Staff memo

How much is priced in? Market expectations of monetary policy lift

Ana Maria Ceh

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Contents

Staff Memo

A Staff Memo provides members of the Riksbank's staff with the opportunity to publish advanced analyses of relevant issues. It is a publication for civil servants that is free of policy conclusions and individual standpoints on current policy issues. Publication is approved by the appropriate Head of Department. The opinions expressed in staff memos are those of the authors and are not to be seen as the Riksbank's standpoint.

Summary

Market participants carefully monitor changes in the Riksbank's policy rate and the signals about future policy intentions communicated by the Riksbank. Their expectations of the average policy rate for the next eight quarters are reflected in so-called Riba futures contracts. The objective of this memo is to describe a method for extracting the expected policy actions at upcoming policy meetings embedded into those contracts, as well as the implied hike probabilities.

Author: Ana Maria Ceh, working at the Monetary Policy Department of the Riksbank.[1]

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1 Introduction

Market participants carefully monitor actions taken by central banks. There is abundant media coverage on the topic of central banks and interest rates, in particular during the days around the policy decision meetings. In addition to surveys, that directly report market participants' policy rate expectations, we may infer the anticipated interest rate levels from market participants' actions and extract expectations straight from market prices.[2] Accordingly, many newspaper articles are written trying to interpret specific movements in market prices.[3] Various academic articles have been published aimed at identifying the financial market instruments that are best at predicting the future path of monetary policy.[4] In addition, different tools have been developed and are widely used to try to extract the information from market prices.[5]

But why are these expectations important? As pointed out by Bush et al. 2017: "It is essential for policymakers and financial market participants to understand market expectations for the path of future policy rates because these expectations can have important implications for financial markets and the broader economy." Conveying information is essential for transparency reasons and the efficiency of monetary policy. Understanding the degree of pass-through of monetary policy decisions and communication to the market participants' expectations, may have important effects both on financial markets and on the broader economy. The size of the surprise on decision days will govern the strength of the financial market reaction and may determine the line between orderly and disorderly response.

Ahead of every policy decision, numerous market letters are written trying to predict the upcoming policy action. But insights from analyzing market expectations surpass the purpose of simply predicting the future monetary policy action. The policymaker, despite knowing the action to be taken, is also interested in learning, estimating and interpreting market expectations of policy rate changes at the upcoming meetings. Knowing the perceived likelihood of a certain policy action helps in assessing the market reaction.

The main focus of this memo is on inferring market participants' perceived probability of a policy-rate change at individual Riksbank monetary policy meetings. From raw market prices we can distil the likely policy rate level expected to prevail after each upcoming policy meeting.[6] Riba futures prices, that is prices of the contracts settled against

[2] Kantar Sifo's monthly Prospera survey commissioned by the Sveriges Riksbank (Central Bank of Sweden) reports expectations by Money Market Players of the Riksbank's policy rate 3, 12, 24, 60 months ahead.

[3] See for instance FT Explainer: Interpreting Fed funds futures.

[4] See for example Gürkaynak et al. (2007).

[5] See CME FedWatch Tool or Bloomberg's World Interest Rate Probability (WIRP) function.

[6] In a similar manner as the smooth forward curve allows, we can arrive at the expected policy rate level at any single point in time. Unlike with the fitted forward curve, that assumes the continuous constantly evolving process for the underlying policy rate, allowing for any sized policy change, here we assume the policy rate can shift solely at the very few predetermined dates -- implementation dates, and in predetermined, regular, discrete sized steps, and is therefore better suited to be described by the step-function instead of via the smooth curve.

the average policy rate over the coming eight quarters, can therefore be used to gauge market expectations of the future course for the Riksbank's monetary policy.[7]

The objective of this memo is, more specifically, to describe a method for deconstructing the quarterly averages portrayed in market rates into expected policy actions at the upcoming meetings, embedded into the prices of those traded contracts. This makes it easy to monitor the markets' assessment of the probability of reaching a certain policy rate level at a specific meeting, as well as inferring the underlying probability of a policy rate change.[8] However, the fact that there are four Riba contracts traded during the calendar year and the Riksbank holds five policy decision meetings per year, complicates the process of translating the prices into the expected policy actions.

The remainder of this memo is organized as follows. Section 2 introduces the framework and justifies some of the assumptions needed to extract the information from market prices. Section 3 explains the methodology via a simple illustrative example, while the section 4 reports the results and offers interpretation. Finally, section 5 concludes.

2 Probability tree representation

In order to start deconstructing the expectations behind the prices, we first need to define the decision-making process and policymakers' alternatives. What is a reasonable course of action in terms of direction and size of the policy rate change over the near-term future? Can the policymaker move the policy rate in any direction at any point in time? Is the policymaker in any way constrained, that is, can any policy rate level be reached or only certain levels (is there a lower bound)? To define the framework and narrow down the alternatives, some identifying assumptions are needed and these will be clarified below.[9]

Regarding the near-term future, one reasonable simplification may be to consider only policy rate moves in a specific direction.[10] At present, this assumption suggests considering only the moves in the direction of reducing the policy accommodation provided by the long period of low interest rates.[11] Another assumption is to consider only policy

[7] The Riba futures contract base is a fictitious loan with a term corresponding to the period between two IMM dates, with final settlement occurring against the average repo rate for the period concerned. With the contracts defined in this manner, and the constant availability of eight contracts at every time point, the series duration is approximately twenty-four months. IMM stands for the International Monetary Market. IMM dates refer to quarterly expiration dates of futures contracts. These contracts stop trading the Monday preceding the third Wednesday of each quarterly cycle. This means the third Wednesday of March, June, September, and December. More details about specifics of Riba contracts can be found at Nasdaq.com, Riksbank Futures Product Sheet.

[8] This method is simply one of few used at the Riksbank to analyze movements in market expectations and the implied policy rate levels.

[9] Similar analysis is conducted by Bloomberg and CME Group in their central bank watch tools available for the Federal Reserve. The same analysis is not, however, available for the Riksbank.

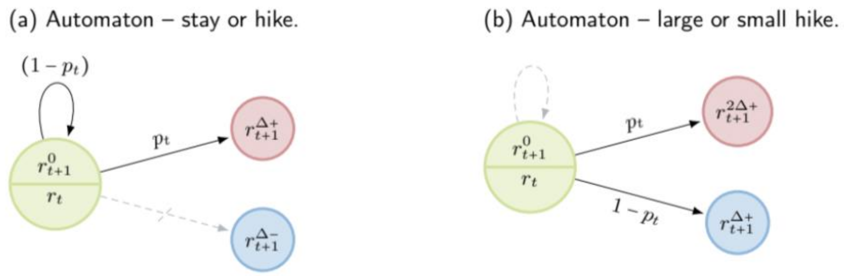
[10] Historically, policy tightening cycles have been highly irregular and very much state dependent, both in terms of the size of the delivered policy tightening as well as being unevenly spaced in time. Recent cycles have, in contrast, had almost a mechanical pace of tightening (in 25 basis points steps). Being patient, orderly and persistent was considered to be both transparent, moderate and state dependent (see Bullard (2015)). The observed regularity of recent cycles offers rationalization to think of the upcoming normalization cycle in a more systematic manner.

[11] This assumption will be relaxed later on.

rate changes of a fixed step size, or multiples thereof. Under the assumptions we make, the policymaker can either keep the policy rate unchanged or raise it by Δ = 25 basis points, or by any multiple of 25 basis points.

Since the policy actions will be taken at a future date, there is no full certainty upon the future course of action, giving rise to a probabilistic view of the policymaker's decision-making process.[12] Figure 1 offers a visual representation of the described decision problem.

Figure 1. States, actions and transition probabilities.



Note. The Figure plots the transition process at each node t = 0, 1, … T of the path tree and the corresponding probabilities of {0, Δ} (left side chart) or {Δ, 2Δ} (right side chart) shift conditional on being at r_t at the end of period t.

The figure shows the permitted set of actions and transition probabilities in a (simplified) case with only two states $\{r^0_{t+1} \equiv r_t; r^{\Delta}_{t+1}\}$, current level of the policy rate r_t and the next period level, r_{t+1}, indicating a policy rate increase of Δ. The permitted set of actions is $\{0; +\Delta\}$, indicating the only two possible actions: *to stay* or *to hike* (Figure 1a). The transition probability p_t is defined as the probability of a rate hike. By assumption, in this simple scenario $-\Delta$ action, that is a rate cut, is not permitted.[13]

In case a large policy hike is warranted, we would want to expand the set of permitted actions. It is possible, however, to set up a somewhat less restrictive process and still allow the ample action space, if instead we focus on delivering a small versus large policy rate shift (Figure 2b), both in the same single direction, making it more suitable to handle current events with central banks delivering large rate hikes.

Building on this setup, we can represent every policy meeting by a set of achievable nodes in a probability tree (at that single point in time) linked together according to the transition properties depicted in Figure 1. At every attainable level (node in the tree), two additional branches in the tree open up – *to stay* and *to hike* (Figure 2).[14] By assumption, at every policy meeting and at every node in the tree, the hike size equals Δ.

[12] In a similar manner that the Riksbank's policy path is the best assessment, it is a forecast, and not a commitment to future policy rate levels.

[13] If rate hikes would be allowed at the same time as rate cuts, the probability of *no change* is undetermined in a process specified in this manner (e.g. expected policy rate level is the same whether the decision maker decides to *stay* with probability 1 or is undecided between hiking and cutting by Δ with equal probabilities).

[14] In case of an expanded action set, the number of branches in the tree that open up at every node should be increased accordingly. In case of large vs. small hikes, we may create the Δ spaced lattice, and move across the nodes in small or large steps. Restricting our attention to certain sized steps assigns zero probability to all other possibilities, and effectively cuts off a number of branches in the tree.

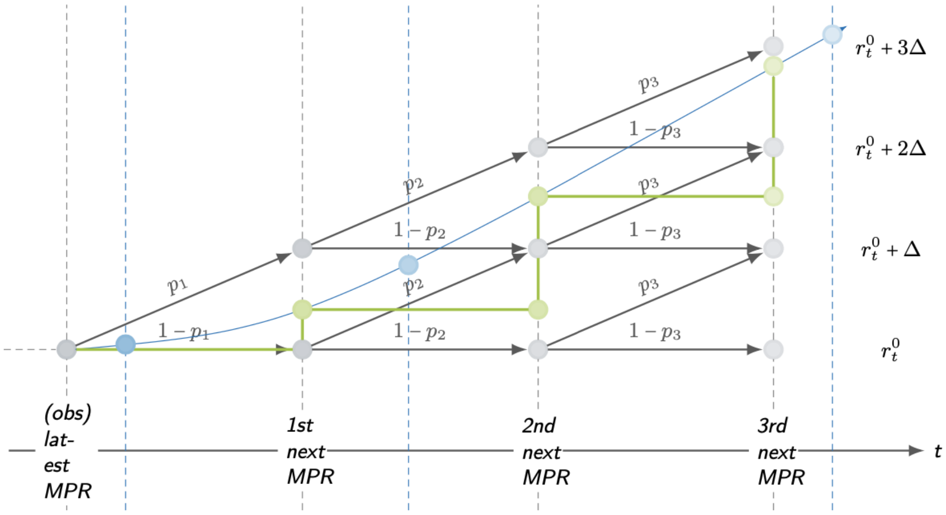
In addition, to simplify the computations, we assume that the probability of any action is independent of the actual policy rate level. This assumption reduces the hike probabilities to a function of time (policy meeting), but not a prevailing level at that point in time. Put differently, at a specific policy meeting, there is a single probability of hiking the policy rate regardless of the prevailing level just ahead of the meeting. Even though this may be a very restrictive assumption, it follows an intuition of focusing on relative rate changes from quarter to quarter to determine the hike probabilities from one meeting to the next.[15] Indirectly, the steepness of the path, together with the constraint on the probability measure to be between zero and one, automatically precludes small hikes, in case large hikes are warranted. Therefore, linking the individual contracts together provides an additional identifying assumption.

To simplify the graphical exposition of the problem, we will focus on the two action-set binary tree, with only a single decision to be made at each of the upcoming policy meetings. This implies that as soon as we decide on the probability of hike p_t, we immediately pin down the probability of the only other possible action. It should also be clear that this simple representation implicitly assumes no room for breaking the policy tightening cycle - reversing any of the decisions made is not possible in this framework.

Nevertheless, this setup can be easily extended to a somewhat more realistic situation corresponding to recent developments, multiple Δ hikes in a single direction (Figure 1b), with implementation details described in more details in Appendix B. Note that in addition, the flexibility of moving in a single direction may be enough to cover the general case. Any curve that changes the slope mid-range can be split into multiple parts so as to cover only monotone sloping curves. Every part of the curve can then be analyzed separately as an upward or downward sloping curve using the decomposition employing single direction moves. Therefore, splitting the Riba curve into monotone sloping parts can help to address the issue at hand by allowing for the direction of policy change to be switched as the slope changes sign.

[15] Due to the independence assumption, we can assign a single probability p_t to every node in the tree that is attainable at time t.

Figure 2. Probability tree and implied policy rate paths.



Note. The Figure plots the policy rate path tree at times $t = -1, 0, 1, 2, \dots$ with corresponding transition probabilities and implied policy rate levels.

These linked action scenarios can be illustrated with paths through a (binary) tree. We start from the current policy rate level (decided at the previous policy meeting) and an unobservable continuous curve representing market expectations for future policy rates (blue curve in Figure 2). At the next meeting we expect no change in the policy rate with probability $(1 - p_1)$ and a policy shift of magnitude Δwith probability p_1. Moving forward to subsequent meetings, at each tree node resulting from the possible action taken at the meeting before, another decision has to be made. The probability of staying at each level is $(1 - p_2)$, regardless of the level itself, while the probability of a shift is p_2.[16]

The same argument is repeated at every step going forward, making it possible to build a tree spanning any arbitrary number of meetings. This specific tree structure, equally-spaced nodes together with the assumption of level-independence for transition probabilities, allows for a recursive representation of the transition probability process. The recursion and the formal derivation are given in Appendix A, but can be explained intuitively as follows. To infer the probability of the policy rate being at a certain level at a specific meeting, it is enough to know the probability of reaching the previous level in one less meeting, together with the probability that the policy rate was changed prior to the current meeting.[17]

Knowing the current policy rate level with full certainty, the recursive argument described above allows us to compute the probability of the policy rate reaching any level at the time of a specific policy decision meeting. Finally, the probability of a rate hike at

[16] Here, an additional identifying assumption is needed in order to be able to uniquely pin down the implied probabilities at this node. While different assumptions could be rationalized, the independence assumption made before corresponds to an agnostic prior assuming that the hike probability at this point in time remains identical regardless of the node level, that is regardless of the outcome of the previous meeting.

[17] See the probability independence assumption and the previous footnote.

that specific meeting is calculated by adding up the probabilities of all the rate levels above the current policy rate.

The end goal is to find the transition probabilities at every meeting and the implied step function (green curve in Figure 2) that is allowed to shift only on implementation dates and is such that the probability weighted average of the attainable nodes aggregated over each quarter (grey tree nodes in figure 2) is equal to the corresponding quarterly Riba price.[18] This can be formulated as a probabilistic optimization problem with a predetermined step size (details are given in Appendix B). The solution directly gives the probability of a rate shift at each of the upcoming policy meetings, while an unconditional probability of a rate hike is calculated by adding up the probabilities of all policy rate levels above the current rate.

It is important to note that although the problem described results in the distribution over the attainable nodes, this distribution is identified under very restrictive assumptions imposed. This distribution should be interpreted as a distribution over likely levels to be attained under the restrictive process describing the architecture of the Riba contracts (see footnote 7), and not as the underlying distribution that market participants had in mind when entering into the trading agreements of those contracts.

3 Input: Riba contracts

In this exercise we will be using financial instruments that are settled against the policy rate for the period under consideration, so called Riba contracts. Riba futures prices reflect the average expected policy rate for the coming two years.[19] The market prices of Riba contracts can thus be seen as a market equivalent to the Riksbank's projected policy rate path, ultimately portraying market participants' expectations of the Riksbank's upcoming policy actions.[20,21]

The goal of this exercise is to find the probability weighted combination of the beginning and end of quarter policy rate levels corresponding to the average represented by the price of the Riba futures contract. Since Riba contracts are settled quarterly and span the period over one IMM-quarter, there are four active contracts traded throughout the calendar year.[22] However, the Riksbank holds five regular monetary policy meetings per year at which the policy rate is set. Therefore, at least one Riba contract

[18] Alternatively, the step function could be defined as a solution to a problem to minimize the distance to the fictitious continuous curve (blue curve in Figure 2) constructed as some interpolation between nodes representing quarterly averages.

[19] This is based on the architecture of Riba contracts. For details see footnote 7.

[20] The Riksbank's Executive Board is responsible for determining the official interest rate levels in Sweden. The Board meets five times per year to set the Riksbank's official policy rate, the base rate according to which all other interest rates in the economy will be adjusted. In addition to the communication by the Board members, the Riksbank complements its forward guidance by publishing the policy rate forecast. The forecast indicates the optimal policy rate and stance needed to reach the inflation target in the upcoming years, while not compromising the growth potential.

[21] Similar analysis can be executed with the Riksbank's published policy rate path. In that case, instead, the analysis conveys probabilities of the policy rate levels implied by the Riksbank's own published path, therefore portraying its own expectations of the best future course of action that will be needed to bring inflation to target and help the economy achieve its potential in the medium run.

[22] See footnote 7 for definition of IMM-quarters.

inevitably covers a period with two policy rate meetings.[23] Based on the current schedule with five monetary policy meetings per year,[24] the typical decision dates are in mid-February, late April, early July, late September and late November. Implementation dates for the policy rate decisions made at those meetings occur on the following working-day Wednesday.[25]

In addition to market prices and contract details such as settlement date and start of the contract, we will also need the specific dates of the Executive Board meetings. In practice, the relevant dates are the implementation dates, corresponding to the dates when the announced policy rate is effectively implemented.[26] All information and necessary input are summarized in Table 1 with further details described below.

Table 1. Riba futures and implied policy path on November 22, 2022 (with supplementary information).

IMM quarter (k)	average rate ($riba_k$)	IMM settlement date ($k0$)	Publication day 1 ⟶ Implementation ($k1$)	Publication day 2 ⟶ Implementation ($k2$)	Days in quarter (N_k)	Meeting day 1 (M_{k1})	Meeting day 2 (M_{k2})	Remaining days in quarter ($N_k - M_{k1} - M_{k2}$)	quarter-start rate ($start_k$)	k-implied rate 1 (mid_{k1})	k-implied rate 2 (mid_{k2})	quarter-end rate (end_k)
-	-	2022-Sep-20	Sep-20 → Sep-21, 2022				$policyrate^{now}$: 1.75					
1	1.93	2022-Dec-20	Sep-20 → Sep-21, 2022	Nov-24 → Nov-30, 2022	91	00	70	21	1.75	1.75	2.52	2.52
2	2.68	2023-Mar-14	Feb-09 → Feb-15, 2023	-	84	56	00	28	2.52	3.00	-	3.00
3	3.09	2023-Jun-20	Apr-26 → May-03, 2023	-	98	49	00	49	3.00	3.18	-	3.18
4	3.20	2023-Sep-19	Jun-29 → Jul-05, 2023	-	91	14	00	77	3.18	3.21	3.14	3.21
5	3.18	2023-Dec-19	Sep-20 → Sep-27, 2023	Nov-24 → Nov-29, 2023	91	07	63	21	3.21	3.19	-	3.14
6	3.12	2024-Mar-19	Feb-09 → Feb-14, 2024	-	91	56	00	35	3.14	3.08	-	3.08
7	3.06	2024-Jun-18	Apr-26 → May-08, 2024	-	91	42	00	49	3.08	3.03	-	3.03
8	3.03	2024-Sep-17	Jun-29 → Apr-03, 2024	-	91	14	00	77	3.03	3.02	-	3.02

Note. The table shows publicly available policy publication/implementation dates, augmented with the assumed publication/implementation dates spanned by the forecasting horizon. In addition, it reports Riba futures prices and the implied policy rate levels across IMM-quarters.

Source: Bloomberg L.P. and author's calculations.

[23] In contrast, Fed Funds Futures are settled monthly, and every monthly Fed Fund Future contract covers at most one FOMC meeting. As the FOMC decides eight times per year, some contracts naturally cover the period without any meetings.

[24] Prior to 2020, the Executive Board of the Riksbank held six regular monetary policy meetings per year at which the policy rate could be adjusted.

[25] The majority of decisions delivered at the respective meetings are reflected in the corresponding IMM-quarter Riba contract, with the exception of the September meeting, which is due to the late implementation date embedded into the December Riba contract, together with the November meeting.

[26] Publication dates are typically published on the Riksbank's website for the coming year. However, Riba contracts cover the period of two years ahead. To cover the two-year period spanned by Riba contracts, the available Riksbank calendar is extended by adding an additional year on currently published meeting dates, and focus on the following working-day Wednesday as the implementation date. Typically, publication dates set in this manner may be a day or two out compared to the actual meeting dates. Implementation dates, that are relevant in our valuation exercise, are closely matched by this algorithm.

Let $(\boldsymbol{riba_t^k}, k = 1, \dots, 8)$ denote daily price fix (at date t) for the corresponding eight Riba contracts. Let $\boldsymbol{start_k}$ and $\boldsymbol{end_k}$ denote the unobserved policy rate level implied by the Riba price at the beginning and end of k-th Riba quarter. Let $\boldsymbol{mid_{k1}}$ be the implied policy rate level after the first meeting date within k-th Riba quarter, $\boldsymbol{mid_{k2}}$ implied policy rate level after the second meeting date within k-th Riba quarter (could also be empty, in case of only single meeting per quarter). Define N_k to be day-length of k-th riba quarter (day-distance between two IMM-quarters), M_{k1} day-distance between start of k-th Riba quarter and the first policy meeting within the same quarter and M_{k2} day-distance between the two consecutive policy meetings within the k-th Riba quarter (or zero). Then, $N_k - M_{k1} - M_{k2}$ is the remaining day-distance between end of k-th Riba quarter and the last policy meeting within the same quarter. The initial value entering into the first contract is the latest known and observed policy rate level. All future policy rate levels at all the upcoming meeting outcomes are unobserved and are to be determined.

The Table 1 systematically collects observable data, together with the unobserved information computed via the algorithm described in Appendix B.

We will use change in prices of eight consecutive Riba contracts to construct a meeting-to-meeting probability matrix of interest rate changes. Combining these together, via the recursive representation derived in Appendix A, it is possible to obtain the probability of a rate change at each meeting covered by those traded contracts.

According to our assumptions, any upcoming meeting has two possible outcomes: either the rate will be kept unchanged or, if it changes, it will be lifted by Δ = 25 basis points (or some multiple of Δ). In addition, the structure of the probability tree implies that meetings further ahead in time have a higher number of attainable level outcomes (see Figure 2).[27]

The algorithm, capturing the reasoning above, prescribes the routine to determine the unobserved values, coloured in **blue** or **green** in the Table 1. The data printed in **black** is publicly available information either published on the Riksbank's website (publication dates), computed via simple algorithm (implementation dates) or given by the Riba contract term sheet and financial market standards (IMM quarters and settlement dates).

On any given date, a set of eight Riba contracts are traded, representing the average expected policy rate over the following eight IMM quarters, coded in **red** in Table 1. The blue and green values are unobserved and correspond to implied policy rate levels on IMM-quarter-end dates and on implementation dates between those dates. These levels are derived via the algorithm described in the Appendix B.[28]

[27] CME FedWatch Tool in a similar fashion analyses the probability of FOMC rate moves for upcoming meetings using 30-Day Fed Fund futures pricing data.

[28] By replacing the market rates captured in Riba futures with central bank's projected policy rate path, in the same fashion one can extract policymaker's expectations and signals directed towards market participants.

4 Results: Riba-implied policy path

The optimization programme employed to process the information collected Table 1 is described in details in Appendix B, and is used to extract probabilities of policy rate change at every meeting.

The goal is to find the probability vector p_t and step size vector n_t (corresponding to large shift as a n_t multiple of Δ) to minimize the distance between the quarterly average implied by the step function shifting only on implementation dates and the quarterly average implied by Riba prices. The solution to the programming problem is the vector of hike probabilities at every meeting, together with the size vector of the implied Δ-hike.

As a result, all implied beginning, end and mid-of-quarter nodes ($\boldsymbol{start_k}, \boldsymbol{mid_{k1}}, \boldsymbol{mid_{k2}}, \boldsymbol{end_k}, k = 1, \ldots, 8$) are such that the following holds,

$$riba_k = \frac{M_{k1}\, start_k + M_{k2}\, mid_{k1} + (N_k - M_{k1} - M_{k2}) \left(\mathbf{1}_{\{M_{k2}>0\}} mid_{k2} + \mathbf{1}_{\{M_{k2}=0\}} mid_{k1} \right)}{N_k}, k = 1, \ldots, 8.$$

In addition, implied policy rate levels at individual nodes are defined as follows,

$$start_k = \begin{cases} policyrate^{outc}, & \text{if } k = 1 \\ end_{k-1}, & \text{if } k = 2, \ldots, 8 \end{cases}$$

$$end_k = \begin{cases} start_k + (n_{k1} + n_{k2})\Delta, & \text{if } M_{k2} > 0, \\ \quad \text{with } mid_{k1} = start_k + n_{k1}\Delta, & \\ \quad \text{and } mid_{k2} = start_k + (n_{k1} + n_{k2})\Delta, & \\ start_k + n_{k1}\Delta, & \text{if } M_{k2} = 0, \\ \quad \text{with } mid_{k1} = start_k + n_{k1}\Delta, & \\ \quad \text{and } mid_{k2} = \varnothing & \end{cases}$$

due to assuming step function interpolation between the nodes.

After having extracted the transition probabilities, the implied best-fit step function is a convex combination of neighbouring Δ-nodes.[29] The extracted probabilities corresponding to the attainable levels at every meeting are given in the heatmap in Figure 3.[30]

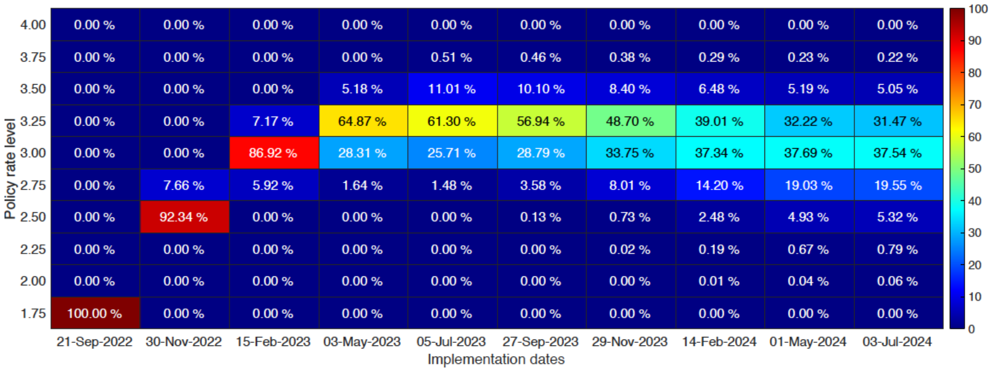
One caveat to interpreting these paths as a pure market expectation is the risk premium component embedded into Riba prices (possibly the term premium and/or liquidity

[29] The implied best-fit step function is the solution to the optimization program set in Appendix B. Effectively, this translates into finding the implied step function that is minimizing the average quadratic deviations from Riba quarterly averages.

[30] The heatmap label is used here to denote the color-coded probability matrix, making it a helpful tool in the visual representation of the results.

premium), which, though small, may still have an effect, especially further ahead in time, and can potentially skew the true underlying distribution.[31,32]

Figure 3. Heatmap. Probability of policy rate levels at the upcoming meetings for the Riba implied policy path (implied by Riba futures on November 22, 2022.)



Note. Information in the heatmap above should be read as follows: at every upcoming policy meeting (implementation date), the probability of being at the implied policy rate level is given at the corresponding row-column cross-section in the matrix. Probability of a rate hike at a certain meeting is calculated by adding up the probabilities of all policy rate levels above the current rate level. Probabilities of possible policy target rates are based on Riba futures prices assuming that the policy rate can either be left unchanged or increased by either25 basis points, or a multiple of 25 basis points, and ignoring the possibility of any inter-meeting moves in the target range.

Source: Bloomberg L.P. and author's calculations.

The heatmap chart above reports the whole implied distribution across all attainable levels at every policy meeting over the next two years (the time range covered by Riba contracts).[33] At each row-column cross-section, one can read the probability of reaching a certain level (row) by the time of the implied policy meeting (column). The probability of a rate hike at a certain meeting is then calculated by adding up the probabilities of all policy rate levels above the current meeting rate level.

The information displayed in Figure 3, on November 22, 2022, tells the following story: the policy rate had been lifted in September to 1.75 percent. At the next decision meeting in November, the policy rate change is a mere certainty, with above 92 percent probability of a 75 basis points hike and close to 8 percent probability of a 100 basis points hike. By the end of the year, the policy rate will likely end at 2.50 percent, with

[31] The same premium is not embedded into survey expectations, which are stripped of those by construction. See Kim and Tanaka (2016).

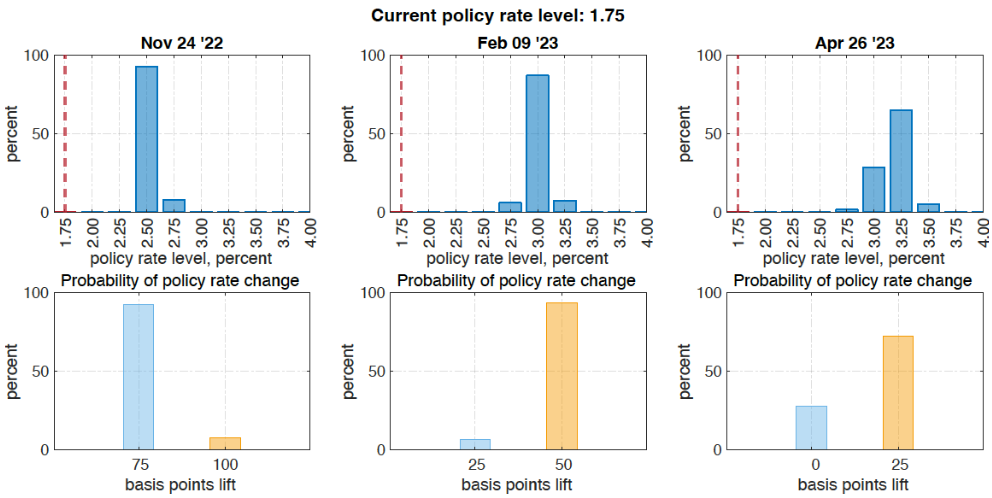
[32] In comparison to the Fed Fund Futures market, Riba futures are not frequently traded contracts. Even though Riba futures prices are regularly quoted, actual transactions are taking place about once per week. Using the number of daily reports submitted by the Riksbank's counterparties as a proxy for the number of trades recorded per day, we could compare the "liquidity" between various money market instruments. In comparison to Riba futures, where in 2022 we have on average one recorded transaction every four days, trading in interest swap contracts is reported three to four times per day. For more details, see the Riksbank's reporting of turnover statistics in the Swedish money and bond market from the Riksbank's counterparties.

[33] Due to the imposed tree structure (see Figure 2 in section 2), at every attainable node per meeting, another two open up as a result of an uncertain future action. Therefore, unless the outcome of every future meeting is expected with full certainty, the distribution captures more than just two level outcomes beyond the first coming meeting.

a smaller probability in favour of higher level, and is unlikely to stay at 1.75 percent (the final decision meeting of 2022 is in November). Additional 50 and 25 basis points shifts are expected in the first half of 2023, with a maintenance of the policy rate in the 3.00 – 3.25 range in the late 2023 and early 2024.

The Figure 4 reports the content extracted from the heatmap with a focus on the next few decision meetings. The most likely policy action sequence expected by market participants across the coming few meetings is to lift the policy rate by 75 basis points in November, and to follow this up by an additional 50 basis points tightening in February and another 25 basis points in April (blue and yellow columns in the bottom row charts in Figure 4). In addition, at November meeting there is a positive probability for even more policy tightening than what is expected in this most likely case (yellow columns in the bottom row charts in Figure 4 indicating the probability of larger policy rate shift) which is then counterbalanced by a positive probability over smaller policy lifts in the first half of 2023. Redistributing these actions across the next few meetings results in the distributions shown in the top row charts in Figure 4, suggesting that by the spring of 2023 the rates are expected to reach the 3.00-3.25 level range.

Figure 4. Probability of monetary policy lift over the upcoming few policy meetings (implied by Riba futures on November 22, 2022).

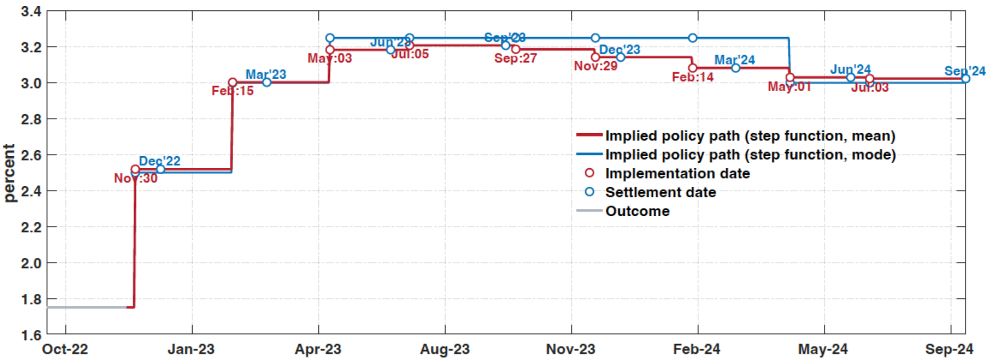


Note. Red vertical line (in the top row charts) denotes the current policy rate level. Blue bars in the top row charts represent the probability of likely policy rate levels on the respective decision date. Bars in the bottom row charts report on the probability of policy rate lift of a certain size (implied by Riba contracts on the respective date).

Source: Bloomberg L.P. and author's calculations.

In addition, the information content behind Figure 3 is summarized as a probability-weighted average of attainable states and is given by the implied step function in Figure 5, corresponding to the expected policy rate path.

Figure 5. Policy rate lift implied by mean and modal path (implied by Riba futures on November 22, 2022).



Note. Policy rate lift implied by mean and modal path indicates the difference between the expected and the most likely outcome at each meeting.

Source: Bloomberg L.P. and author's calculations.

Alternatively, instead of focusing on the expected path we could think of the most likely outcome at each meeting, simply choosing the level that is assigned the highest probability at each point in time (as suggested in Figure 4). Such an exercise will result in the modal implied path, and the resulting modal step function in Figure 5. Jointly, the two step functions in Figure 5 tell the story of the distribution in Figure 3 being skewed towards higher levels, as the mean (red curve) is almost always below the mode (blue curve).[34]

To sum up, the whole profile of the implied policy rate path (corresponding to the heatmap in Figure 3) suggests front loaded hikes over the next few meetings, only to maintain the policy rate in the neighbourhood of 3.25 percent level afterwards. According to the most likely outcome scenario (in Figure 5), 2022 is expected to end with the policy rate at 2.50 percent with the additional tightening throughout the first half of 2023, after which the policy rate should stabilize in the range of 3.25 percent over the following year.

5 Summary

This memo discusses a method to extract market participants' expectations of the policy action at the upcoming decision meetings, and their perceived probability of a policy rate shift, directly from market prices. The methodology is set jointly with assumptions needed to identify the probabilities in question. A caveat is that premiums likely embedded in Riba contracts may cloud the pure measure of expectations. Nevertheless, since the focus is typically on the short to medium term, these premiums should generally be small.[35]

[34] It is important to note here that the resulting distribution is largely influenced by the problem design and the identifying assumptions that were made along the way.

[35] The premiums contained in futures are typically small, with an order of magnitude of just a few basis points. Fed's estimates of the front-end term premiums have been about 1 basis point per month, but have flipped the sign to -1 basis point in recent years. See Dierkcs and Karl (2019).

The method is described via a simple example and is applied to Riba pricing on November 22, 2022, few days ahead of the November monetary policy meeting. The decomposition into discrete step shifts allows for a discussion about hike probabilities at a specific meeting, but even more, the likelihood of reaching a certain level by the specific date. While discussion on hike probabilities seems more relevant for the few upcoming policy meetings, over the medium to longer term the likelihood of reaching a certain level seems more appropriate. Statements as *market participants expect the policy rate to reach certain level by the end of 2022* are directly discernible from the heatmap while simultaneously pointing to the path time profile and speed required to reach that level. In addition, the heatmap illustrates the implied distribution over discrete rate levels at every meeting, which makes it suitable to infer the *implied probability of a hike at the meeting of interest*. The heatmap is therefore a useful tool in dissecting a simple quarterly average number into pointed conclusions on policy rate level expectations coming directly from market pricing.

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A APPENDIX – Transition probabilities

Let T be the number of policy meetings covered by traded Riba contracts and let $\{X_t, t = 1,2, \dots, T\}$ be the process counting rate hikes across meetings, p_t the probability of a larger policy rate shift at each meeting, and $(1 - p_t)$ the residual probability of a smaller shift.[36] Then $p_t = P(rate_{t+1} = rate_t + X_t\Delta)$ and the *probability of having exactly k hikes at time t* is given by

$$\mathbb{P}(X_t = k) = \mathbb{P}(rate_{t+1} - rate_t = k\Delta) = \begin{cases} p_t, & k = n_t, \text{ (large hike)}, \\ 1 - p_t, & k = n_t - 1, \text{ (small hike)}, \\ 0, & k \notin \{n_t - 1, n_t\}, \text{ (otherwise)}. \end{cases}$$

with $(p_t, n_t), t = 1, \dots, T$ as the probability of hike and the optimal large hike size at time t (solution to the optimization problem in Appendix B). In a general case, probabilities p_t could depend on the current state (policy rate level at the time of the meeting t) but for identification purposes, we assumed it to be only a function of time/meeting, and independent of the current policy rate level.

Now we can compute the *probability of having exactly k hikes by the time t*, $P(\{Y_t = k\}) = P\left(\left\{\sum_{j=1}^{t} X_j = k\right\}\right)$. First, we initialize the probabilities at time t = 1 (i.e. the first coming policy meeting),

$$s_1^k = \mathbb{P}(\{Y_1 = k\}) = \mathbb{P}(\{X_1 = k\}) = \begin{cases} p_1, & k = n_1, \\ 1 - p_1, & k = n_1 - 1, \\ 0, & k \notin \{n_1 - 1, n_1\} \end{cases}$$

which we will use to compute the desired probability of having exactly k hikes at the time of the t policy meetings ahead

$$\begin{aligned} s_{t+1}^k = \mathbb{P}(\{Y_{t+1} = k\}) &= \mathbb{P}\left(\{X_t = n_t, Y_t = k - n_t\} \cup \{X_t = n_t - 1, Y_t = k - n_t + 1\}\right) \\ &= \mathbb{P}(\{X_t = n_t\}) \cdot \mathbb{P}(\{Y_t = k - n_t\}) + \mathbb{P}(\{X_t = n_t - 1\}) \cdot \mathbb{P}(\{Y_t = k - n_t + 1\}) \\ &= p_t \cdot \mathbb{P}(\{Y_t = k - n_t\}) + (1 - p_t) \cdot \mathbb{P}(\{Y_t = k - n_t + 1\}) \\ &= p_t \cdot s_t^{k-n_t} + (1 - p_t) \cdot s_t^{k-n_t+1} \end{aligned}$$

[36] Alternatively, p_t can be the probability of a policy rate change at every meeting, with $(1 - p_t)$ being a probability of no change. This can be viewed as a special case of the process described, with the large hike size, n_t, equal to one, and the resulting small hike size, $n_t - 1$, equal to zero.

B APPENDIX – Riba-implied policy rate path

The algorithm imports quarterly Riba contracts and produces an implied policy rate path by preserving quarterly average suggested by Riba prices.[37] All the intermediate results are collected in Table 1 in the main text.

Step 1. Input - import data:

- Riba futures average rates over the following eight IMM quarters (set of eight contracts, $\boldsymbol{riba_t^k}, k = 1, \dots, 8$) at a fixed date t,
- policy rate (daily) outcome (last observation),
- publication dates (current year calendar published on the Riksbank's website),
- settlement dates (IMM end-of-quarter) and business dates array (trading calendar published by NASDAQ OMX and adhering to international money market standards),
- implementation dates (for missing dates, add a year to the currently available publication dates, extend until the final IMM end-quarter date, and set the first following business day Wednesday as an implementation date),
- define start, end and mid-of-quarter (implementation) dates $\boldsymbol{k}, \boldsymbol{k1}, \boldsymbol{k2}$ for $k = 1, \dots, 8$.

Step 2. Optimization - fitting the curve:

Goal: find the probability vector p_t and step size vector n_t (corresponding to large shift as a n_t multiple of Δ) to minimize the distance between the quarterly average implied by the step function shifting only on implementation dates and the quarterly average implied by Riba prices,[38]

$$\begin{cases} (p_t, n_t) = \underset{\{p_t\}\in[0,1],\,\{n_t\}\in\mathbb{N}_0}{\arg\min} \sum_{k=1}^{8} \left(r_t^k - riba_t^k\right)^2, \ t = \{\text{implementation dates}\}, \\ \text{st. } r_t = (1-p_t)\cdot(r_{t-1} + (n_t - 1)\Delta) + p_t \cdot (r_{t-1} + n_t\Delta), \ \Delta = 25 \text{ basis points}, \\ r_t^k \dots \text{ average rate } r_t \text{ between the end-of-quarter dates}, \ k = 1, \dots, 8, \\ riba_t^k \dots \text{ k-th riba contract (expected average policy rate between the end-of-quarter dates)}, \ k = 1, \dots, 8. \end{cases}$$

Step 3. Implied policy rate - continuous daily curve:

- set start, end and mid-of-quarter rate values ($\boldsymbol{start_k}, \boldsymbol{mid_{k1}}, \boldsymbol{mid_{k2}}, \boldsymbol{end_k}$) for $k = 1, \dots, 8$ as implied by the outcome of the optimization programme in Step 2,
- define the continuous daily policy rate as the *step function* interpolation between the implementation dates (or some other alternate interpolation method).

[37] The algorithm is written in Matlab, and the code is available upon request.

[38] Given the numerical nature of solution to our optimization problem, we tested the ability of two numerical methods in our toolbox to find the global optimum. The first one, *fmincon* (based on the standard Newton-Raphson algorithm), performs well as long as we do not have to deal with the discrete parameter constraints. In the case of integer optimization (with the step size constraints) *fmincon* fails, but the second method, *ga* (based on the genetic algorithm), is able to find the optimum.

SVERIGES RIKSBANK
Tel +46 8 - 787 00 00
registratorn@riksbank.se
www.riksbank.se

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