Staff memo

Assessing the foreign linkages in MAJA - a conditional forecast evaluation approach

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Summary

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Sweden is a small open economy which is strongly influenced by global economic developments. The ability to forecast Swedish macroeconomic variables accurately therefore relies both on good predictions of the foreign economy and an understanding of how developments abroad affect the Swedish economy. One aspect of the latter concerns the construction of economic models which can adequately capture the strong correlations between many Swedish and foreign variables found in the data.

The Riksbank's new dynamic stochastic general equilibrium (DSGE) model, MAJA, was developed with the purpose of better capturing the dependence between Sweden and the foreign economy, as represented by Sweden's main trading partners; see Corbo and Strid (2020). In this memo we evaluate how well the foreign-domestic link in the model works through a forecast evaluation. Our main result is that the accuracy of MAJA forecasts of Swedish variables are generally improved, and for many variables substantially so, when they are conditioned on ”perfect” forecasts of the foreign economic variables. Also, the improvement in forecast accuracy is larger with MAJA compared to the Riksbank's earlier DSGE model, RAMSES II, and a Bayesian vector autoregressive model. This suggests that the influence of the foreign economy on Sweden is better captured in MAJA. Information on foreign developments is particularly helpful in forecasting Swedish GDP (and its components) and the policy rate but less so for inflation, wages and unemployment. For most Swedish variables in MAJA, the forecasting performance improves with the amount of information on foreign developments, i.e. when more foreign variables are conditioned on. However, conditioning only on foreign GDP, inflation and the policy rate appears “good enough” for most purposes. The choice of foreign variables to condition on when MAJA is used for forecasting at the Riksbank could to some extent depend on the objective and the current economic situation, e.g. forecast accuracy versus storytelling or conditioning on the foreign corporate spread in times of financial turmoil.

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1. Introduction

Sweden is a small economy with open goods and capital markets which is strongly influenced by global economic developments. This is reflected in the generally strong comovement between Swedish and foreign-economy macroeconomic variables, see Lindé and Reslow (2017). The Riksbank has recently developed a new two-country dynamic stochastic general equilibrium (DSGE) model for Sweden and its main trading partners – MAJA (Modell för Allmän Jämviktsanalys) - to better capture these dependencies; see Corbo and Strid (2020). The authors assess the cross-country dependencies in MAJA through the study of cross-correlations, variance decompositions and historical decompositions. Here we instead focus on the forecast performance of the model and provide complementary evidence on how well the cross-country links are captured in MAJA.

The purpose of this memo is to study the forecasting performance of MAJA, with an emphasis on forecasts of Swedish variables which are conditioned on information about future developments of foreign (i.e. trade-weighted) variables, henceforth *conditional forecasts*. The Riksbank, as well as other institutions, are routinely forecasting global economic developments and judgemental forecasts of foreign macroeconomic variables are used in MAJA and other models to generate conditional forecasts of Swedish variables. These model forecasts are then inputs in the construction of the Riksbank's judgemental forecasts of domestic variables.[2]

The accuracy of the conditional (on foreign variables) model forecasts of Swedish variables depends mainly on two factors. First, the accuracy of the foreign forecasts which are used as conditioning information when generating the domestic forecasts and, second, how well the model captures the linkages between the foreign and domestic economies.[3] To learn from a forecast evaluation these two possible sources of error of the conditional forecasts should be studied separately. Since we are primarily interested in evaluating the foreign-domestic linkages in the model, rather than the quality of (e.g. the Riksbank's judgemental) foreign variable forecasts, we choose to condition on the actual outcomes, or *realisations,* of the foreign variables in generating conditional forecasts of the Swedish variables. That is, if the model is provided with "perfect" forecasts of the foreign variables, how well can it forecast the Swedish variables?[4]

The paper proceeds by posing and answering a set of questions related to the forecasting ability of MAJA. *First, is information on future foreign developments at all helpful in forecasting domestic variables?* This question is answered by contrasting the forecast accuracy of unconditional and conditional forecasts, where an "unconditional forecast" is conditioned only on data available at the

[2] MAJAs unconditional, i.e. its own, forecasts of foreign variables can of course also be used as an input in the construction of a judgmental forecast of foreign developments. However, in this paper we do not discuss foreign variable forecasting using MAJA.

[3] Maih (2010) shows in an application to a small open economy DSGE model that the benefits of conditioning depend crucially on the ability of the model to capture the correlation between the conditioning information and the variable of interest. In our case this means the correlations between foreign and domestic variables.

[4] Our paper is not the first to evaluate Riksbank DSGE model based forecasts. Adolfson et. al. (2007) evaluate forecasts from the DSGE model RAMSES 1, and versions thereof, and VAR and VECM models for the period 1994-2002. Iversen et. al. (2016) evaluate real-time forecasts from the DSGE models RAMSES 1, RAMSES 2 and a BVAR model and compare these forecasts with the Riksbank's published forecasts for the period 2007-2013.

point in time when the forecast is made.[5] If the model captures the foreign-domestic relationships in the data in a reasonably good way, perfect knowledge of the future development of the foreign economy should improve the forecasts of Swedish variables, i.e. the conditional forecasts should be more accurate than the unconditional forecasts. For example, foreign and Swedish GDP growth are strongly positively correlated and, in a model which captures this data relationship reasonably well, counterfactual knowledge of the sharp decline in foreign (trade-weighted) GDP growth during the financial crisis in 2008-09 should have been helpful in forecasting Swedish GDP during this period. If, on the other hand, the conditional forecast of a Swedish variable does not improve on the unconditional forecast it could be due to either of two broad explanations. First, it could be due to a “poor” model of the linkages between the foreign and domestic economies and, second, it could be because knowledge of foreign developments are not very informative for the behaviour of the domestic variable in question, i.e. the data relationships are weak.[6]

Once it has been established that information on the future developments of the foreign economy is indeed helpful in forecasting *most*, but not all, domestic variables we study the conditional forecasts in some more detail. The second question we ask is: *which of the foreign variables are particularly useful to have information on when forecasting Swedish variables?* MAJA was estimated using ten foreign variables and here we investigate whether using more information on foreign developments, i.e. conditioning on a larger set of variables, improves the forecasts. We also investigate which ones of the foreign variables that are more important to include in the conditioning set.

The third and final question is: *is information on foreign developments particularly helpful in forecasting certain domestic variables? (And, if so, what characterises these variables, e.g. are these typically variables which are highly correlated with foreign variables?)*. As discussed by Corbo and Strid (2020), some Swedish variables, e.g. GDP, labour market variables and the policy rate (the repo rate), are strongly correlated with foreign variables, while other variables, e.g. wage inflation and CPIF inflation are less correlated with their foreign counterparts. One could suspect that the forecasting performance of the strongly correlated variables improves more when we condition on the foreign variables. This is, however, not entirely clear since it has been shown that it is difficult for open economy DSGE models to capture the strong cross-country relationships between variables in the data; see e.g. Justiniano and Preston (2010).

The main results of the analysis are the following. Regarding the first question, “*Is information on future foreign developments at all helpful in forecasting domestic variables?*”, it is found that conditioning on foreign variables generally improves the accuracy of domestic variable forecasts using MAJA. Both the biases and RMSFEs of the conditional forecasts are generally lower in comparison with the unconditional forecasts. The accuracy of the Swedish GDP growth and policy rate forecasts improve significantly when information on

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[5] An unconditional forecast of a variable for the forecast period t+1 to t+h is conditioned on information up to quarter t where t+1 is the first forecast quarter and t+h is the longest forecast horizon. A conditional forecast of a variable is further conditioned on the outlook for some subset of variables, in our case some group of foreign variables, in the forecast period t+1 to t+h.

[6] In the appendix we provide a simple framework to discuss how the strength of the relationships in the data between foreign and domestic variables, and how well these are captured in a model, affect the accuracy of the unconditional and conditional forecasts. The purpose is to provide intuition for the results on the forecasting performance of the dynamic multivariate models presented in the paper.

foreign developments is utilised, while this is not the case for CPIF inflation.[7] Further, the gain from conditioning is substantially larger with MAJA in comparison with the Riksbank's previous DSGE model, RAMSES II, and a two-country block exogenous Bayesian vector autoregressive (BVAR) model. This suggests that the influence of the foreign economy on Sweden is better captured in MAJA than in the other models.

As for the second question, "*which of the foreign variables are particularly useful to have information on when forecasting Swedish variables?*", it appears important to include foreign GDP, CPI inflation and the policy rate in the conditioning set. However, the gain of conditioning on a large set of foreign variables in MAJA, compared to only the three "key" variables, is generally rather small. This implies that in most situations it is probably "good enough" to condition on the smaller set of variables.[8]

Regarding the third question, "*Is information on foreign developments particularly helpful in forecasting certain domestic variables?*", results indicate that conditioning on foreign variables is particularly helpful in forecasting Swedish GDP (and its components) and the policy rate but less so (or actually sometimes even worsens the accuracy of the forecasts) for inflation, wages and unemployment.

The rest of this paper is organised as follows. Section 2 briefly describes MAJA and the data used for our analysis. Methodological issues and the setup of the forecast evaluation are discussed in Section 3. Results are presented in Section 4 and finally, Section 5 concludes.

2. Model and data

The DSGE model MAJA is presented and estimated in Corbo and Strid (2020). Here we only provide a brief description of the model and the reader is referred to the aforementioned paper for more detail. MAJA is a two-region model consisting of a domestic economy (Sweden) and a foreign economy (a trade-weighted aggregate of Sweden's main trading partners). The model of the foreign economy is very similar to the DSGE model of Christiano et al. (2005), which was estimated using Bayesian methods on euro area and US data by Smets and Wouters (2003, 2007). The model of the domestic economy share many similarities with the previous Riksbank models, RAMSES I and RAMSES II; see Adolfsson et al. (2007) and Adolfson et al. (2013). By comparison, in the earlier Riksbank DSGE models, the foreign economy is modelled through a small vector autoregressive (VAR) model for foreign GDP, inflation and the policy rate. Nominal rigidities in MAJA are included through Calvo style sticky prices and wages. The real frictions include habit formation in consumption, investment adjustment costs and a working capital channel. It is assumed that the foreign economy is exogenous, i.e. domestic shocks have no influence on foreign variables.

[7] The CPIF index is the consumer price index with fixed interest rates and is since September 2017 the inflation target variable at the Riksbank.

[8] This is not a new phenomenon in the DSGE literature. Herbst and Schorfheide (2012) found that adding additional features to a simple three-equation New Keynesian model on average do not lead to improvements in the quality of point forecasts, density forecasts and predictions of comovements of US output, inflation, and interest rates.

Since the work of Justiniano and Preston (2010) it is well known that it is difficult for standard open economy DSGE models to generate meaningful cross-country spillovers. In MAJA the influence of foreign developments on the Swedish economy is increased mainly through two features. First, the demand for Swedish exports is more dependent on foreign investments (rather than foreign GDP). Second, the model allows for global shocks to e.g. technology, real interest rates, firm and household confidence and interest rate spreads. These shocks affect the foreign and domestic economies simultaneously and in similar ways and therefore create comovement between foreign and domestic variables. Model comparisons based on the marginal likelihood show that the inclusion of these features in MAJA is supported by the data; see Corbo and Strid (2020).

MAJA is estimated on quarterly data for Sweden (the domestic economy), and a trade-weighted measure of the foreign economy (KIX2), where the euro area has a weight of around 85% and the United States a weight of 15%.[9] The sample period begins roughly when inflation targeting was introduced in Sweden, in 1995Q2, and ends in 2018Q4.[10] The model is estimated using data on 25 variables (10 foreign and 15 domestic variables), see Table 1. Many of the variables are transformed into annualised quarterly growth rates (AQG) as indicated in the table. The data are also shown in Figure A1 (domestic variables) and Figure A2 (foreign variables) in Appendix A. Table A1 in Appendix A reports the sample averages, standard deviations and steady states of the variables.[11]

In the forecast evaluations presented below the forecasts of those variables which appear in annualised quarterly growth rates in the model are transformed into annual growth rates.

[9] The version of MAJA used for the forecast evaluation is a baseline version where foreign and domestic parameters are estimated jointly using Bayesian methods; see Corbo and Strid (2020). We consider the mean forecasts generated with the posterior mode estimates of the DSGE model parameters, i.e. we do not consider parameter uncertainty and we do not evaluate density forecasts.
[10] Thus, we employ the same data sample as in Corbo and Strid (2020).
[11] See the Appendix to Corbo and Strid (2020) for the data sources, SDMX codes, and bivariate correlations between the variables.

Table 1. Measurement variables in MAJA. AQG=the variable is transformed into annualised quarterly growth rates. The variables are expressed in percent, percentage points (*), or percentage deviation from trend (**).

Foreign variables	Domestic variables
KIX2 GDP, aqg	GDP, aqg
KIX2 Consumption, aqg	Consumption, aqg
KIX2 Investments, aqg	Investments, aqg
KIX2 CPI, aqg	Exports, aqg
KIX2 CPI excl. energy, aqg	Imports, aqg
KIX2 Policy rate	CPIF, aqg
KIX2 Wages, aqg	CPIF excl. energy, aqg
KIX2 Corporate spread*	Import prices excl. energy, aqg
KIX2 Unemployment rate	Policy rate
KIX2 Employment gap**	Unemployment rate
	Employment gap**
	Wages, aqg
	Capacity utilisation
	Corporate spread*
	Real exchange rate, aqg

3. Method

3.1 Pseudo out-of-sample forecasts

The forecasts which are evaluated in this paper are pseudo out-of-sample forecasts and they differ from real time forecasts in three ways. First, the evaluation is based on data available in April 2019 rather than real time data (for which revisions of the data are made sequentially over time). Second, the parameters of MAJA, which are used to generate recursive forecasts, are estimated for the full sample 1995Q2-2018Q4, i.e. the model is not recursively estimated. A single vintage of the data and a single set of estimated parameters simplifies the forecast evaluation. Finally, we consider forecasts conditional on *realisations,* and not e.g. actual real-time Riksbank forecasts, of the foreign variables. This choice relates to the main purpose of the paper, which is to evaluate how well MAJA and the other models capture the dependencies between the foreign and Swedish economies. Conditioning instead on e.g. the Riksbank's real time forecasts of the foreign variables would imply that also the accuracy of these forecasts would matter for the accuracy of the models' forecasts of the domestic variables. In order to learn from the evaluation it is, we believe, important to study the two possible sources of domestic variables' forecast errors separately.[12] While an evaluation of the real time foreign variable forecasts is certainly of interest, it is not the objective of this paper.

3.2 Models

The forecasting performance of MAJA is compared to the Riksbank's previous main DSGE model, RAMSES II, which is described in Adolfson et al. 2013. Henceforth we will refer to the latter model simply as "RAMSES". The version of RAMSES used here

[12] If the purpose had instead been to evaluate how well the models' conditional forecasts perform in a real time forecasting situation they should be conditioned on the real time foreign forecasts. See also appendix D for further discussion.

is the one that was active in forecasting and policy work at the Riksbank in 2018.[13, 14] Both DSGE models are evaluated on the sample 1999Q4-2018Q4, for which forecasts for 1-12 quarters ahead are generated. Based on data until 1999Q4, forecasts are generated for the period 2000Q1-2002Q4, and so on. This implies that a total of 76 one-quarter-ahead forecasts and 65 twelve-quarter-ahead forecasts are produced.[15]

Comparisons are also made with a Bayesian VAR (BVAR) model with a prior distribution on the model's steady state; see Villani (2009). The model is briefly described in Appendix B.[16] The specification used here consists of 9 variables – 3 foreign and 6 domestic. The variables, together with their steady-state priors, are listed in Table A2 in Appendix A (in the same order as they enter the model). Since our sample begins in 1995Q2 and since (unlike the DSGE models) the BVAR is recursively estimated, the forecast evaluation statistics for the BVAR model are computed for the shorter sample 2007Q1-2018Q4. Since we use different evaluation periods for the DSGE models and the BVAR model the comparison with the BVAR is imperfect.[17] In the analysis that follows the emphasis is therefore on the comparison between the two DSGE models.

In addition, comparisons with three simple and standard benchmark models are also performed. These are naïve (random walk) forecasts, recursive mean forecasts and AR(1) forecasts.

3.3 Conditional forecasts

A conditional forecast is a forecast where the future paths of a subset of the variables in the model are fixed, or 'treated as data'. In this paper attention is restricted to forecasts of Swedish economic variables conditional on foreign economy variables. To fix the conditioning paths of a group of variables, shocks are assigned values in the forecasting period such that the desired paths are obtained. Beyond choosing which variables to condition on, this means that one needs to decide on the set of shocks to use for conditioning. In the simplest case the number of shocks equals the number of conditioning variables and the values of the shocks can be obtained by solving a linear equation system.[18] If, for example, the conditioning path for the foreign policy rate is lower than the unconditional forecast generated by the model, one way to impose the former path is through a sequence of negative, i.e. expansionary, monetary policy shocks. In the widely used approach proposed by Waggoner and Zha (1999), *all* available shocks in a model are employed and their values are selected optimally, i.e. the shock values are chosen in the "least

[13] DSGE models used for forecasting and policy are regularly updated with new features. The main difference between the version of RAMSES described in Adolfson et al. (2013) and the more recent version is that the latter incorporates a time-varying neutral rate; see Corbo and Strid (2020). In the context of this paper, the main implication is that the policy rate forecasts from the recent version of RAMSES are significantly more accurate than those using the model presented in Adolfson et al. (2013).
[14] Waggoner-Zha type of forecasts (all foreign shocks active) are evaluated in the DSGE models. RAMSES 2 is estimated for the sample period 1995Q1-2017Q2.
[15] Note that the rather long evaluation period is made possible by the fact that the DSGE models are not recursively estimated but instead estimated on the full sample of data.
[16] The priors on the parameters and the hyperparameters of the model used in this document follow those in Villani (2009).
[17] In this memo we wanted to compare the forecasting performance of MAJA with the same, recursively estimated, BVAR model that is used at a daily basis at the Riksbank. Our assessment is that the different evaluation periods puts the BVAR at a disadvantage, e.g. since the financial crisis period gets a larger weight when computing RMSFE:s. An alternative would be to shorten the evaluation period.
[18] For example, conditioning on 3 foreign variables in 12 forecast quarters means that one solves for the values of 3 shocks in these quarters. A linear equation system in 3*12=36 unknowns is then solved. Warne (2018) refers to this case as "values for shocks".

costly way" avoiding e.g. very large values of the shocks.[19] In this paper we use the latter, Waggoner and Zha (1999), approach.[20]
The conditioning shocks can further be used to provide economic interpretations of the forecasts. Since we condition on the realisations of foreign variables the shocks will capture the difference between the model's forecasts of foreign variables and their realisations, i.e. the forecast errors associated with the unconditional forecasts. These shocks will in turn affect the forecasts of the domestic variables. For example, during the financial crisis in 2008-2009 the conditional forecast of Swedish GDP growth forecast is lower than the corresponding unconditional forecast and this is mainly attributed to a combination of negative foreign supply and demand shocks. In generating conditional forecasts sequentially across time, the values of the foreign shocks which are used to condition in the foreign variable forecast paths will change. In other words, the drivers of the forecasts and hence the economic interpretation of the forecasts change across time with each forecast. It is beyond the scope of this memo to provide an extensive discussion of how these drivers have evolved over time. However, forecast error variance decompositions (FEVD) reported in Corbo and Strid (2020) provide information on how important the foreign shocks are in explaining the variation in domestic variables *on average* and historical decompositions illustrate the importance of foreign shocks in particular episodes of the sample. For example, global technology shocks (supply shocks) and shocks to the marginal efficiency of investment (demand shocks) in the foreign economy are the most important foreign/global drivers of Swedish GDP growth. And shocks to the global neutral rate of interest are an important driver of both the foreign and Swedish policy interest rates in MAJA; for further discussion see Corbo and Strid (2020).

3.4 Forecasts evaluation measures

Forecast accuracy is assessed using root mean squared forecast errors (RMSFE), thus assuming a quadratic loss function for the forecasts. Using mean absolute errors (MAE, assuming a linear loss function) instead yields very similar results and does not change the conclusions from the evaluation substantially.[21] To get a sense of the size of the differences of the models' forecast accuracy, the Diebold-Mariano (Diebold and Mariano, 1995) test for equal forecasting precision of two models is applied.

The RMSFE and MAE are measures of forecast accuracy and cannot say anything about whether one model tends to over- or underpredict the realisations (i.e. systematic errors, or bias). Because of this we also include the mean error (ME), or bias, in the analysis in this study.[22]

4. Results

In this chapter we first show MAJA's unconditional and conditional forecasts of three key macroeconomic variables – annual GDP growth, annual CPIF inflation and the policy rate - across time (Section 4.1). Next, we study the accuracy of the forecasts

[19] Under some additional assumptions this approach is identical to the standard approach to conditional forecasting in state-space models, i.e. to treat conditional forecasting as a missing data problem and obtain the forecasts using the state smoother, see e.g. Durbin and Koopman (2001) for more details.
[20] The large number of possibilities concerning the set of conditioning variables and the set of conditioning shocks means that the number of possible forecasts conditional on foreign variables is very large. In MAJA we choose among 10 foreign variables which means that there are 1024 possible selections of conditioning variables.
[21] Results from the MAE are available from the authors upon request.
[22] See Appendix C for a description of the forecasting error measures used in this study.

more formally using the root mean squared forecast error (RMSFE) (Section 4.2) and compare the accuracy of conditional forecasts generated using different foreign variable conditioning sets (Section 4.3). Finally, Section 4.4 includes an analysis of the systematic errors in MAJA.

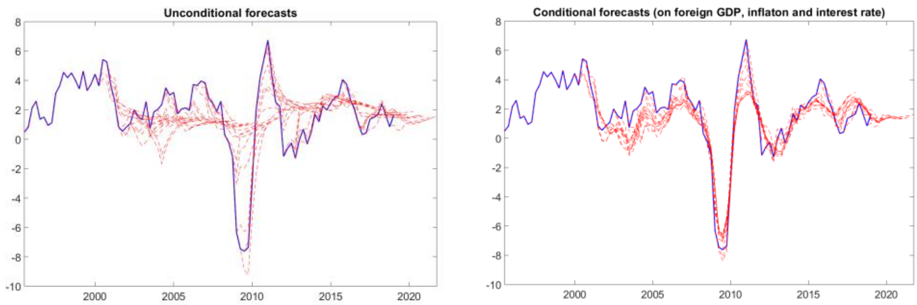
4.1 MAJA forecasts across time

Unconditional forecasts of Swedish GDP growth and forecasts conditional on foreign GDP growth, CPI inflation and the policy rate from MAJA for the period 1999Q4-2018Q4 are shown in Figure 1. It is evident that the conditional GDP growth forecasts are more accurate than the unconditional ones. This is particularly apparent during the financial crisis in 2008-2009 but it is the case also in the period before and after the crisis.

The benefits of conditioning depend crucially on the ability of the model to capture the correlation between the conditioning information and the variable of interest; see Maih (2010). The Swedish GDP growth rate is strongly correlated with the foreign, i.e. trade-weighted, GDP growth rate and the accuracy of the conditional forecasts reflects that this correlation appears to be reasonably well captured in the model.[23]

We also note that the forecasts underpredict growth prior to the financial crisis, while there does not appear to be a bias in the forecasts in the period after the crisis. This is to some extent related to the calibration of a constant steady state growth rate in the model, which makes it difficult to account for the fact that the average growth rate in the data was higher pre-crisis.

Figure 1. Recursive MAJA forecasts (red) and realisations (blue). Swedish GDP per capita, annual growth rate in percent.

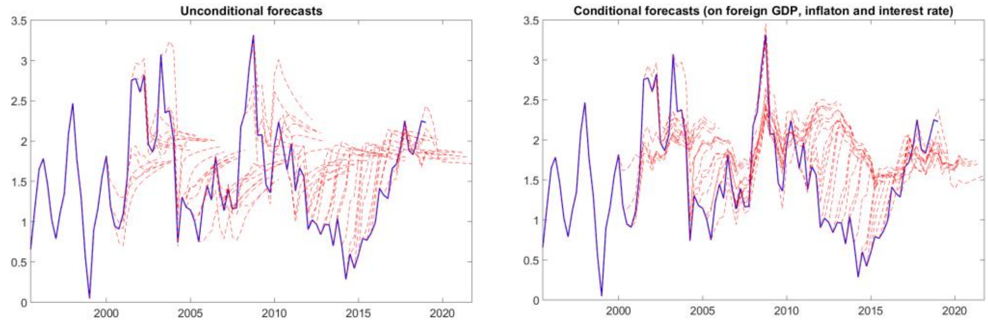


The corresponding forecasts of CPIF inflation are shown in Figure 2. In contrast to the GDP forecasts there appear to be no apparent benefits from conditioning on the three foreign variables. In particular, inflation appears difficult to forecast in the period of low inflation from 2012 to 2016 where both the unconditional and conditional MAJA forecasts overpredict inflation. Overall, i.e. for the entire evaluation period, it does not appear to be the case that the conditional inflation forecasts improve on the unconditional ones (this observation is supported by the formal analysis that follows below).

[23] The correlation between the foreign and Swedish quarterly GDP growth rates in the sample is 0.7. Two foreign/global shocks are particularly important in accounting for the correlation between the two variables in MAJA. A common unit root technology shock captures a common stochastic trend in Swedish and foreign GDP (and also other variables such as real wages). Foreign shocks to the marginal efficiency of investment affect the Swedish economy mainly through trade, but also through an effect on consumer confidence abroad and in Sweden (through correlations between shocks). Both shocks generate a strong positive correlation between foreign and Swedish GDP growth, and between foreign and Swedish real quantities more generally. For further details; see Corbo and Strid (2020).

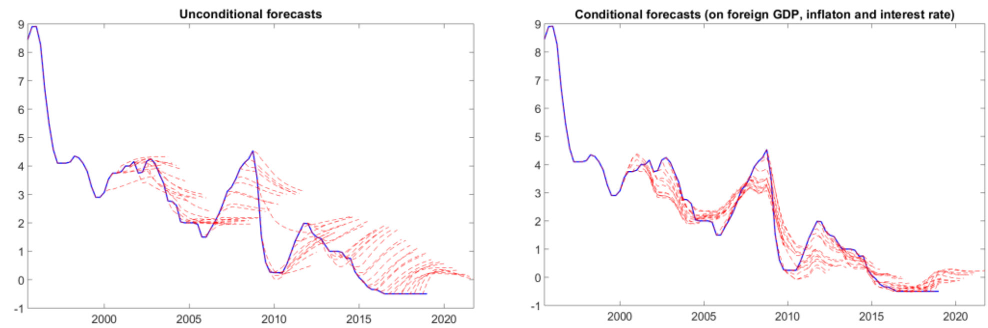
CPIF inflation is generally weakly correlated with the foreign variables in the model, which suggests that it may be difficult for forecasts conditional on foreign variables to improve on the unconditional forecast.[24] The contemporaneous correlation between quarterly CPIF inflation and foreign CPI inflation is 0.4 in the sample period, and it is largely driven by common high frequency movements in the oil price. For example, in 2012-2013 it appears that the relationship with foreign inflation actually pointed to a higher CPIF inflation rate, since in these years the conditional forecasts exceed the unconditional ones.

Figure 2. Recursive MAJA forecasts (red) and realisations (blue). Swedish annual CPIF inflation in percent.



In Figure 3 forecasts of the repo rate, i.e. the Swedish policy rate, are displayed. Similar to the GDP growth forecasts, the conditional forecasts of the policy rate appear to be more accurate than the unconditional forecasts. The foreign and Swedish policy rates are trending downwards in the sample period and they are strongly correlated. To account for the interest rate trends MAJA has been equipped with a time-varying global neutral interest rate; see Corbo and Strid (2020). This feature improves both the unconditional and conditional repo rate forecasts. First, the high persistence of the neutral rate adds a “random walk element” which increases the accuracy of the forecasts. Second, the assumption of a global, i.e. common, neutral rate increases the correlation between the foreign and domestic policy rates in MAJA and this further improves the conditional forecast relative to the unconditional forecast.

Figure 3. Recursive MAJA forecasts (red) and realisations (blue). Swedish policy rate in percent.



In summary the visual inspection of the recursive forecasts suggests that the MAJA conditional forecasts of GDP growth and the policy rate are substantially more accurate than their unconditional counterparts, while this is not the case for the CPIF inflation forecasts. The forecast errors associated with the unconditional and conditional forecasts of Swedish GDP growth, CPIF inflation and the policy rate at a

[24] Note that this statement concerns the foreign variables which are included in MAJA. It may of course be the case that there are foreign variables which are more strongly correlated with CPIF inflation but which are not included in the model.

forecast horizon of 8 quarters are shown in Figures A3-A5 in Appendix A. These graphs provide a complementary viewpoint on the accuracy of the forecasts across time.

Before moving on to the formal examination of forecast accuracy, in the remainder of this section we discuss the factors which determine the relative accuracy of the unconditional and conditional forecasts.[25] In order for the conditional forecasts to improve on the unconditional forecasts we broadly need two features of the data and the model, respectively. First, we need some degree of strength in the empirical relationships between the variable of interest, i.e. the domestic variable to be forecasted and the foreign variables included in the conditioning set. Second, these relationships need to be reasonably well captured by the model. Justiniano and Prestons (2010) critique against open-economy DSGE models is based on the observation that while cross-country correlations are often strong in the data they are generally weak in these models.

The forecasts displayed above are generated using the baseline version of MAJA, where global shocks have been included to strengthen the relationships between Swedish and foreign (mainly euro area) variables. Model comparisons based on marginal likelihoods illustrate that the baseline version is strongly favoured to versions of the model without global shocks; see Corbo and Strid (2020). These results strongly suggest that the inclusion of global shocks in the model is important also for the improvement in accuracy of conditional forecasts relative to unconditional forecasts, by increasing the correlations between foreign and domestic variables.[26]

In order to understand this further, consider again the GDP growth forecasts displayed in Figure 1. The model's unconditional forecasts of foreign GDP growth prior to the financial crisis in 2008-2009 did not anticipate the sharp drop in growth (the forecasts of foreign growth are not shown here). Therefore, when we condition on the realisations of foreign GDP growth during the crisis shocks which contribute to lower growth are required. It turns out that two foreign shocks are particularly important to achieve this conditioning: negative productivity shocks (negative supply shocks) and negative shocks to the marginal efficiency of investment (negative demand shocks). Both these shocks induce a strong positive correlation between foreign and Swedish growth. As a result the conditional forecasts of Swedish GDP growth become substantially lower than the unconditional forecasts.

4.2 Forecast accuracy of MAJA, RAMSES and the BVAR model

In this section forecast accuracy is evaluated based on a comparison of the root mean squared forecast errors (RMSFEs) of the model forecasts. The RMSFEs of the unconditional and conditional forecasts of GDP growth, inflation and the policy rate from MAJA, RAMSES and the BVAR model are reported where the main focus is on the comparison of the two DSGE models. The conditional forecasts are based on forecasts for foreign GDP growth, inflation and the policy rate (where these forecasts are assumed to equal the realisations of the variables). This could be viewed as a "standard conditional forecast" since it has been the most usual type of

[25] In appendix D we provide a simple framework to discuss this further.

[26] While we do not report forecasts from a version of MAJA without global or correlated shocks here the relative accuracy of the unconditional and conditional forecasts from RAMSES, a similar DSGE model but (largely) without global or correlated shocks, illustrates the importance of these features. In Corbo and Strid (2020) the importance of including global shocks is illustrated through by studying cross-correlations between foreign and domestic variables and the foreign variance share in forecast error variance decompositions of domestic variables.

foreign variable conditioning when macroeconomic models have been used for forecasting at the Riksbank. The main reason it is used here is simply that all three models include these three foreign variables, i.e. it enables a comparison of the models.

In the following the main focus is on the comparison between the two DSGE models. In Figure 4 the RMSFEs of the unconditional and conditional forecasts of annual GDP growth from MAJA, RAMSES and the BVAR models are displayed. First, the RMSFEs of the MAJA and RAMSES unconditional GDP growth forecasts are quite similar. Second, for both models the conditional forecasts improve on the unconditional forecasts, i.e. perfect knowledge of the future paths of the three foreign variables leads to better forecasts of Swedish GDP growth. Finally, the improvement in forecast accuracy when conditioning on the foreign variables is much larger for MAJA, where the RMSFEs at longer forecast horizons are roughly halved, in comparison with RAMSES. This reflects that the data relationships between Swedish GDP and the foreign variables are better captured in MAJA compared to RAMSES.[27] And since Swedish GDP growth is strongly related to foreign variables, in particular foreign GDP growth, developments abroad generally contain a lot of information on the Swedish growth outlook. The gains in forecasting accuracy from knowledge of foreign developments in MAJA carries over to the components of GDP, i.e. consumption, investments, exports and imports; see Tables A3-A6 in Appendix A.

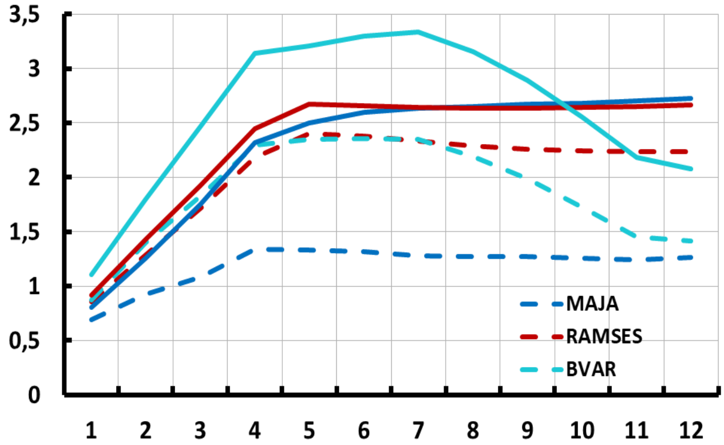
The accuracy of the forecasts conditional on foreign variable realisations, as measured by the RMSFE, can be interpreted as a “measure of consistency” between the Swedish GDP forecast and the foreign forecasts, in the following sense. The results suggest that if we have good forecasts of foreign developments, then MAJA will generally deliver a decent forecast of Swedish GDP growth. If, on the other hand, foreign GDP growth is, say, overpredicted then MAJA will tend to overpredict also Swedish GDP growth, i.e. the model would deliver forecast errors which are in line with the strong relationship between the two variables in the data. For comparison, in RAMSES, the Swedish GDP forecast is less affected by the forecasts of the foreign variables. Alternatively, foreign shocks have larger effects on Swedish variables in MAJA in comparison with RAMSES.

Whether it is preferable to capture the strong cross-country data relationships in the model or not depends on the perspective. In MAJA, systematic errors in the foreign variable forecasts would translate into systematic errors in the domestic GDP growth forecast, i.e. it does not necessarily imply that the forecast accuracy is improved in an actual real-time forecasting environment. On the other hand, better multivariate consistency between the forecasts would hopefully imply that it would be easier to understand and learn from the forecast errors.[28]

[27] In terms of model features, the effects of the foreign shock to the marginal efficiency of investment in MAJA is presumably an importance difference between MAJA and RAMSES. The shock is quite important in accounting for the variation in Swedish GDP growth in MAJA and it induces a positive correlation between Swedish and foreign growth. The shock affects Swedish demand mainly through exports but also through effects on consumer confidence abroad and in Sweden. For more details, see Corbo and Strid (2020).

[28] To simplify, here we take the perspective that it is better to be “consistently wrong” than to be “right by chance”.

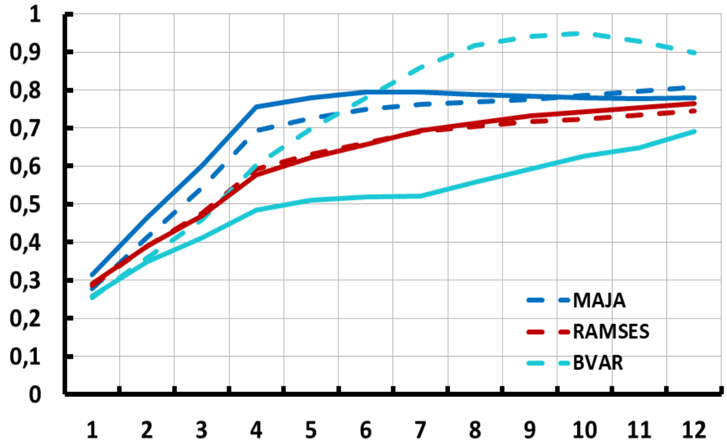
Figure 4. RMSFEs for unconditional (solid lines) and conditional (dashed lines) forecasts of annual GDP growth. Based on forecasts generated for the period 1999Q4-2018Q4 for the DSGE models and 2007Q1-2018Q4 for the BVAR model. Forecast horizons 1-12 quarters.



The RMSFEs of the annual CPIF inflation forecasts are shown in Figure 5. First, the RMSFEs of the unconditional inflation forecast are somewhat lower for RAMSES than for MAJA. Second, the improvements in RMSFE when conditioning on the foreign variables in MAJA are quite small, and in RAMSES the RMSFEs of the unconditional and conditional forecasts are almost identical.
These results for the conditional inflation forecast in MAJA to some extent reflect the generally weak relationships in the data between Swedish CPIF inflation and foreign variables. The contemporaneous correlation between CPIF inflation and foreign CPI inflation in the sample period (1995Q2-2018Q4) is close to 0.4, and this correlation is mainly due to the common effects on inflation of movements in global energy prices. While this relationship is well-captured in MAJA it does not lead to a large improvement in the RMSFE of the conditional forecast relative to the unconditional forecast. The correlations between CPIF inflation and other foreign variables included in the model are generally even lower.

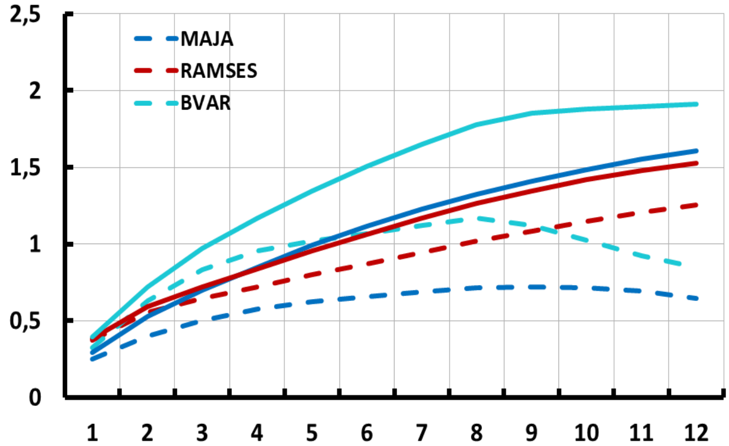
Domestic inflation in the model is determined by current and future costs of firms, which are dominated by wage costs. In Table A11 in Appendix A it is shown that the conditional forecast of Swedish wages does not improve significantly on the unconditional forecast. A weak link between foreign and Swedish wage inflation in the model presumably contributes to the rather weak link between the inflation rates.

Figure 5. RMSFEs for unconditional (solid lines) and conditional (dashed lines) forecasts of annual CPIF inflation. Based on forecasts generated for the period 1999Q4-2018Q4 for the DSGE models and 2007Q1-2018Q4 for the BVAR model. Forecast horizons 1-12 quarters.



Finally, in Figure 6 the RMSFEs of the policy rate forecasts are shown. First, the RMSFEs of the unconditional MAJA and RAMSES forecasts are very similar.[29] Second, the conditional policy rate forecasts in MAJA and the BVAR model improve substantially on the respective unconditional forecasts, while the improvement is smaller for RAMSES. The reason is that MAJA and the BVAR model capture the strong correlation between foreign and Swedish policy rates quite well, while the correlation implied by RAMSES appears to be somewhat smaller, judged by the relatively smaller improvement of the conditional forecast.

Figure 6. RMSFEs from unconditional (solid lines) and conditional (dashed lines) policy rate forecasts. Based on forecasts generated for the period 1999Q4-2018Q4 for the DSGE models and 2007Q1-2018Q4 for the BVAR model. Forecast horizons 1-12 quarters.

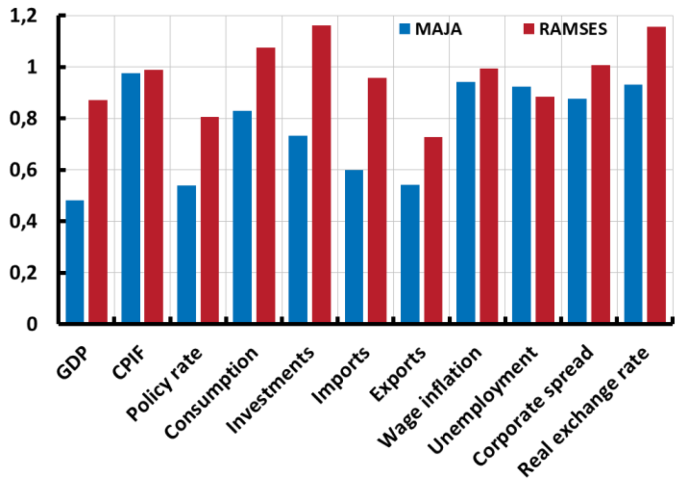


So far we have focused on the forecasts of three key macroeconomic variables: GDP growth, inflation and the policy rate. In Figure 7 the RMSFEs for the standard conditional forecasts relative to the unconditional ones at a forecast horizon of 8 quarters is reported for a larger set of domestic variables which are included in both MAJA and RAMSES. First, for MAJA the RMSFE of the conditional forecast is lower than the RMSFE of the unconditional forecast, i.e. the ratio is below 1, for all eleven variables examined. Second, the gain from conditioning on foreign variables is on average larger in MAJA than in RAMSESand the relative RMFS is lower in MAJA for

[29] The feature of a time-varying neutral rate in the model is important for the performance of the unconditional policy rate forecast. This feature is included both in MAJA and the version of RAMSES used here.

all variables except unemployment. The gains are particularly evident for the forecasts of Swedish GDP, exports and imports, and for the policy rate. These results also hold for forecasts at the horizons of 4 and 12 quarters; see Figures A6 and A7 in Appendix A.

Figure 7. RMSFEs of conditional forecasts relative to the unconditional ones in MAJA and RAMSES. Forecast horizon 8 quarters.



4.3 Foreign conditioning variables in MAJA

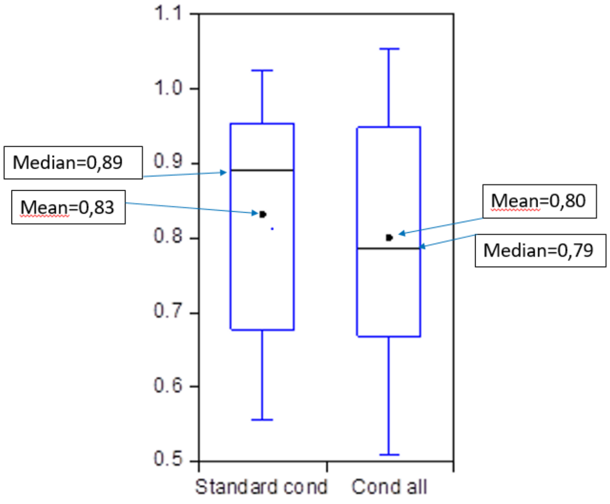
Conditioning on foreign GDP growth, inflation and the policy policy rate generally, i.e. for most variables and most forecast horizons, improves forecast accuracy compared to the unconditional forecast. But, as mentioned in Section 2, MAJA contains a structural model of the foreign economy and ten foreign data series are used in the estimation of MAJA, while there are only three foreign variables in RAMSES and in the BVAR model. Therefore, the set of possible foreign variable conditional forecasts in MAJA is larger than in the other two models.[30] A natural question arises: does forecast accuracy improve further if more information contained in the foreign variables is used for the conditional forecasts?

Figure 8 shows that the forecast accuracy across all forecast horizons and all Swedish variables in MAJA is improved by 17 percent on average for the standard conditional forecast relative to the unconditional forecast. Conditioning instead on all ten observed foreign variables in MAJA ("Cond all") improves forecast accuracy by 20 percent on average. The corresponding median improvements are 11% and 21%, respectively.[31] Conditioning on a larger set of foreign variables therefore appears to improve the overall forecast accuracy somewhat, but apparently not much, relative to the standard foreign conditional forecast.

[30] The number of possible conditioning sets with 10 foreign variables equals 1024. There are 10 possible conditional forecasts where one conditions on a single foreign variable, 45 possibilities involving two variables, and so on. With 3 foreign variables there are instead a total of 7 different conditioning sets.

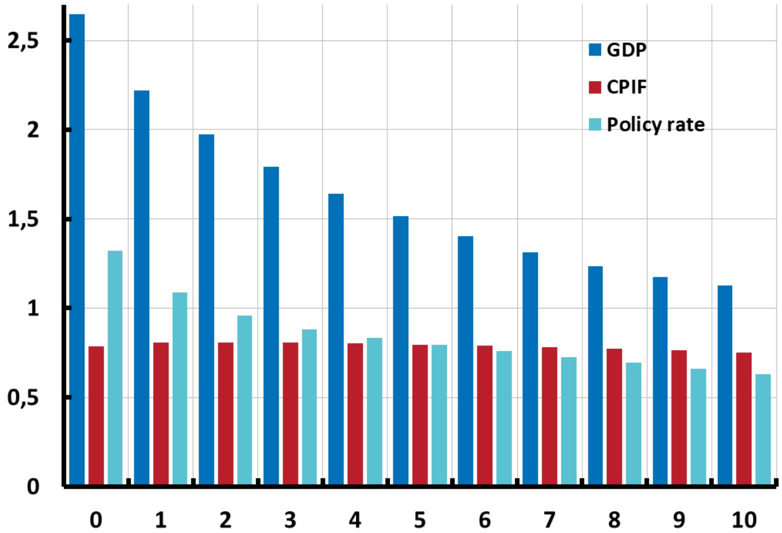
[31] The reason why the difference is bigger using the median instead of the mean depends on the distribution of the relative RMSFEs. The "Cond all" conditioning has a relatively symmetric distribution (where the mean is close to the median) whereas the distribution of the relative RMSFEs of the standard conditioning is skewed to the left (negatively skewed).

Figure 8. Boxplot of RMSFEs of the conditional forecast relative to the unconditional forecast. Based on all 15 observed domestic variables in MAJA and all forecast horizons, 1-12 quarters.[32]



There are, however, large differences in how the forecast accuracy of the different Swedish variables is affected by the number of variables included in the foreign conditioning set. In Figure 9 the average RMSFE at a forecast horizon of 8 quarters is shown, where the average is computed across all possible conditional forecasts with a given number of foreign conditioning variables, i.e. as we move to the right in the graph more information on the foreign economy is used for conditioning.

Figure 9. Average RMSFEs for forecasts based on different numbers of foreign conditioning variables. GDP growth, CPIF inflation and policy rate. Forecast horizon 8 quarters.[33]



The figure illustrates that the accuracy of the forecasts of Swedish GDP growth and the policy rate improve with a larger set of foreign variables in the conditioning set, while this is not the case for Swedish CPIF inflation. These results are in line with

[32] The boxes of the boxplots denote the first and third quartiles, and the whiskers show the minimum and maximum relative RMSFEs.
[33] The figure shows the mean RMSFEs of the forecasts where the conditioning set contains 0,1,..., 10 foreign variables where "0" is simply the unconditional forecast. For example, there are 10 forecasts with 1 conditioning variable, 45 forecasts with two conditioning variables, 120 forecasts with 3 conditioning variables etc. The results are similar also at horizons t+4 and t+12 quarters (available from the authors upon request).

those presented above for the standard foreign conditional forecast relative to the unconditional forecast.

For most of the other Swedish variables in MAJA – consumption, investments, export and import growth, real exchange rate growth, capacity utilisation, the employment gap and the corporate spread – the forecasts are improved when conditioning on a larger set of foreign variables; see Figures A8a and A8b in Appendix A. On the other hand, just as for CPIF inflation, the forecasts of wage inflation and unemployment do not seem to benefit much from conditioning on foreign variables; see Figure A9 in Appendix A.

So far we have investigated how *the size* of the foreign conditioning set affects the accuracy of Swedish variable forecasts. But clearly it matters which foreign variables one conditions on, and furthermore the best set of foreign conditioning variables differs across the Swedish variables. In Table 2 we report the set of foreign conditioning variables which delivers the conditional forecast with the lowest average RMSFE (averaged across all forecasting horizons 1 to 12) for each of the Swedish variables. This could be described as the Swedish variable specific *optimal conditional forecast* using MAJA. For example, the best conditional forecast for Swedish GDP growth includes six foreign variables in the conditioning set. These variables are foreign GDP growth, investment growth, CPI excluding energy inflation, wage inflation, the employment gap and the corporate spread.

We make the following broad observations on the optimal conditional forecasts. First, for the majority of the Swedish variables the optimal conditioning set includes the foreign variable counterpart, e.g. in forecasting Swedish consumption growth it is important to include foreign consumption growth. One reason is that the Swedish variables are often relatively strongly correlated with their foreign counterparts. Second, the optimal conditioning sets differ across the Swedish variables. For Swedish GDP and its components and the labour market variables it is important to include foreign GDP growth. For Swedish headline, core and import inflation it is important to include foreign headline and/or core inflation. Third, the optimal number of variables to condition on varies between two and seven for the Swedish variables. Generally, the Swedish variables with a larger optimal conditioning set are those for which the conditional forecasts improve substantially on the unconditional ones, e.g. GDP growth and the policy rate. On the contrary, the price and wage inflation variables have smaller optimal conditioning sets, presumably reflecting that their correlations with foreign variables are generally weaker. This is also in line with the result that, for these variables, the accuracy of conditional forecasts (no matter how they are constructed) is not much better than the accuracy of the unconditional forecast.

One use of the results on optimal conditioning sets, we believe, is to identify foreign variables which appear to be useful in forecasting several “important” Swedish variables. Two examples are provided to exemplify this. First, if we again restrict attention to Swedish GDP growth, CPIF inflation and the repo rate (see the top three rows in Table 2) we note that foreign investment growth, the foreign employment gap and the foreign corporate spread belongs to the optimal conditioning sets of all three domestic variables. Second, we note that the foreign corporate spread is included in the optimal conditioning sets of 11 of the 15 domestic variables. This result probably reflects that information on this variable (and financial variables more generally) is particularly useful in periods of financial turbulence when forecasting becomes more difficult.

Table 2: Set of foreign conditioning variables (in columns) which yields the lowest RMSFE (average RMSFE across forecast horizons 1-12Q) for each of the Swedish variables (in rows)

		Foreign variables									
		GDP	Infl.	Policy rate	Cons.	Inv.	Infl. excl energy	Wages	Unempl.	Empl. gap	Spread
Swedish variables	GDP	x				x	x	x		x	x
	CPIF		x			x				x	x
	Policy rate			x	x	x	x		x	x	x
	Consumption	x		x	x			x		x	
	Investments	x	x	x		x		x		x	x
	Imports	x				x	x	x	x		x
	Exports	x	x	x	x		x	x			
	CPIF excl energy			x		x	x			x	x
	Import prices excl energy		x				x				
	Wage inflation				x			x			x
	Unemployment	x									x
	Employment gap	x			x			x	x		
	Capacity utilisation	x		x		x	x		x	x	x
	Corporate spread				x	x					x
	Real exchange rate	x	x				x		x		x
	Number of occurencies	**9**	**5**	**6**	**6**	**8**	**8**	**7**	**5**	**7**	**11**

Note: See Table A1 in Appendix A for a description of each variable.

We conclude the discussion on conditional forecasts in MAJA by studying different forecasts of three key variables, GDP growth, CPIF inflation and the policy rate, in some more detail. In Tables 3-5 the RMSFEs at forecast horizons one, two and three years are reported for a number of forecasts, including some simple forecasting approaches used as benchmarks.

In Table 3 it is shown that the RMSFEs for the conditional forecasts of Swedish GDP growth are roughly half of the unconditional forecast RMSFEs and about 30 to 40 percent of the RMSFEs for the simple benchmarks. Also, the conditional forecasts of GDP growth based on three or all ten foreign variables, or the optimal set of foreign variables, are all significantly more accurate than the unconditional forecasts according to the Diebold-Mariano (1995) test of equal forecasting accuracy.

Table 3: Forecast RMSFEs for Swedish GDP growth (sample std = 2,65).[34]

	t+4	t+8	t+12
Unconditional	2,32	2,65	2,73
Conditional on three for. variables	1,34*	1,28*	1,27*
Optimal conditioning	1,23**	1,12**	1,07**
Conditional on all foreign variables	1,24*	1,13**	1,10**
Naïve forecast	4,01	5,44	5,07
Iterative mean forecast	2,87	2,95	2,97
AR(1) model forecast	2,88	2,96	2,97

The RMSFEs for the forecasts of CPIF inflation are reported in Table 4. The standard conditional forecast and forecasts conditional on all foreign variable do not significantly improve on the unconditional forecast while the optimal conditional forecast is significantly more accurate. However, the RMSFEs of the optimal forecast at various forecast horizons are not much lower than the RMSFEs associated with the simple approaches, e.g. an AR(1) model.

Our results on the optimal conditional forecast for CPIF inflation appear to be well in line with observations made by Andersson and Jonung (2020) in a discussion about the appropriate specification of a Swedish Phillips curve relationship. These authors show that including euro area unemployment and inflation in a Phillips curve regression for CPIF inflation increases the explanatory power of the model somewhat relative to a specification which instead includes only the Swedish unemployment rate. The optimal conditioning set for CPIF inflation according to our analysis includes both foreign CPI inflation and the foreign employment gap (which is almost perfectly negatively correlated with the unemployment rate). Our results also suggest that including the foreign corporate spread (or, more generally, some foreign financial variable) could increase the fit of such a regression-based Phillips curve further.[35]

Table 4: RMSFEs for annual CPIF inflation (sample std = 0,68).

	t+4	t+8	t+12
Unconditional	0,76	0,79	0,78
Conditional on foreign Y, PiC and R	0,69	0,77	0,81
Optimal conditioning	0,62**	0,67**	0,71*
Conditional on all foreign variables	0,70	0,75	0,79
Naïve forecast	1,32	1,36	1,49
Iterative mean forecast	0,72	0,72	0,70
AR(1) model forecast	0,73	0,72	0,71

[34] The null hypothesis of the Diebold-Mariano test is that two forecasts are equally accurate (Diebold and Mariano, 1995) . *,** and *** denotes that the respective conditional forecasts are significantly more accurate than the unconditional forecast at the 10, 5 and 1 percent level respectively according to the Diebold-Mariano test with Newey-West standard errors. The row "optimal conditioning" shows the RMSFEs from (out of all possible combinations of variables) the best model at the specific horizon. The last three rows show the RMSFEs from three frequently employed benchmarks where the naïve forecast (or random-walk forecast) is equal to the last known realisation. The forecast using the iterative mean equals the mean up until the last realisation. The AR(1) model forecasts is the forecast from an recursively estimated autoregressive model of order 1.

[35] In the domestic New Keynesian price and wage Phillips curves in MAJA domestic inflation and wage inflation are related to domestic marginal costs and domestic unemployment, respectively, i.e. these equations do not include foreign variables directly. However, in MAJA global shocks to e.g. firm and consumer confidence generate largely simultaneous and similar effects on foreign and domestic real quantities. For example, such shocks will generate the strong correlation between foreign and Swedish unemployment noted by Andersson and Jonung (2020). But both the foreign and domestic Phillips curves in MAJA are estimated to be rather flat, i.e. the relationships between real and nominal quantities are generally rather weak. While the optimal conditioning set for CPIF inflation includes the foreign employment gap the overall improvement in forecast accuracy when conditioning on this variable is perhaps not large. Corbo and Strid (2020) contains a discussion of some of the challenges of modelling inflation and the apparently complex relationship between foreign and Swedish inflation.

Finally, the conditional policy rate forecasts improve significantly on the unconditional forecast at the 2- and 3-year forecast horizons; see Table 5. The RMSFEs of the conditional forecasts are roughly half and 30-40% of the RMSFE of the unconditional forecast at the 2- and 3-year horizons, respectively.

Table 5: RMSFEs for the policy rate (sample std = 1,64)

	t+4	t+8	t+12
Unconditional	0,85	1,32	1,61
Conditional on foreign Y, PiC and R	0,57	0,71***	0,65***
Optimal conditioning	0,48*	0,60***	0,50***
Conditional on all foreign variables	0,57	0,63***	0,54***
Naïve forecast	1,09	1,59	1,68
Iterative mean forecast	2,37	2,54	2,72
AR(1) model forecast	1,06	1,51	1,57

Results for the other 12 domestic variables in MAJA are reported in Appendix A (Tables A3-A14). For most Swedish variables, the standard conditioning and/or conditioning on all foreign variables forecasts are significantly more accurate than the unconditional forecasts. The exceptions are unemployment, wage inflation and CPIF inflation for which the unconditional forecasts are about equally accurate as the forecasts conditional on all foreign variables. The conditional forecasts for import prices excl. energy are actually less accurate than the unconditional forecasts. This is also the only variable for which the optimal conditional forecast is not significantly better than the unconditional forecast.

4.4 Analysis of systematic errors in MAJA (bias)

Squaring the forecast errors as in the RMSFE measure (or taking the absolute value of the forecast errors as in the MAE measure) ruins the possibility to say anything about whether one model tends to overpredict or underpredict the realisations (i.e. systematic errors or bias).

In this section we therefore study the mean forecast error, i.e. the average deviation between the forecasts and the realisations. The mean error thus gives an indication of whether the forecasting model systematically underpredict or overpredict the realisations. Table 6 shows the mean errors for each of the Swedish variables, for the unconditional forecast, the standard conditioning and the conditioning on all foreign variables for the horizons of 4, 8 and 12 quarters.

Table 6: Mean errors, computed as realisation minus forecast, of the unconditional and conditional forecasts. A positive (negative) value means that the forecast underpredicted (overpredicted) the realisation.

	Unconditional			Conditional on foreign Y, PiC and R			Conditional on all foreign variables		
Variables	**T+4**	**T+8**	**T+12**	**T+4**	**T+8**	**T+12**	**T+4**	**T+8**	**T+12**
GDP	-0,11	-0,34	-0,36	0,52	0,45	0,24	0,40	0,27	0,10
CPIF	-0,20	-0,26	-0,33	-0,29	-0,38	-0,45	-0,27	-0,36	-0,47
Policy rate	-0,21	-0,57	-0,91	0,08	0,15	0,10	0,14	0,19	0,12
Consumption	-0,29	-0,46	-0,41	-0,12	-0,12	-0,03	0,03	0,10	0,19
Investments	2,14	1,30	0,78	3,08	2,43	1,53	2,69	2,05	1,47
Imports	-1,08	-1,16	-0,68	-0,07	-0,00	0,08	-0,04	0,12	0,34
Exports	-2,33	-2,00	-1,29	-1,25	-0,61	-0,39	-1,05	-0,62	-0,35
CPIF excl. energy	-0,28	-0,32	-0,40	-0,34	-0,43	-0,53	-0,32	-0,41	-0,51
Import prices excl. energy	-0,58	-0,47	-0,48	-0,65	-0,60	-0,64	-0,68	-0,67	-0,74
Wages	-0,25	-0,43	-0,58	-0,19	-0,30	-0,41	-0,19	-0,32	-0,44
Unemployment	-0,07	-0,01	0,08	-0,17	-0,39	-0,51	-0,26	-0,46	-0,57
Employment gap	0,14	0,02	-0,16	0,27	0,49	0,56	0,38	0,57	0,63
Capacity utilisation	0,61	0,47	0,17	1,21	1,59	1,57	1,11	1,47	1,50
Corporate spread	-0,04	-0,02	0,10	-0,06	-0,08	-0,07	-0,05	-0,07	-0,08
Real exch. rate	1,38	0,99	0,90	1,00	0,85	0,85	0,89	0,54	0,61

Note: See Table A1 in Appendix A for a description of each variable.

An important factor in accounting for the forecast biases reported in the table is the relation between the steady state, or mean level, of a variable in MAJA and the sample mean of the variable in the forecast evaluation period. For example, steady state CPIF inflation in the model equals the 2 percent inflation target of the Riksbank, while the sample mean has been 1.5 percent. Import price inflation and real exchange rate growth are other examples where the discrepancy between the assumed steady state and the data sample mean is an important factor in accounting for the bias.

It is also important to consider how large the bias has been in different periods of the sample. For example, the conditional forecasts have underpredicted GDP growth, but in Figure 1 we note that this bias appears to be mainly a characteristic of the pre-crisis period, while the bias is smaller in the post-crisis period.

Additionally, the relative biases (as well as the forecasting precision) have to be interpreted in the light of the volatility of each variable. I.e. the relative large underpredictions of e.g. investments have to be viewed in relation to the fact that the volatility of investments is more than twice the size of the volatility of GDP (and about 10 times larger than the volatility of inflation), see Table A1 in Appendix A.

Deviations between the purely model-implied steady states of the variables and their sample means have in some cases been handled by the incorporation of so called "excess parameters"; see Corbo and Strid (2020). One striking example is, again, import inflation. The model-implied steady state of import inflation is identical to the steady state of CPIF inflation, i.e. 2 percent. But the sample mean of import inflation is below zero and in the absence of an excess parameter capturing this discrepancy the bias would become very large. The excess parameter for import inflation equals minus 2 percent, implying that average import inflation in the model equals zero. Since the sample mean is lower than zero, however, the forecasts on average still overpredict the realisations.

The possible tendency to over- or underpredict the realisations can also be graphically spotted from the complete distributions for the forecast errors, see Figure 10.[36]

Figure 10. The distributions of the forecast errors (conditioned on all foreign variables) of Swedish GDP

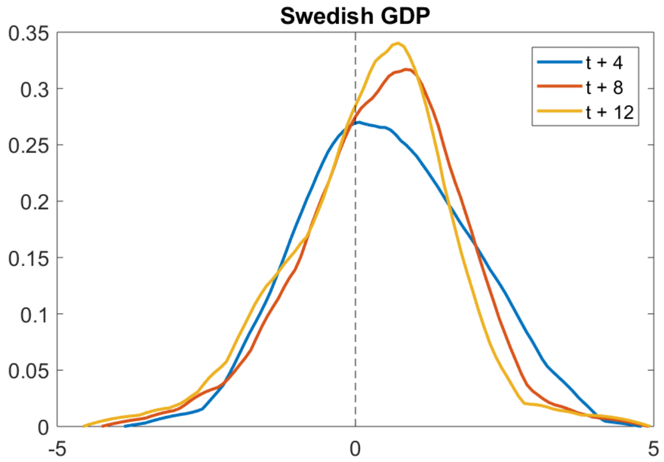


Figure 10 clearly shows that the forecast errors of Swedish GDP have been positive on average (underprediction of the realisations), but also that quite a large share of the distribution is on the negative side. Surprisingly, the variation in the forecast errors at the 4-quarters horizons seems larger than the ones at longer forecast horizons. The averages of the distributions are, however, relatively similar (see Table 6). The forecast error distributions of CPIF inflation and the policy rate are shown in Figure A10 in Appendix A.

5. Discussion/Conclusion

Being a small open economy, Sweden is strongly influenced by global economic developments, which is reflected in the generally strong comovement between Swedish and foreign-economy macroeconomic variables. The Riksbank's new DSGE model, MAJA, has been developed with the purpose of capturing these dependencies in a better way than the Riksbank's previous DSGE models; see Corbo and Strid (2020). The purpose of this memo has been to evaluate the forecasting performance of MAJA, with special emphasis on forecasts of Swedish variables conditional on realisations of foreign variables. In particular, we have tried to answer three questions:

First, is information on future foreign developments helpful in forecasting domestic variables?

The answer to this question is "yes". The conditional forecasts for most of the variables are often significantly more accurate than the unconditional forecasts. Also the biases are generally lower. In comparison with the Riksbank's previous DSGE model, RAMSES II, and a two-country block exogenous Bayesian vector autoregressive model, the gain from conditioning, i.e. the improvement in

[36] The distributions have been smoothed using so called Gaussian Kernel functions.

forecast accuracy when conditioning on foreign variables, is larger with MAJA. This suggests that the influence of the foreign economy on Sweden is better captured in MAJA.

Which of the foreign variables are particularly useful in forecasting Swedish variables?

The optimal conditional forecast of a Swedish variable in most cases involves conditioning on its foreign counterpart, e.g. in forecasting Swedish GDP growth it is important to include foreign GDP growth in the conditioning set. While using the variable-specific optimal conditional forecasts could be advocated from a pure RMSFE-based forecasting perspective this approach is impractical and, more importantly, it means that the multivariate consistency of the forecast gets lost.

Conditioning on the "standard" set of foreign variables containing GDP growth, CPI inflation and the policy rate, yields conditional forecasts which are generally difficult to substantially improve upon. A smaller set of conditioning variables is probably also preferable in a real-time forecasting situation since a larger set entails larger risks of foreign variable forecast errors and inconsistencies among the foreign variable forecasts. Also, given current practices, more emphasis is put on forecasting these three key variables compared to the other seven foreign variables in MAJA.

The results on optimal conditioning sets suggests that the foreign corporate spread adds value in forecasting many of the Swedish variables (it is included in the optimal set for 11 out of the 15 variables). Further, in our experience, information on financial variables is particularly important in times of financial turbulence, e.g. in 2008-2009 or in the current coronavirus crisis.

While it may be difficult to motivate a large foreign variable conditioning set, e.g. including all ten foreign variables, from a forecasting perspective it could serve other purposes. Conditioning on the full set of foreign variable forecasts the model should deliver a story in terms of shocks which is more consistent with the story of the foreign-economy forecaster. On the other hand, if a smaller conditioning set is used, the model will produce forecasts for the other (i.e. those not conditioned upon) foreign variables which are based on the historical patterns in the data. This may be desirable in general, but not necessarily in all situations. For example, in the current corona crisis many forecasts (both for Sweden and other countries) involve a relatively large decrease in consumption growth, larger than what the historical relationships in the data would suggest given the forecasted drop in GDP growth. In this type of situation one would presumably want to incorporate foreign consumption growth into the conditioning set.

In summary, these considerations lead to three broad conclusions. First, the standard conditional forecast appears to be a good and relatively simple benchmark, and "good enough" for most purposes. Second, the choice of conditioning set could to some extent depend on the objective, e.g. forecast accuracy versus storytelling. Third, there is no need to mechanically adapt a certain type of conditional forecast, e.g. in times of financial market turbulence forecast accuracy is probably improved by incorporating information on financial variables while the issue may be largely unimportant in more normal times.

Is information on foreign developments particularly helpful in forecasting certain domestic variables?

The results reported in this paper suggest that the Swedish variables can be broadly placed in two categories. One set of variables are strongly related to foreign variables in the data and their conditional forecasts improve quite substantially on the unconditional forecast. These variables are the growth rates of GDP and its components (consumption, investments, exports and imports), capacity utilisation, the corporate spread and the policy rate. For most of these variables more information on the foreign economy, through more foreign variables in the conditioning set, tends to improve forecast accuracy.

The other group of variables involve variables which are generally more weakly correlated with the foreign variables in the data, and hence also in the model. The conditional forecasts of these variables do not appear to improve substantially on the unconditional forecasts. The price inflation rates (CPIF inflation, CPIF excl. energy inflation and import inflation) and wage inflation fall into this category.

The two labour market variables, the employment gap and the unemployment rate are perhaps somewhere in between the two groups. While the optimal conditional forecasts of these variables improve significantly on the unconditional forecasts, the standard and conditional-on-all forecasts do not lead to significantly improved forecast accuracy.

Finally, it is well known that exchange rates are difficult to forecast, see e.g. Askestad et al. (2019). The optimal conditional forecast of the real exchange rate is significantly more accurate than the unconditional forecast and the various simple benchmarks. However, the absolute values of the RMSFEs of all forecasting approaches are large and certain episodes, e.g. the depreciation of the Krona in 2008, presumably have a very large influence on the RMSFEs. Therefore, in our view the RMSFEs of the real exchange rate forecasts reported here are probably of very limited value in learning how to better predict exchange rates.

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Appendix A. Data and results

Table A1. The measurement variables[37]

	Transformation	Mean	Std	Steady state
KIX2 GDP	Per capita, annualised, log QoQ. SA	1,10	1,79	1,3
KIX2 Consumption	Per capita, annualised, log QoQ. SA	0,90	1,10	1,3
KIX2 Investments	Per capita, annualised, log QoQ. SA	0,79	4,15	1,3
KIX2 Inflation	Annualised, log QoQ. SA	1,79	0,93	2,0
KIX2 Inflation, excl. energy	Annualised, log QoQ. SA	1,61	0,52	2,0
KIX2 Policy rate		1,61	1,69	3,0
KIX2 Wage inflation	Nominal, annualised, log QoQ. SA	2,18	0,60	2,5
KIX2 Corporate spread	Percentage points	1,83	0,46	1,8
KIX2 Unemployment rate		8,88	1,30	8,0
KIX2 Employment gap	Per capita	-1,41	2,05	0,0
GDP	Per capita, annualised, log QoQ. SA	1,50	2,65	1,8
Consumption	Per capita, annualised, log QoQ. SA	1,48	1,55	1,8
Investments	Per capita, annualised, log QoQ. SA	2,34	5,69	2,6
Exports	Per capita, annualised, log QoQ. SA	2,01	6,01	2,8
Imports	Per capita, annualised, log QoQ. SA	2,05	6,31	2,8
CPIF inflation	Annualised, log QoQ. SA	1,55	0,68	2,0
CPIF inflation excl. energy	Annualised, log QoQ. SA	1,34	0,56	2,0
Import prices excl. energy	Annualised, log QoQ. SA	-0,47	1,03	0,0
Policy rate		1,72	1,64	3,0
Unemployment rate		7,12	0,86	7,2
Employment gap	Per capita	-0,08	1,38	0,0
Wage inflation	Nominal, annualised, log QoQ. SA	3,06	0,66	3,8
Capacity utilisation		83,59	3,64	85,0
Corporate spread	Percentage points	1,74	0,42	1,8
Real exchange rate	Annualised, log QoQ. SA	0,92	5,62	0,0

Table A2. BVAR variables and their steady-state prior intervals

	Prior interval
KIX2 GDP	(1; 2)
KIX2 inflation	(1,5; 2,5)
KIX2 policy rate	(4,5; 5,5)
Employment rate	(0; 0,5)
GDP	(1,9; 2,1)
Wage inflation	(3,5; 4,5)
CPIF inflation	(1,99; 2,01)
Policy rate	(4,2; 4,3)
Real exchange rate	(log(120); log(135))

Note: 95 percent prior probability intervals for parameters determining the unconditional means. Prior distributions are all assumed to be normal. Variables are defined in Section 2.

[37] Percent unless otherwise stated. For the employment, unemployment and per capita measures, a population of 15-74 years old are used. Means and standard deviations have been calculated for the entire evaluation sample 2000Q1-2018Q4. See the Data and Estimation Appendix to Corbo and Strid (2020) for the data sources, SDMX codes bivariate correlations between the variables.

Table A3: RMSFEs for consumption (sample std = 1,55)

	t+4	t+8	t+12
Unconditional	1,16	1,43	1,37
Conditional on foreign Y, PiC and R	0,91	1,18	1,15
Optimal conditioning	0,85*	1,01*	0,98*
Conditional on all foreign variables	0,89*	1,07*	1,06
Naïve forecast	3,03	3,46	3,01
Iterative mean forecast	1,65	1,63	1,60
AR(1) model forecast	1,66	1,62	1,60

Table A4: RMSFEs for investments (sample std = 5,69)

	t+4	t+8	t+12
Unconditional	5,39	5,95	5,90
Conditional on foreign Y, PiC and R	4,57	4,35*	3,80
Optimal conditioning	4,29**	3,70**	3,26**
Conditional on all foreign variables	4,36*	3,78**	3,32*
Naïve forecast	12,05	13,87	13,93
Iterative mean forecast	6,16	6,35	6,20
AR(1) model forecast	6,29	6,35	6,19

Table A5: RMSFEs for exports (sample std = 6,01)

	t+4	t+8	t+12
Unconditional	5,40	6,09	6,23
Conditional on foreign Y, PiC and R	3,41	3,32*	3,34*
Optimal conditioning	3,24*	3,22**	3,30*
Conditional on all foreign variables	3,46	3,75*	3,86
Naïve forecast	10,71	12,48	12,56
Iterative mean forecast	6,57	6,59	6,44
AR(1) model forecast	6,62	6,60	6,44

Table A6: RMSFEs for imports (sample std = 6,31)

	t+4	t+8	t+12
Unconditional	4,95	5,96	6,12
Conditional on foreign Y, PiC and R	3,14*	3,55*	3,44
Optimal conditioning	2,97**	3,18**	3,08*
Conditional on all foreign variables	3,11*	3,34*	3,22*
Naïve forecast	10,72	13,36	12,52
Iterative mean forecast	6,82	6,70	6,25
AR(1) model forecast	6,99	6,77	6,27

Table A7: RMSFEs for CPIF inflation excl. energy (sample std = 0,56)

	t+4	t+8	t+12
Unconditional	0,64	0,73	0,69
Conditional on foreign Y, PiC and R	0,64	0,75	0,77
Optimal conditioning	0,57**	0,61**	0,63*
Conditional on all foreign variables	0,61	0,68	0,70
Naïve forecast	0,84	1,00	1,11
Iterative mean forecast	0,57	0,58	0,50
AR(1) model forecast	0,55	0,58	0,50

Table A8: RMSFEs for import prices excl. energy (sample std = 1,03)

	t+4	t+8	t+12
Unconditional	1,10	1,24	1,18
Conditional on foreign Y, PiC and R	1,13	1,25	1,22
Optimal conditioning	1,05	1,16	1,12
Conditional on all foreign variables	1,13	1,24	1,25
Naïve forecast	1,92	2,15	2,19
Iterative mean forecast	1,10	1,12	1,08
AR(1) model forecast	1,09	1,12	1,09

Table A9: RMSFEs for unemployment rate (sample std = 0,86)

	t+4	t+8	t+12
Unconditional	0,48	0,78	0,93
Conditional on foreign Y, PiC and R	0,52	0,72	0,79
Optimal conditioning	0,39*	0,51**	0,59**
Conditional on all foreign variables	0,58	0,79	0,86
Naïve forecast	0,78	1,19	1,29
Iterative mean forecast	1,53	1,46	1,34
AR(1) model forecast	0,83	1,38	1,70

Table A10: RMSFEs for employment gap (sample std = 1,38)

	t+4	t+8	t+12
Unconditional	0,82	1,23	1,37
Conditional on foreign Y, PiC and R	0,80	1,11	1,16
Optimal conditioning	0,62**	0,85**	0,85***
Conditional on all foreign variables	0,78	1,02*	1,04*
Naïve forecast	1,22	1,85	1,99
Iterative mean forecast	2,18	2,07	1,87
AR(1) model forecast	1,28	2,10	2,57

Table A11: RMSFEs for wage inflation (sample std = 0,66)

	t+4	t+8	t+12
Unconditional	0,55	0,63	0,77
Conditional on foreign Y, PiC and R	0,54	0,60	0,66
Optimal conditioning	0,52	0,55*	0,60*
Conditional on all foreign variables	0,55	0,62	0,70
Naïve forecast	0,82	0,89	0,96
Iterative mean forecast	0,90	0,97	1,04
AR(1) model forecast	0,78	0,95	1,03

Table A12: RMSFEs for capacity utilisation (sample std = 3,64)

	t+4	t+8	t+12
Unconditional	3,85	4,58	4,37
Conditional on foreign Y, PiC and R	2,93*	3,36*	3,14***
Optimal conditioning	2,78**	3,16***	2,97***
Conditional on all foreign variables	2,79**	3,19***	3,03***
Naïve forecast	4,27	5,57	5,69
Iterative mean forecast	3,88	4,02	4,08
AR(1) model forecast	4,08	5,02	4,81

Table A13: RMSFEs for corporate spread (sample std = 0,42)

	t+4	t+8	t+12
Unconditional	0,21	0,28	0,34
Conditional on foreign Y, PiC and R	0,19**	0,24**	0,28**
Optimal conditioning	0,16***	0,18***	0,18***
Conditional on all foreign variables	0,16***	0,18***	0,18***
Naïve forecast	0,25	0,37	0,46
Iterative mean forecast	0,45	0,47	0,49
AR(1) model forecast	0,28	0,39	0,45

Table A14: RMSFEs for real exchange rate (sample std = 5,62)

	t+4	t+8	t+12
Unconditional	5,18	5,22	5,66
Conditional on foreign Y, PiC and R	4,84	4,85	5,07
Optimal conditioning	4,36**	4,36**	4,66*
Conditional on all foreign variables	4,52*	4,59*	4,90
Naïve forecast	10,42	11,56	10,90
Iterative mean forecast	5,95	5,66	5,50
AR(1) model forecast	5,98	5,67	5,50

Figure A1. Swedish data

Figure A2. Foreign data

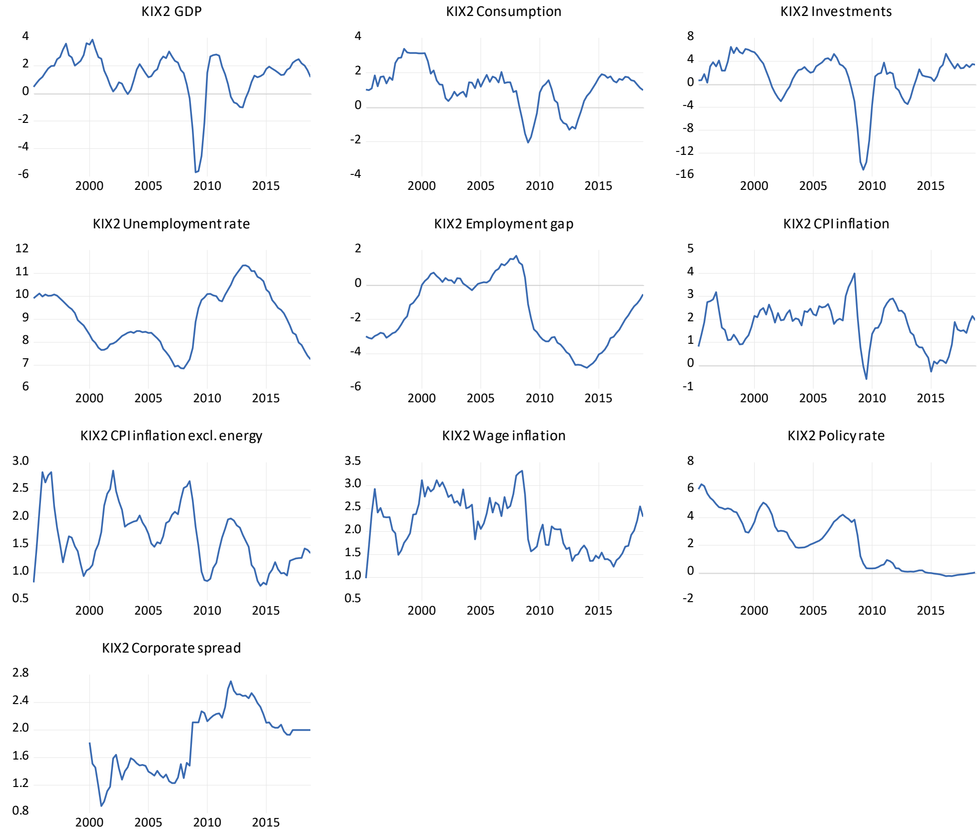


Figure A3. Unconditional and conditional (on all foreign variables) forecast errors for Swedish GDP. Horizon t+8 quarters.

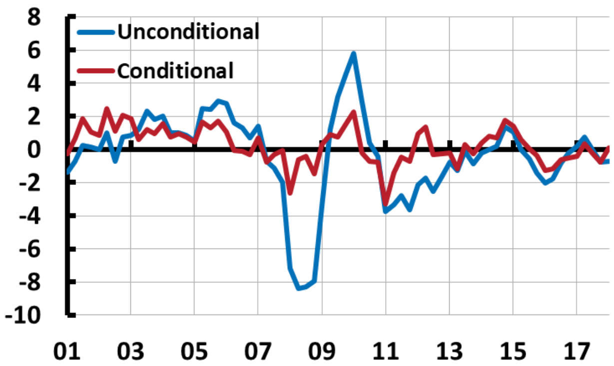


Figure A4. Unconditional and conditional (on all foreign variables) forecast errors for CPIF inflation. Horizon t+8 quarters.

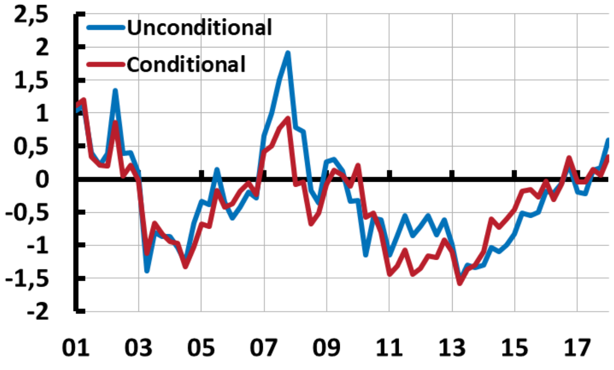


Figure A5. Unconditional and conditional (on all foreign variables) forecast errors for the policy rate. Horizon t+8 quarters.

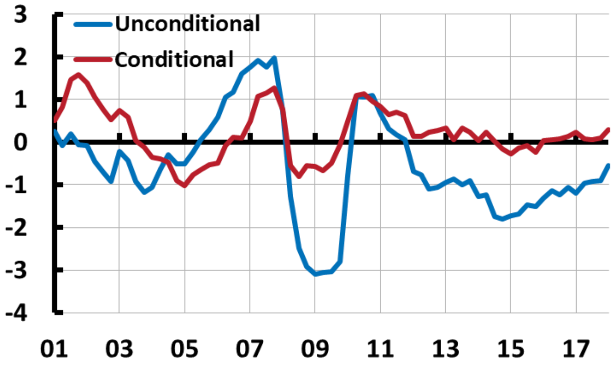


Figure A6. Conditional (standard conditioning) RMSFEs relative to the unconditional ones. Horizon t+4 quarters.

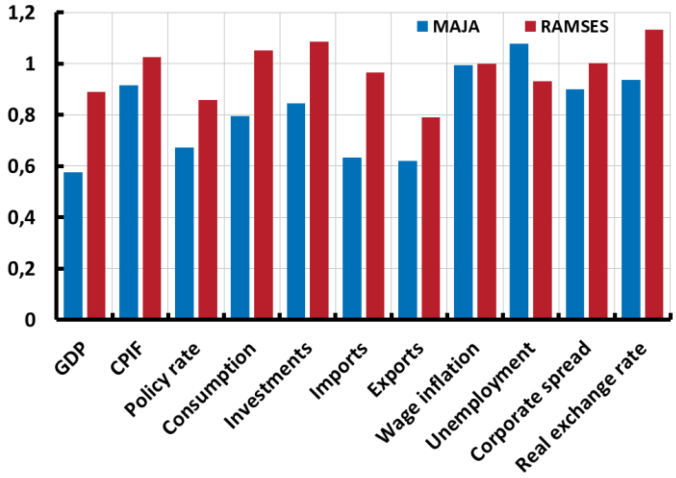


Figure A7. Conditional (standard conditioning) RMSFEs relative to the unconditional ones. Horizon t+12 quarters.

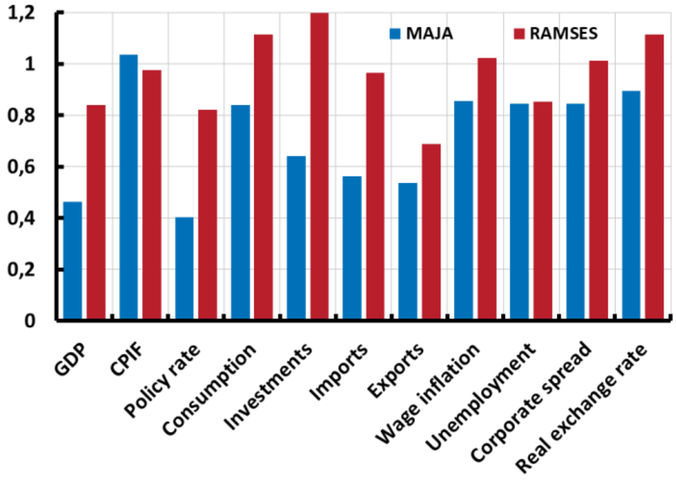


Figure A8a. Mean RMSFEs across the numbers of foreign variables. The improved ones. Horizon 8 quarters.

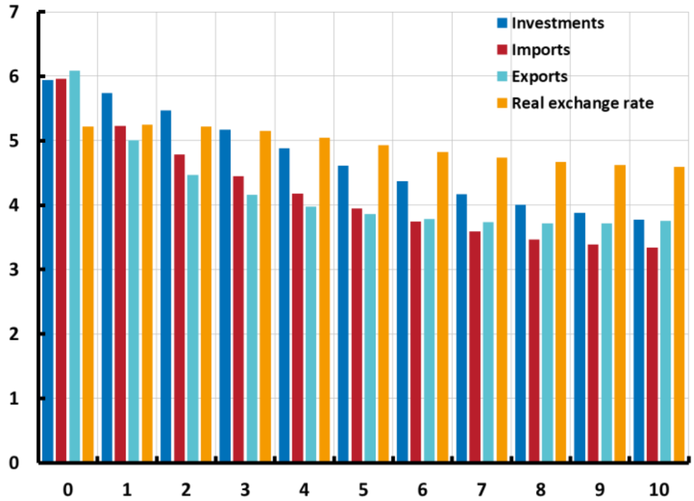


Figure A8b. Mean RMSFEs across the numbers of foreign variables. The improved ones. Horizon 8 quarters.

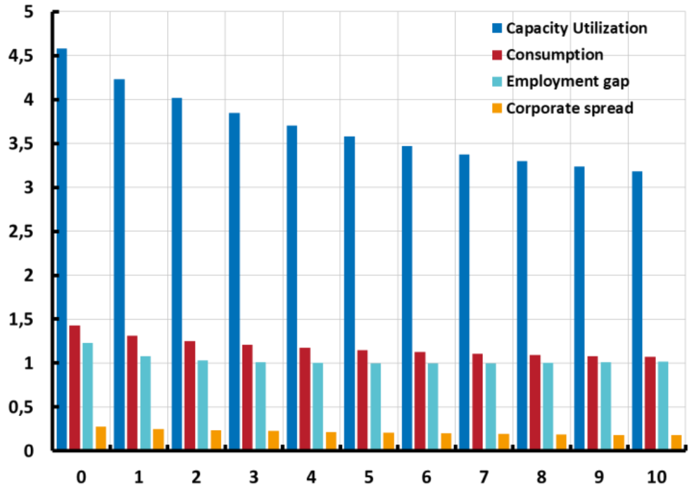


Figure A9. Mean RMSFEs across the numbers of foreign variables. The less improved ones. Horizon 8 quarters.

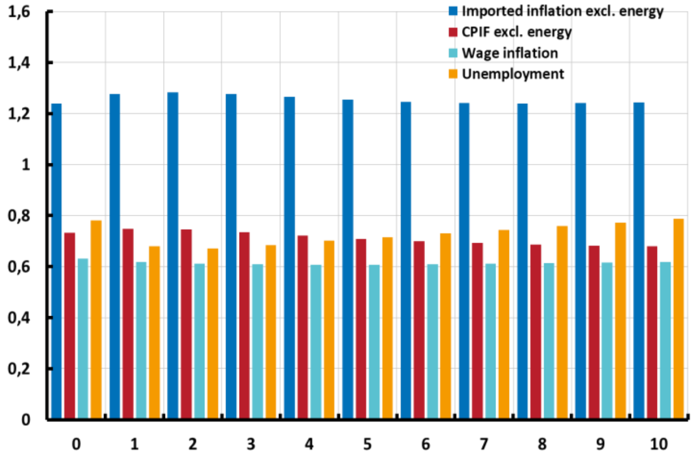
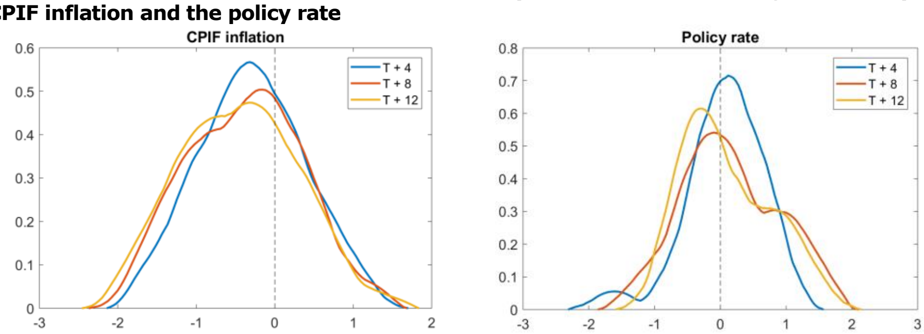


Figure A10. The distributions of the forecast errors (conditioned on all foreign variables) of CPIF inflation and the policy rate



Appendix B. The Bayesian VAR model

We use the Bayesian VAR model given by:

$$\mathbf{G}(L)(\mathbf{x}_t - \boldsymbol{\mu}) = \boldsymbol{\eta}_t,$$

As can be seen from the equation above, the model is expressed in deviations from its steady state. This feature was introduced by Villani (2009) and has the benefit that an informative prior distribution for the steady-state values of the variables in the system – the *n*x1 vector $\boldsymbol{\mu}$ – can be specified. Obviously, this can be particularly useful when forecasting e.g. Swedish CPIF inflation seeing that the Riksbank has an explicitly stated inflation target.[38]

The rest of the model is defined as follows: $\mathbf{G}(L) = \mathbf{I} - \mathbf{G}_1 L - \ldots - \mathbf{G}_m L^m$ is a lag polynomial of order *m*; the lag length of the model is in all cases set to $m = 4$. $\mathbf{x}_t$ is an *n*x1 vector of stationary variables and $\boldsymbol{\eta}_t$ is an *n*x1 vector of *iid* error terms fulfilling $E(\boldsymbol{\eta}_t) = \mathbf{0}$ and $E(\boldsymbol{\eta}_t \boldsymbol{\eta}_t') = \boldsymbol{\Sigma}$.

The priors of the model largely follow the convention in the literature. For $\boldsymbol{\Sigma}$ the prior is given by $p(\boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-(n+1)/2}$ and the prior on $vec(\mathbf{G})$, where $\mathbf{G} = (\mathbf{G}_1 \quad \ldots \quad \mathbf{G}_m)'$, is given by $vec(\mathbf{G}) \sim N_{mn^2}(\boldsymbol{\theta}_{\mathbf{G}}, \boldsymbol{\Omega}_{\mathbf{G}})$. It should be noted that the priors on the dynamics have been modified somewhat relative to the traditional Minnesota prior; this is standard when using Villani's specification.[39] The prior on $\boldsymbol{\mu}$ is given by $\boldsymbol{\mu} \sim N_n(\boldsymbol{\theta}_{\boldsymbol{\mu}}, \boldsymbol{\Omega}_{\boldsymbol{\mu}})$ and is specified in detail in Table A2 in Appendix A. The hyperparameters of the model are also in line with mainstream choices in the literature: we set the overall tightness to 0.2, the cross-variable tightness to 0.5 and the lag decay parameter to 1.

[38] Villani's specification of the BVAR can improve forecast accuracy when it comes to inflation. This has been shown by, for example, Beechey and Österholm (2010).

[39] The prior mean on the first own lag for each variable is here set equal to 0.9 and all other coefficients in **G** have a prior mean of zero.

Appendix C. Forecast errors and measures for forecasting precision and bias

Realisations and forecast errors

Let y_t and $\hat{y}_t$ be the *realisation* and *forecast* for the same variable. The forecast error, e_t, is defined as: $e_t = y_t - \hat{y}_t$. Thus, a positive forecast error means that the model underpredicted the realisation and a negative forecast error implies an overestimation. The *mean error*, ME, (often referred to as *bias*) is the arithmetic mean of the forecast errors:

$$ME = \frac{1}{n}\sum_{t=1}^{n} e_t$$

where *n* is the number of forecasts. The mean error shows how much the forecasts deviated from the realisations on average. The mean error thus gives an indication of whether the forecast model systematically under- or overpredicted the realisations. Since large over- and underpredictions can cancel each other and generate a small mean error, it cannot be used to assess forecast precision.

Forecast accuracy measures

The mean absolute error, MAE, is the arithmetic mean of the absolute values of the forecast errors:

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |e_t|$$

Thus, this measure ignores whether the forecast errors are positive or negative. The size of the forecast errors affects the mean absolute error in a linear manner. That is, an increase in the forecast error from three to four percentage points penalises the forecast accuracy as much as an increase in the forecast error from one to two percentage points.

The mean square error, MSE, is the arithmetic mean of the squared forecast errors. Like the mean absolute error, this measure ignores whether the forecast errors are positive or negative. The fact that the errors in this measure are squared means that large forecast errors, unlike in the case of the mean absolute error, contribute more than proportionally to the mean value. An increase in the forecast error from three to four percentage points then penalises the forecast precision more than an increase from one to two percentage points. The root mean square forecast error, RMSFE, is the square root of the mean square error:

$$RMSFE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$$

It thus contains the same information as the mean square error but is comparable in size to the mean error and the mean absolute error.

Appendix D. A simple framework to aid interpretation

Here we use a simple framework to provide some intuition for the discussions on the relative performance of unconditional and conditional forecasts in different models in the memo. The models in the paper are dynamic multivariate models and it may therefore be difficult to explain exactly why one model performs better or worse in comparison with some other model in a forecast evaluation. Here we therefore consider a simpler framework - a static bivariate model - to convey some basic ideas about how the relative accuracy of a model's unconditional and conditional forecasts provides information both on the strength of the (empirical) dependencies among variables and how well these dependencies are captured by a certain model.

Consider a pair of variables x and y which are distributed according to a bivariate normal distribution (this is the assumed data-generating process, DGP). Relating to the discussion in the memo, variable y would represent a domestic variable and the variable x would represent a foreign variable. We assume that x is known and that we are interested in forecasting the value of y.

The unconditional mean forecast of y is provided by

$$E(y) = \mu_y$$

while the conditional mean forecast of y given x is provided by

$$E(y|x) = \mu_y + \frac{\sigma_y}{\sigma_x}\rho(x - \mu_x).$$

The conditional variance is given by

$$V(y|x) = (1 - \rho^2)\sigma_y^2,$$

where

$$E(x) = \mu_x\,, V(x) = \sigma_x^2, E(y) = \mu_y \text{ and } V(y) = \sigma_y^2$$

are the respective means and variances of the two variables and where ρ is the correlation coefficient between x and y (which describes the linear dependence between the two variables). While five parameters describe the properties of x and y we will assume that uncertainty is restricted to the parameter ρ. That is, our model (indexed by m) of x and y equals the DGP with the only exception that the model´s value of the correlation coefficient, ρ_m, does not necessarily equal its true value, ρ. The parameter ρ thus describes the true relationship between x and y while ρ_m describes the relationship between the two variables in the model. To simplify the framework further we assume (without loss of generality) the following values

$$\mu_x = 0, \sigma_x^2 = 1,\ \sigma_y^2 = 1$$

while the value of μ_y is left undetermined.

Now, we are interested in how different combinations of ρ (the true correlation) and ρ_m (the model correlation) affect the unconditional and conditional model forecasts of y, $E_m(y)$ and $E_m(y|x)$ respectively. Since the unconditional forecast does not depend on the correlation between the variables it is always the case that $E_m(y) = E(y) = \mu_y$ since μ_y is assumed to be known to the modeller/forecaster.
Below we consider four cases where ρ and ρ_m are assigned values to reflect the cases of "weak" ($\rho = 0$) and "strong" ($1 \geq \rho \gg 0$) correlations between the variables, respectively.[40] The four cases are:

- Case 1: Weak correlation between x and y which is captured by the model, $\rho_m = \rho = 0$.
- Case 2: Strong correlation between x and y which is captured by the model, $1 \geq \rho_m = \rho \gg 0$.
- Case 3: Strong correlation between x and y which is not captured by the model, $1 \geq \rho \gg \rho_m = 0$.
- Case 4: Weak correlation between x and y which is not captured by the model, $1 \geq \rho_m \gg \rho = 0$.

As in the main part of the memo we will assume that a "perfect forecast" of the foreign variable x is available, i.e. its value is known, and focus on the forecast of y conditional on x. The optimal forecast of y given x is the conditional expectation $E(y|x) = \mu_y + \rho x$.[41] The model unconditional and conditional forecasts are provided by $E_m(y) = E(y) = \mu_y$ and $E_m(y|x) = \mu_y + \rho_m x$, respectively.

To summarise we will then study three forecasts

- The optimal forecast: $E(y|x) = \mu_y + \rho x$
- The model's unconditional forecast: $E_m(y) = \mu_y$
- The model's conditional forecast: $E_m(y|x) = \mu_y + \rho_m x$

with the four different assumptions on ρ and ρ_m listed above.

CASE 1: no correlation between x and y which is captured by the model, $\rho_m = \rho = 0$.
In this case there is no correlation between x and y, $\rho = 0$, and it is furthermore assumed to be captured by the model since $\rho_m = \rho$. Since the correlation is zero, knowledge of the foreign variable x is not helpful in forecasting y. The conditional forecast of y given x, which is the optimal forecast, equals the unconditional forecast and the model conditional forecast equals the optimal forecast. All three forecasts of interest are equal:

$$E_m(y|x) = E_m(y) = E(y|x)$$

In a forecast evaluation this case would be recognized through a similarity of the unconditional and conditional forecasts, and hence also a similarity of statistics computed based on the forecasts or forecast errors, e.g. RMSFEs. For example, this case provides intuition for why the conditional on foreign variables MAJA forecast of CPIF inflation does not improve much on the unconditional forecast. First, the relationship between CPIF

[40] The only reason we assume $\rho \gg 0$ rather than $\rho = 1$ is to illustrate how the difference between forecasts depend on ρ. Note also that the correlation coefficient may just as well be strongly negative, instead of positive. It would not affect the discussion here.
[41] This forecast is optimal in the sense that it minimises the squared expected forecast error.

inflation and the foreign variables in the data is generally rather weak (here very coarsely approximated by $\rho = 0$) and, second, the weak correlations are presumably reasonably well captured by the model (here coarsely approximated by $\rho_m = \rho$).

CASE 2: strong correlation between x and y which is captured by the model, $1 \geq \rho_m = \rho \gg 0$.
In this case knowledge of the variable x improves the forecast, i.e. the unconditional forecast is no longer optimal (which it was in case 1). The model conditional forecast equals the optimal forecast, $E_m(y|x) = E(y|x)$, and the deviation of the unconditional forecast from the optimal forecast equals

$$E(y|x) - \mu_y = \rho x.$$

We note that the "bias" (or "error") of the unconditional forecast is larger i) when the true correlation, ρ, between the variables is large and ii) when the deviation between the foreign variable x and its mean, $\mu_x = 0$, is large (which is expected to be the case in e.g. a crisis).

This case can be used to understand why the conditional forecasts of variables such as GDP growth and the repo rate improves on the unconditional forecasts using MAJA. First, there are strong relationships between these domestic variables and foreign variables in the data ($1 \geq \rho \gg 0$) and, second, these relationships appear reasonably well captured by MAJA ($\rho_m = \rho$). This case further illustrates that the error of the unconditional forecast will be larger in extreme situations, i.e. when x deviates a lot from its mean, μ_x. As the GDP forecasts displayed in the paper show, the differences between the unconditional and conditional forecasts are particularly large in the period 2008-09 and these observations are obviously influential for the computations of the RMSFE:s of the unconditional forecasts.

CASE 3: strong correlation between x and y which is not captured by the model, $1 \geq \rho \gg \rho_m = 0$.
This case captures in a simplified way the concerns raised by Justiniano and Preston (2010) regarding the relationships between foreign and domestic variables in open-economy DSGE models. While there is a strong relationship between the two variables, $1 \geq \rho \gg 0$, it is not captured by the model, $\rho_m = 0$. The model unconditional and conditional forecasts are equal, $E_m(y|x) = E_m(y) = \mu_y$, and the deviation from the optimal forecast equals ρx.

We note that this case may be difficult to separate from case 1 above since in both cases 1 and 3 the conditional model forecasts equal the unconditional forecasts, $E_m(y|x) = E_m(y)$. In a forecast evaluation we would presumably observe that the RMFSE:s of the two forecasts are similar. But while "all is well" if we are in case 1, being in case 3 instead indicates that the model is "poor". One way to separate the two cases is by introducing a second, competing, model. Assume that there is a second model (indexed by n) and that it is found that the conditional forecast of model n is more accurate than the conditional forecast of model m. This would provide evidence that there is a strong correlation between x and y but that it is not captured by model m (case 3). On the other hand, if it is difficult to find a competing model with an improved conditional forecasting performance it would indicate that case 1 is a better description of model m.[42]

[42] Whether a lack of improvement of the conditional forecast upon the unconditional forecast is a concern can of course be further studied e.g. by comparing the cross-correlations between variables in the model and in the data.

To illustrate this reasoning, consider the Ramses forecasts of GDP growth in the main part of the memo. The RMFSE of the Ramses conditional GDP forecast improves somewhat on the RMFSE of the unconditional forecast but maybe not by much so here we will consider this situation as described by either case 1 (weak correlation which is captured by the model) or case 3 (strong correlation which is not captured by the model). But next we observe that the conditional GDP growth forecast of MAJA improves substantially on the Ramses conditional forecast, and hence we conclude that forecasting GDP growth using Ramses is best described by case 3. At the same time we conclude that forecasting GDP growth using MAJA is best described by case 2 (strong correlation which is captured by the model). In summary we conclude that the strong correlations between Swedish GDP growth and foreign variables in the data are not adequately captured in Ramses.

CASE 4: no correlation between x and y but a large correlation in the model, $\rho = 0, \rho_m \gg 0$. In this case the model correlation "exaggerates" the true (non-existent) relationship, a situation which appears unlikely to arise in an estimated model.[43] Here the model unconditional forecast equals the optimal forecast while the deviation of the model conditional forecast from the optimal forecast equals $E_m(y|x) - \mu_y = \rho_m x$. This case would presumably be recognized by a larger RMSFE of the conditional forecast in comparison with the RMFSE of the unconditional forecast. This is probably the worst case for the modeller/forecaster since the model contains a strong relationship with no support in the data.

Finally, we illustrate a potential difficulty in interpreting a forecast evaluation in the more common setting where one conditions on actual forecasts rather than on realisations of the variables (as has been assumed so far). We assume that the forecaster both has the "wrong" model of the relationship between x and y, i.e. $\rho_m \neq \rho$, and furthermore the "wrong" forecast of the conditioning variable x, $x_m \neq x$. The optimal forecast of y given x is (as always) provided by $E(y|x) = \mu_y + \rho x$ and the model forecast using the wrong value for the conditioning variable x is given by $E_m(y|x) = \mu_y + \rho_m x_m$. One intriguing possibility here is that even if both the model *and* the conditioning information are poor the forecast could yet turn out to be accurate, which happens if $\rho_m x_m = \rho$x. Here the two "errors" cancel and the resulting forecast becomes "right". But there is then a risk that the forecaster perceives the situation incorrectly and draws the conclusion that the forecast of the foreign variable x is accurate and that the relationship between x and y is well described by the model.

More importantly this example illustrates that in order to learn something useful from an evaluation the sources of forecast error should be isolated and studied separately. In the main paper, by conditioning on realisations of the foreign variables it is possible to focus on the pure conditional forecasting performance of the respective models.

[43] Corbo and Strid (2020) compare the correlations in the data with the corresponding correlations implied by MAJA for the 300 pairs of observed variables in the model. The comparison illustrates that the model implied correlations are generally lower than those in the data (in absolute values). This suggests that case 4 is probably less relevant in practice, at least in the context of medium or large scale estimated DSGE models.

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