contracts are defined in real terms.

5 Alternative borrowing constraints

There can be other aspects of how loan contracts are designed that affect the transmission from monetary policy to household consumption.

[28] Figure 2G in the appendix shows impulse responses of additional variables such as ex-ante and ex-post real interest rates as well as interest expenses.

[29] See Iacoviello (2005), for the discussion of debt-inflation channel and its implications for supply shocks.

In this section we consider two other types of borrowing constraints and briefly discuss their implications for monetary policy transmission to household consumption.

5.1 Debt-service-to-income constraint

A debt-service-to-income (DSTI) constraint is an example of a constraint that directly links household income to the amount of new debt. A DSTI ratio for households is captured by ω which is a policy parameter that is often set by a macroprudential regulator. Hence, the debt service on new loans B_t^F is not allowed to exceed a share ω of nominal wage income:

$$19.\ i_t B_t^F \leq \omega W_t N_t^F.$$

If we assume that financially constrained households always borrow up to the borrowing limit, $i_t B_t^F = \omega W_t N_t^F$, this specification of the borrowing constraint implies that a change in the nominal interest rate has an immediate effect on the maximum amount of borrowing, for a given nominal wage income. For example, a rise in the nominal interest rate reduces the amount of new loans. Furthermore, a change in the labour income will have a direct effect on the amount of new loans.[30]

Now, we briefly describe the transmission of monetary policy to financially constrained household consumption via this specification of the DSTI constraint. In a general equilibrium framework, a rise in the policy rate has a negative effect on consumption via the interest expense (cash-flow channel) as discussed in Section 2. Given this specification of the borrowing constraint, Eq.(19), monetary policy also has an immediate effect on new loans. Furthermore, monetary policy can have a second-round effect via the borrowing constraint. In particular, the rise in the policy rate will eventually have a negative impact on labour demand and thus lead to a lower labour income. The decrease in the labour income then further reduces the amount of new loans for a given DSTI ratio.[31] The decrease in the amount of borrowing reduces household consumption.

5.2 Loan-to-value constraint

A loan-to-value (LTV) constraint is an example of a constraint that directly links the house value to the amount of new debt. A LTV ratio for households is captured by m which is a policy parameter that often is set by a macroprudential regulator. When borrowing is constrained by a loan-to-value ratio, the amount of new loans may not be greater than the share m of the nominal house value:

[30] For an analysis of DSTI, or payment-to-income (PTI), constraints, and their interaction with LTV constraints, see Greenwald (2018) and Grodecka (2020).

[31] The decrease in the labour income has both direct and indirect impacts on household consumption via labour income and borrowing constraint channels.

$$20.\quad B_t^F \leq mQ_tH_t,$$

where Q_t is nominal house prices and H_t is the housing stock. As in the previous section, we assume that the borrowing constraint is binding, i.e., $B_t^F = mQ_tH_t$. This specification implies that a change in the housing value has a direct effect on the maximum amount of new loans. This effect is called the "housing collateral channel".

Now, we briefly describe the transmission of monetary policy to financially constrained household consumption via this specification of a LTV constraint. In a general equilibrium set-up, a rise in the policy rate directly increases the interest expense (cash-flow channel) and has a negative impact on household consumption. Monetary policy also has a second-round effect via the housing collateral channel. Specifically, a rise in the policy rate will eventually have a negative impact on housing demand and thus lead to a decline in house prices. The effect leads to a decrease in the housing collateral value and this leads to a tightening of the borrowing constraint.

6 Conclusion

We extend the standard New Keynesian framework by incorporating a cash-flow channel and discussing how monetary policy transmits via this channel. We also show how different assumptions on how borrowing constraints are specified can lead to different conclusions about whether nominal or real interest rates matter for household consumption. We further demonstrate that depending on the timing of interest rates in the loan contracts, monetary policy can have a lagged or immediate impact on household consumption. Finally, in these different simple models, the design of financial arrangements (real versus nominal loan contracts) have implications for the central bank's ability to stabilise the economy in response to a supply or demand shock. Particularly, if the economy is hit by a demand shock, monetary policy is more effective in stabilising household consumption if financial contracts are defined in nominal terms, while in the case of a supply shock, monetary policy is more effective in stabilising consumption if financial contracts are set in real terms. Empirically, it therefore matters if loan contracts are specified in nominal or real terms. As we discuss in the introduction, in Sweden, constraints are typically neither purely nominal nor real. If constraints defined relative to house prices or income are infrequently updated, they can for a given time period have the characteristics of a nominal constraint. In that case it is more challenging for monetary policy to stabilise the economy in response to a supply-type shock.

This paper focuses on illustrating the intuition behind the cash-flow channel of monetary policy transmission under different model specifications. Thus, our quantitative implications are based on simple theoretical models and intended for illustrative purposes only. In the future, our aim is to develop a more realistic quantitative model that can be used for monetary policy analyses. For future work, it could also be interesting to analyse the implications of including a cash-flow channel for financial stability and macroprudential policy.

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Appendix

In this appendix, first we present the table with the baseline model calibration. Second, we show details of the theoretical models. Third, we present the impulse responses to a monetary policy shock with varying levels of the household debt-to-GDP ratio. Fourth, we present the impulse responses to demand and supply shocks.

Table 1: Baseline calibration

Parameter	Description	Value	Source
β	Discount factor	0.99	Standard macro-literature
φ	Labor disutility, Frisch elasticity	1	Standard macro-literature
h	Consumption habit	0.75	Standard macro-literature
sf	Share of borrowers	0.35	SELMA
ξ	Indexation to previous inflation	0.75	Standard macro-literature
θ	Calvo probability	0.75	Standard macro-literature
ρ_i	Taylor rule, interest rate smoothing	0.8	Standard macro-literature
ρ_π	Taylor rule, inflation response	1.5	Standard macro-literature
ρ_y	Taylor rule, output reponse	0.125	Standard macro-literature

Appendix A

Model A: Borrowing constraint in real terms

A.1.1 Saver: First-order conditions

This section derives first-order conditions for a saver. Particularly, the household chooses her consumption C_t and labour supply N_t. The utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(lnC_t - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right). \quad \text{(A.1)}$$

The budget constraint in nominal terms is:

$$P_t C_t + i_{t-1} B_{t-1} = B_t + W_t N_t + \mathrm{T}_t. \quad \text{(A.2)}$$

We divide the above budget constraint by the current price level P_t and we have the following budget constraint:

$$C_t + i_{t-1}\frac{B_{t-1}}{P_t} = \frac{B_t}{P_t} + \frac{W_t}{P_t}N_t + \frac{\mathrm{T}_t}{P_t}. \tag{A.3}$$

We let $\frac{B_t}{P_t} = b_t$, $\frac{W_t}{P_t} = w_t$ and $\frac{\mathrm{T}_t}{P_t} = \tau_t$. The household faces the following budget constraint:

$$C_t + \frac{i_{t-1}}{\pi_t}b_{t-1} = b_t + w_t N_t + \tau_t. \tag{A.4}$$

The Lagrangian is:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(lnC_t - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right) + \mu_t \left(b_t + w_t N_t + \tau_t - C_t - \frac{i_{t-1}}{\pi_t} b_{t-1} \right) \right\}.$$

The first-order conditions with respect to $C_{t,}$ b_t, and N_t are as follows, respectively:

$$\frac{\partial \Gamma}{\partial C_t} = \frac{1}{C_t} - \mu_t = 0, \tag{A.5}$$

$$\frac{\partial \Gamma}{\partial b_t} = \mu_t - \beta E_t \left(\mu_{t+1} \frac{i_t}{\pi_{t+1}} \right) = 0, \tag{A.6}$$

and

$$\frac{\partial \Gamma}{\partial N_{U,t}} = -N_t{}^{\varphi} + \mu_t w_t = 0. \tag{A.7}$$

Combining Eq. (A.5) and (A.6), we can find the standard Euler consumption equation:

$$\frac{1}{C_t} = \beta E_t \left(\frac{i_t}{\pi_{t+1}} \frac{1}{C_{t+1}} \right). \tag{A.8}$$

Combining Eq. (A.5) and (A.7), the labor supply equation is written as:

$$\frac{w_t}{C_t} = N_t^{\varphi}. \tag{A.9}$$

A.1.2 Saver: Steady state

Given Eq.(A.8) and dropping the time-subscript, we have the following expression:

$$\frac{1}{\beta} = \frac{i}{\pi}. \tag{A.10}$$

For simplification, we let the gross steady state of inflation π to be 1 and thus the steady state of nominal interest rate can be defined as:

$$i = \frac{1}{\beta}. \tag{A.11}$$

A.1.3 Saver: Log-linearized equations

Financially unconstrained consumption's equation in terms of log-linearization can be written as:

$$\hat{C}_t = E_t(\hat{C}_{t+1}) - E_t(\hat{\imath}_t - \hat{\pi}_{t+1}). \tag{A.12}$$

Financially unconstrained household consumption equation with external habit is expressed as:

$$\hat{C}_t = \frac{h}{(1+h)}\hat{C}_{t-1} + \frac{1}{(1+h)}E_t(\hat{C}_{t+1}) - \frac{(1-h)}{(1+h)}E_t(\hat{\imath}_t - \hat{\pi}_{t+1}). \quad \text{(A.13)}$$

If $h = 0$, we have the standard Euler consumption equation as in (A.12). Note for model simulations, we assume $h > 0$.

Financially unconstrained labour supply's equation in terms of log-linearization can be written as:

$$\widehat{w}_t = \varphi\widehat{N}_t + \hat{C}_t. \quad \text{(A.14)}$$

Financially unconstrained household labor supply equation with external habit is:

$$\widehat{w}_t = \varphi\widehat{N}_t + \frac{1}{(1-h)}(\hat{C}_t - h\hat{C}_{t-1}). \quad \text{(A.15)}$$

If $h = 0$, we have the standard labour supply equation as in (A.14). Note for model simulations, we assume $h > 0$.

A.2.1 Borrower: Consumption

This section derives the borrowers' consumption equation given the borrowing constraint is defined in real terms. The utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta_F^t \left(lnC_t^F - \frac{(N_t^F)^{1+\varphi}}{1+\varphi} \right), \quad \text{(A.16)}$$

subject to the following budget constraint:

$$C_t^F + \frac{i_{t-1}}{\pi_t} b_{t-1}^F = b_t^F + w_t N_t^F. \quad \text{(A.17)}$$

Borrower faces the fixed borrowing constraint\limit in real terms:

$$b_t^F = \overline{b}. \quad \text{(A.18)}$$

Rewrite Eq. (A.17) as:

$$C_t^F = b_t^F - \frac{i_{t-1}}{\pi_t} b_{t-1}^F + w_t N_t^F. \quad \text{(A.19)}$$

Using (A.18) and (A.19), financially constrained household consumption can be written as:

$$C_t^F = \left(1 - \frac{i_{t-1}}{\pi_t}\right)\overline{b} + w_t N_t^F. \quad \text{(A.20)}$$

A.2.2 Borrower: Steady state

Given (A.20) and dropping the time-subscript, we have the following expression:

$$C^F = \left(1 - \frac{i}{\pi}\right)\overline{b} + \mathrm{w}N^F. \quad \text{(A.21)}$$

In steady state, $N = N^F$ and $\frac{1}{\beta} = \frac{i}{\pi}$ .The above equation becomes:

$$C^F = \left(1 - \frac{1}{\beta}\right)\overline{b} + \mathrm{w}N. \tag{A.22}$$

The above equation is divided by Y. The steady state of financially constrained household consumption can be expressed as:

$$\frac{C^F}{Y} = \left(1 - \frac{1}{\beta}\right)\frac{\overline{b}}{Y} + \frac{\mathrm{w}N}{Y}. \tag{A.23}$$

A.2.3 Borrower: Log-linearized consumption equation

Given the financially constrained household consumption (A.20), financially constrained consumption's equation in terms of log-linearization can be derived as follows:

$$C^F \hat{C}_t^F = -\frac{i}{\pi}\overline{b}(\hat{\imath}_{t-1} - \hat{\pi}_t) + wN(\widehat{w}_t + \widehat{N}_t). \tag{A.24}$$

Using $\frac{1}{\beta} = \frac{i}{\pi}$, the above equation can be written as:

$$C^F \hat{C}_t^F = -\frac{1}{\beta}\overline{b}(\hat{\imath}_{t-1} - \hat{\pi}_t) + wN(\widehat{w}_t + \widehat{N}_t). \tag{A.25}$$

We divide Eq. (A25) by output $Y$and we have the following log-linearized version of the borrower's consumption:

$$\frac{C^F}{Y}\hat{C}_t^F = -\frac{1}{\beta}\frac{\overline{b}}{Y}(\hat{\imath}_{t-1} - \hat{\pi}_t) + \frac{wN}{Y}(\widehat{w}_t + \widehat{N}_t). \tag{A.26}$$

A.3 Firm: Log-linearized equations

This section states the log-linearized equations related to firms.

Marginal cost is defined as:

$$\widehat{mc}_t = \widehat{w}_t - \hat{A}_t. \tag{A.27}$$

We assume Calvo sticky prices where a firm (i) is allowed to reset its price with probability $(1-\theta)$ in each period. Moreover, when a firm that is unable to reoptimise, its price is partially indexed to past inflation, i.e. $P_t(i) = P_{t-1}(i)\pi_{t-1}^{\xi}$. Thus, the Phillips curve with inflation inertia equation can be expressed as:

$$\hat{\pi}_t = \frac{\xi}{(1+\xi\beta)}\hat{\pi}_{t-1} + \frac{\beta}{(1+\xi\beta)}E_t\hat{\pi}_{t+1} + \frac{(1-\theta\beta)(1-\theta)}{\theta(1+\zeta\beta)}(\widehat{mc}_t + \hat{\lambda}_t). \tag{A.28}$$

If $\xi = 0$, we have the forward-looking Philips curve. Note for model simulations, we assume $\xi > 0$.

A.4 Monetary policy rule

$$\hat{\imath}_t = \rho_i\hat{\imath}_{t-1} + (1-\rho_i)(\rho_\pi\hat{\pi}_t + \rho_y\hat{Y}_t) + \varepsilon_t^i \tag{A.29}$$

A.5 Market clearing conditions and stochastic processes

Aggregate consumption:

$$\hat{C}_t^{agg} = (1 - sf)\hat{C}_t + sf\hat{C}_t^F \tag{A.30}$$

Aggregate demand:

$$\hat{Y}_t = \hat{C}_t^{agg} + \hat{g}_t \tag{A.31}$$

Aggregate supply:

$$\hat{Y}_t = \hat{A}_t + \hat{N}_t \tag{A.32}$$

Bond market clearing condition:

$$\hat{b}_t + \hat{b}_t^F = 0 \tag{A.33}$$

Shock processes:

Cost-push shock: $\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \hat{\varepsilon}_t^\lambda$ (A.34)

Demand (time-varying discount factor) shock: $\hat{\beta}_t = \rho_\beta \hat{\beta}_{t-1} + \hat{\varepsilon}_t^\beta$ (A.35)

Appendix B

Model B: Borrowing constraint in nominal terms

B1. Borrower: Consumption

This section derives the borrowers' consumption equation given the borrowing constraint is defined in nominal terms.

The budget constraint is:

$$P_t C_t^F + i_{t-1} B_{t-1}^F = B_t^F + W_t N_t^F. \tag{B.1}$$

We divide the budget constraint by the current price level P_t and we have the following budget constraint:

$$C_t^F + i_{t-1}\frac{B_{t-1}^F}{P_t} = \frac{B_t^F}{P_t} + \frac{W_t}{P_t} N_t^F. \tag{B.2}$$

The borrowing limit is fixed in nominal terms:

$$B_t^F = \bar{B}. \tag{B.3}$$

Using (B.2) and (B.3), financially constrained household consumption equation becomes:

$$C_t^F = \frac{\bar{B}}{P_t} - i_{t-1}\frac{\bar{B}}{P_t} + w_t N_t^F. \tag{B.4}$$

B2. Borrower: Steady state

Given (B.4), we assume $N = N^F$ and drop the time-subscript, we have the following expression:

$$C^F = (1-i)\frac{\bar{B}}{P} + wN. \quad \text{(B.5)}$$

We let $\frac{\bar{B}}{P} = \bar{b}$ and Eq. (B.5) can be written as:

$$C^F = (1-i)\bar{b} + wN. \quad \text{(B.6)}$$

Using following relation: $i = \frac{1}{\beta}$ and dividing the above equation by Y, the steady state of financially constrained household consumption is written as:

$$\frac{C^F}{Y} = \left(1 - \frac{1}{\beta}\right)\frac{\bar{b}}{Y} + \frac{wN}{Y}. \quad \text{(B.7)}$$

B3. Borrower: Log-linearized consumption equation

Given the financially constrained household consumption (B.4), financially constrained consumption's equation in terms of log-linearization can be derived as follows:

$$C^F\hat{C}_t^F = \frac{i\bar{B}}{P}\hat{P}_t - \frac{\bar{B}}{P}\hat{P}_t - \frac{i\bar{B}}{P}\hat{\imath}_{t-1} + wN(\hat{w}_t + \hat{N}_t) \quad \text{(B.8)}$$

The above equation can be rewritten as:

$$C^F\hat{C}_t^F = [i-1]\frac{\bar{B}}{P}\hat{P}_t - \frac{i\bar{B}}{P}\hat{\imath}_{t-1} + wN(\hat{w}_t + \hat{N}_t). \quad \text{(B.9)}$$

Using $i = \frac{1}{\beta}$, Eq. (B.9) can be expressed as:

$$C^F\hat{C}_t^F = \left[\frac{1}{\beta} - 1\right]\frac{\bar{B}}{P}\hat{P}_t - \frac{1}{\beta}\frac{\bar{B}}{P}\hat{\imath}_{t-1} + wN(\hat{w}_t + \hat{N}_t). \quad \text{(B.10)}$$

Using $\frac{\bar{B}}{P} = \bar{b}$ and dividing the above equation by Y, we have the following log-linearized version of the borrower's consumption:

$$\frac{C^F}{Y}\hat{C}_t^F = \left[\frac{1}{\beta} - 1\right]\frac{\bar{b}}{Y}\hat{P}_t - \frac{1}{\beta}\frac{\bar{b}}{Y}\hat{\imath}_{t-1} + \frac{wN}{Y}(\hat{w}_t + \hat{N}_t). \quad \text{(B.11)}$$

Appendix C

Model C: Borrowing with the current nominal interest rate

C1. Borrower: Consumption

This section derives the borrowers' consumption equation given the borrowing constraint is defined in nominal terms and is a function of the current nominal rate.

The budget constraint is:

$$P_tC_t^F + i_{t-1}B_{t-1}^F = B_t^F + W_tN_t^F\ . \quad \text{(C.1)}$$

We divide the budget constraint by the current price level P_t and we have the following budget constraint:

$$C_t^F + i_{t-1}\frac{B_{t-1}^F}{P_t} = \frac{B_t^F}{P_t} + \frac{W_t}{P_t}N_t^F. \quad \text{(C.2)}$$

The above equation can be rewritten as

$$C_t^F = \frac{B_t^F}{P_t} - i_{t-1}\frac{B_{t-1}^F}{P_t} + w_t N_t^F. \quad (C.3)$$

The borrowing limit is specified in nominal terms and is varied with the current nominal interest rate:

$$B_t^F = \bar{B}\frac{1}{i_t}. \quad (C.4)$$

C2. Borrower: Steady state

Given (C.3), we assume $N = N^F$ and drop the time-subscript, we have the following expression:

$$C^F = \frac{B^F}{P} - i\frac{B^F}{P} + wN. \quad (C.5)$$

Using $B^F = \frac{\bar{B}}{i}$, the above equation can be written as:

$$C^F = \frac{\bar{B}}{P}\frac{1}{i} - \frac{\bar{B}}{P} + wN. \quad (C.6)$$

We let $\bar{b} = \frac{\bar{B}}{P}$, Eq. (C.6) becomes:

$$C^F = \bar{b}\frac{1}{i} - \bar{b} + wN. \quad (C.7)$$

Using $\frac{1}{i} = \beta$ and dividing the above equation by Y, the steady state of financially constrained household consumption is written as: :

$$\frac{C^F}{Y} = (\beta - 1)\frac{\bar{b}}{Y} + \frac{wN}{Y}. \quad (C.8)$$

C3. Borrower: Log-linearized consumption equation

Given the financially constrained household consumption (C.3), financially constrained consumption's equation in terms of log-linearization can be derived as follows:

$$C^F\hat{C}_t^F = \frac{B^F}{P}\left(\hat{B}_t^F - \hat{P}_t\right) - i\frac{B^F}{P}\left(\hat{\imath}_{t-1} + \hat{B}_{t-1}^F - \hat{P}_t\right) + wN\left(\hat{w}_t + \hat{N}_t\right). \quad (C.9)$$

The above equation can be rewritten as:

$$C^F\hat{C}_t^F = \frac{iB^F}{P}\hat{P}_t - \frac{B^F}{P}\hat{P}_t - \frac{iB^F}{P}\hat{\imath}_{t-1} - \frac{iB^F}{P}\hat{B}_{t-1}^F + \frac{B^F}{P}\hat{B}_t^F + wN\left(\hat{w}_t + \hat{N}_t\right). \quad (C.10)$$

Using $B^F = \bar{B}\frac{1}{i}$, the above equation can be written as:

$$C^F\hat{C}_t^F = \frac{i}{P}\frac{\bar{B}}{i}\hat{P}_t - \frac{1}{P}\frac{\bar{B}}{i}\hat{P}_t - \frac{i}{P}\frac{\bar{B}}{i}\hat{\imath}_{t-1} - \frac{i}{P}\frac{\bar{B}}{i}\hat{B}_{t-1}^F + \frac{1}{P}\frac{\bar{B}}{i}\hat{B}_t^F + wN\left(\hat{w}_t + \hat{N}_t\right), \quad (C.11)$$

and we have the following equation:

$$C^F\hat{C}_t^F = \frac{\bar{B}}{P}\hat{P}_t - \frac{1}{i}\frac{\bar{B}}{P}\hat{P}_t - \frac{\bar{B}}{P}\hat{\imath}_{t-1} - \frac{\bar{B}}{P}\hat{B}_{t-1}^F + \frac{1}{i}\frac{\bar{B}}{P}\hat{B}_t^F + wN\left(\hat{w}_t + \hat{N}_t\right). \quad (C.12)$$

Using $\frac{1}{i} = \beta$, the above equation is written as:

$$C^F\hat{C}_t^F = [1 - \beta]\frac{\bar{B}}{P}\hat{P}_t - \frac{\bar{B}}{P}\hat{\imath}_{t-1} - \frac{\bar{B}}{P}\hat{B}_{t-1}^F + \beta\frac{\bar{B}}{P}\hat{B}_t^F + wN\left(\hat{w}_t + \hat{N}_t\right). \quad (C.13)$$

Given $\hat{B}_t^F = -\hat{\imath}_t$, we can rewrite the above equation as follows:

$$C^F \hat{C}_t^F = [1-\beta]\frac{\bar{B}}{P}\hat{P}_t - \frac{\bar{B}}{P}\hat{\imath}_{t-1} + \frac{\bar{B}}{P}\hat{\imath}_{t-1} - \beta\frac{\bar{B}}{P}\hat{\imath}_t + wN(\widehat{w}_t + \widehat{N}_t), \tag{C.14}$$

and we have the following equation:

$$C^F \hat{C}_t^F = [1-\beta]\frac{\bar{B}}{P}\hat{P}_t - \beta\frac{\bar{B}}{P}\hat{\imath}_t + wN(\widehat{w}_t + \widehat{N}_t). \tag{C.15}$$

We let $\bar{b} = \frac{\bar{B}}{P}$, the above equation becomes:

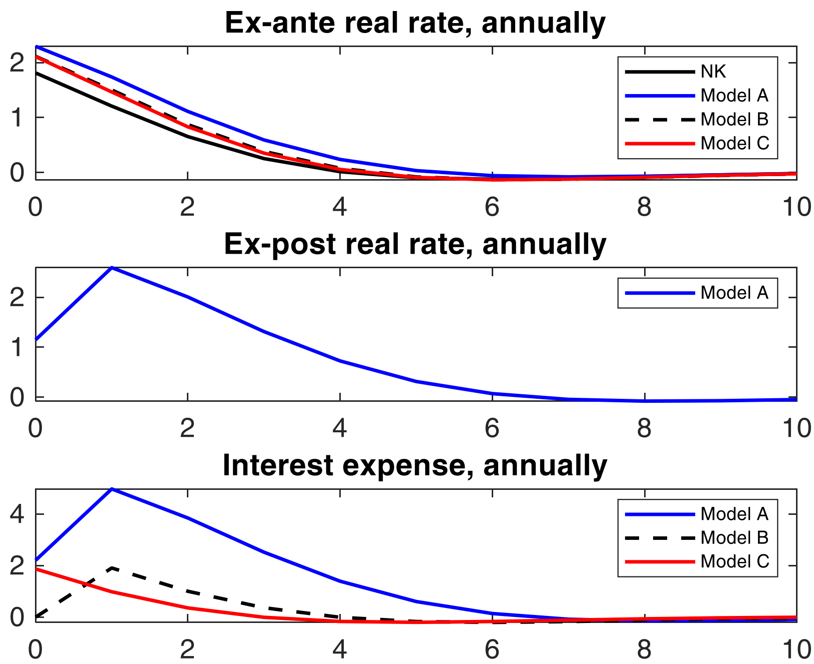
$$C^F \hat{C}_t^F = [1-\beta]\bar{b}\hat{P}_t - \beta\bar{b}\hat{\imath}_t + wN(\widehat{w}_t + \widehat{N}_t). \tag{C.16}$$

We divide the above equation by Y and we have we have the following log-linearized version of the borrower's consumption:

$$\frac{C^F}{Y}\hat{C}_t^F = [1-\beta]\frac{\bar{b}}{Y}\hat{P}_t - \beta\frac{\bar{b}}{Y}\hat{\imath}_t + \frac{wN}{Y}(\widehat{w}_t + \widehat{N}_t). \tag{C.17}$$

Appendix D: Impulse responses to a monetary policy shock

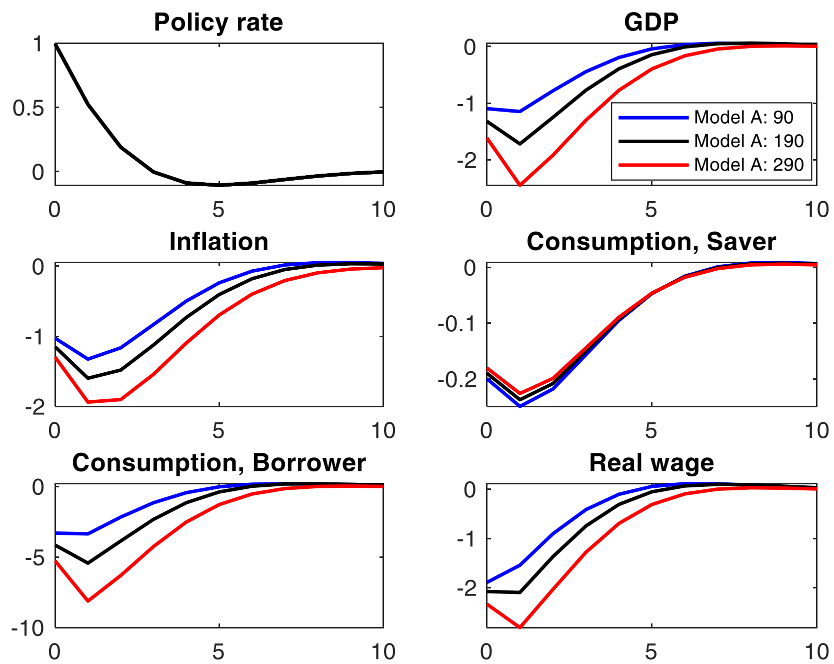
Figure 1D. Impulse responses of additional variables to a monetary policy shock



Source: Own calculations

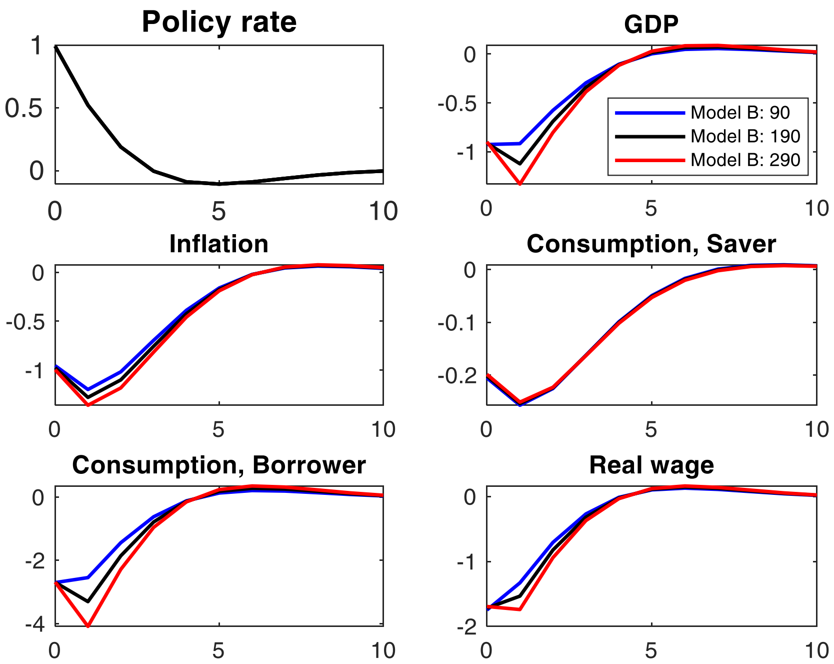
Appendix E: Impulse responses to a monetary policy shock given different levels of debt-to-GDP (90, 190 and 290 household debt as a percentage of GDP)

Figure 1E. Impulse responses for Model A at different levels of debt to GDP



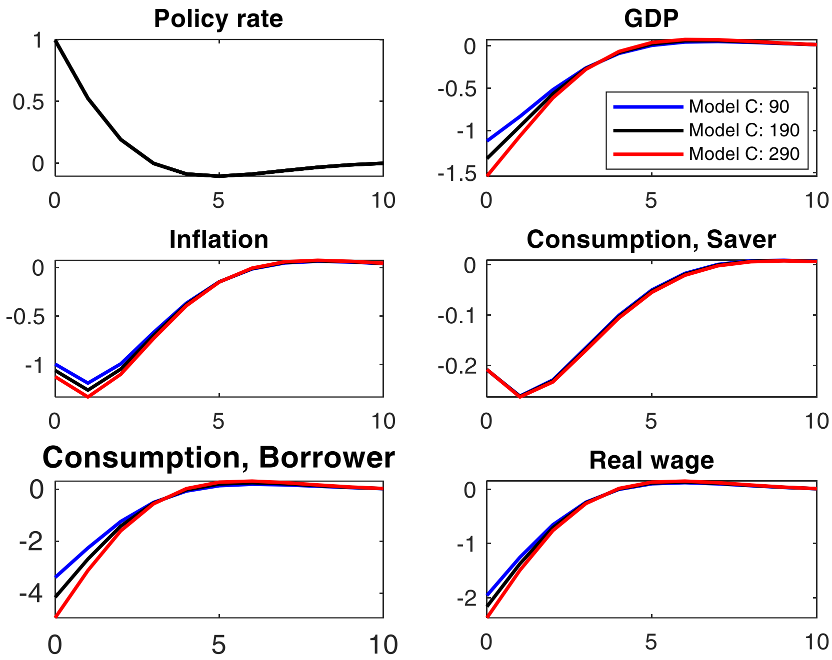
Source: Own calculations

Figure 2E. Impulse responses for Model B at different levels of debt to GDP



Source: Own calculations

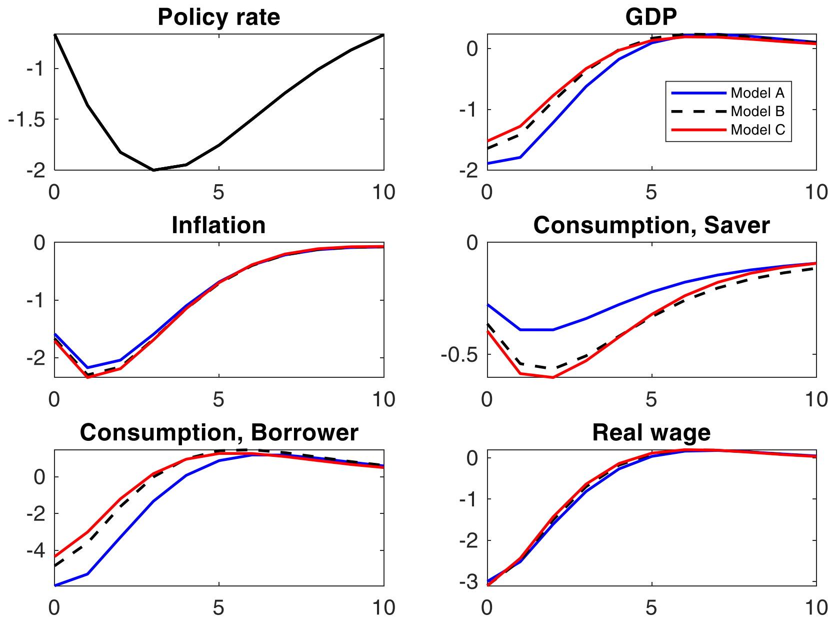
Figure 3E. Impulse responses for Model C at different levels of debt to GDP



Source: Own calculations

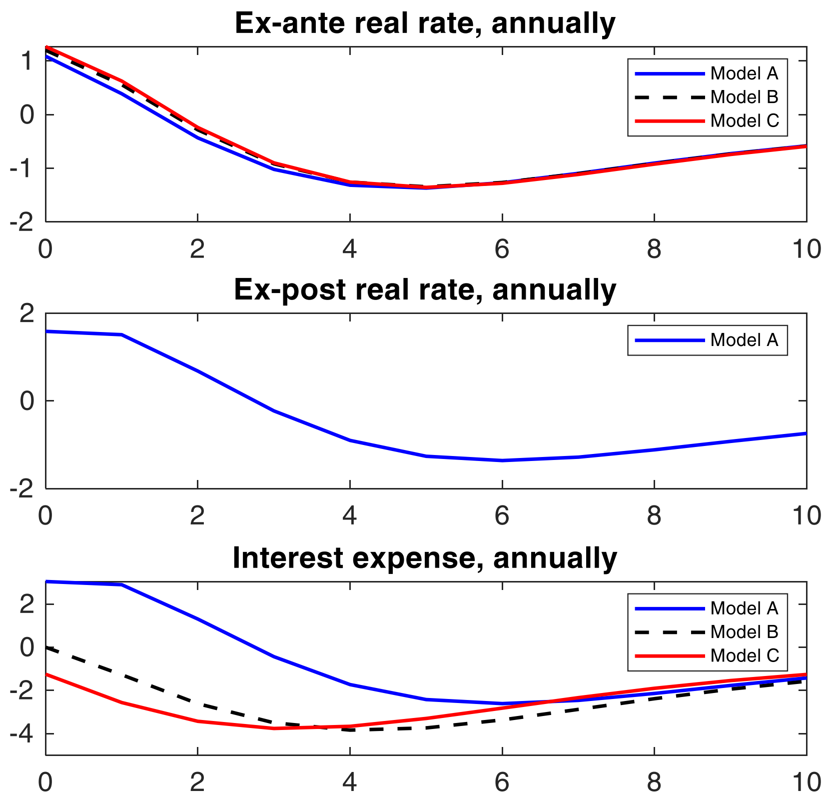
Appendix F: Impulse responses to a demand shock

Figure 1F. Impulse responses of main variables to a demand shock



Source: Own calculations

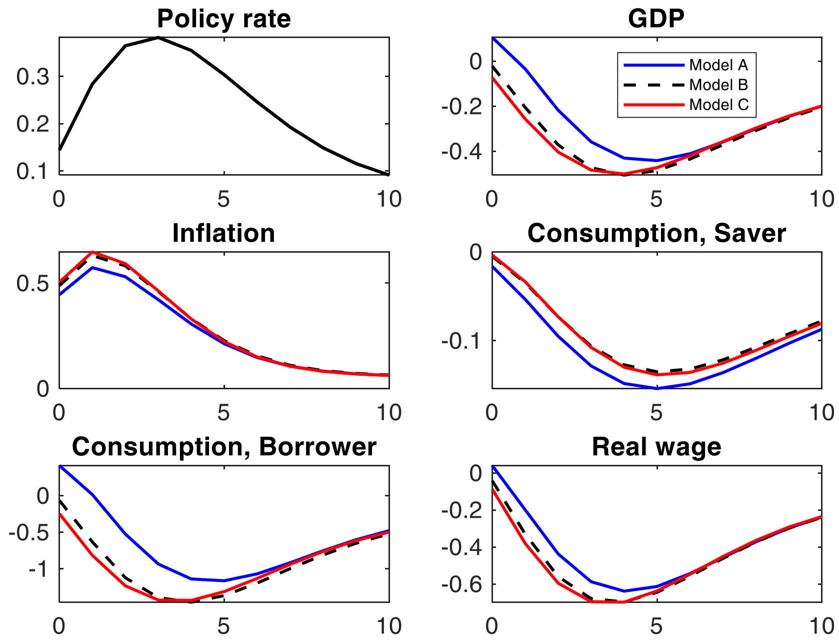
Figure 2F. Impulse responses of additional variables to a demand shock



Source: Own calculations

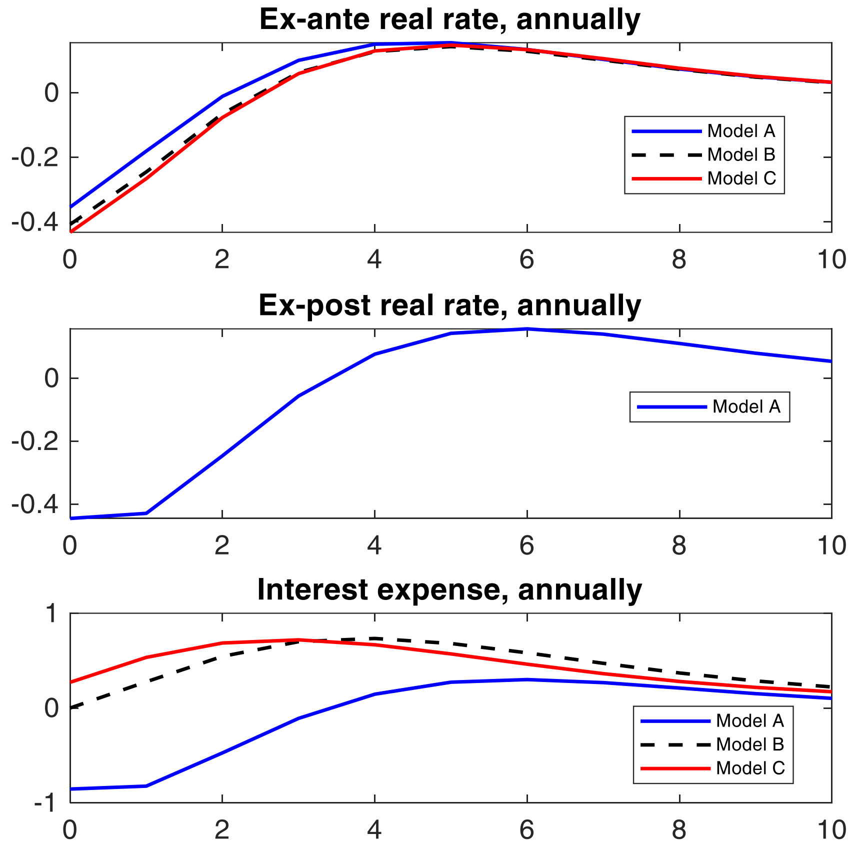
Appendix G: Impulse responses to a supply shock

Figure 1G. Impulse responses of main variables to a supply shock



Source: Own calculations

Figure 2G. Impulse responses of additional variables to a supply shock



Source: Own calculations

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