Staff memo

Expectations, premiums and forward rates

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Summary

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According to the expectations hypothesis of interest rates, longer maturity interest rates are determined by expectations of shorter maturity interest rates. The hypothesis is a usual benchmark when analysing changes in forward rates and is frequently used in macro-economic models.

If the expectations hypothesis is not valid, one cannot analyse longer maturity interest rates solely in terms of expectations. Empirical support for the hypothesis is weak, both in Sweden and internationally. A common explanation for the failure of the hypothesis is the occurrence of various kinds of risk compensation (premiums) in market interest rates. The empirically weak support for the expectations hypothesis means that there are several ways to measure market expectations and it is unclear which the best method is.

Depending on the measure of expectations, the difference between the Riksbank forecast of the future repo rate and market-based measures of expectations during the period 2011-2012 range from 1.5 percentage points to markedly smaller differences of 0.5 percentage points at a two-year horizon. In this staff memo, we discuss methods to model expectations and to separate them from premiums in market interest rates. Different measures of the premiums and hence, different measures of market expectations, are shown and we illustrate some of the uncertainty in this analysis.

An overall conclusion is that forward interest rates are not a pure reflection of expectations of the central bank policy rate. Talking about the markets interest rate expectations requires various assumptions and the analysis of expectations will always be fraught with uncertainty.

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Expectations, premiums and forward rates

Introduction

A common assumption is that financial market prices, and specifically interest rates, to a large extent are determined by expectations. It is also thought that monetary policy, to a large extent, works through expectations. For a central bank, the question of how changes in the monetary policy stance is transmitted to changes in interest rates, financial prices and to the general economic conditions and inflation, is central. However, measuring expectations is not an easy task as they are not directly observable. In this memo, we briefly discus a popular theory of interest rates, the expectations hypothesis, that is often used for interpreting changes in interest rates. We then discuss that the empirical support for this hypothesis is weak, both internationally and in Sweden. If there are no obvious measures of interest rate expectations, one has to analyse several imperfect measures and combine information from different sources. We propose one such method where expectations can be modelled and combined with other measures of interest rate expectations. The method we discuss here clearly separates movements in interest rates into changes of expectations and changes in (risk) premiums. Potentially, this method can give us better information on how changes in the monetary policy stance is affecting various interest rates.

What is the expectations hypothesis of interest rates?

In this section, we discuss the expectations hypothesis, a very popular theory for analysing the yield curve (or the term structure of interest rates, as it is sometimes also called). Simply stated, the yield curve shows how interest rates of different maturities are linked.[2] For a central bank that controls the very short end of the yield curve, the risk-free overnight interest rate, it is of importance to analyse how changes in this interest rate affect the rest of the yield curve, and how best to measure that influence. To that end, the expectations hypothesis is often used.

A simple variant of the expectations hypothesis[3]

A common view on how interest rates of different maturities are linked is based on the expectations hypothesis (henceforth EH). Simplified, the EH states that a two-year interest rate today, $i(2,t)$, is equal to the mean of a one-year interest rate today, $i(1,t)$, and the expected one-year interest rate in one year's time, $E_t i(1,t+1)$:

$$i(2,t) = \frac{1}{2}\{i(1,t) + E_t i(1,t+1)\}$$

The EH implies that an investor buying a two-year bond has the same expected return as another investor who rolls a one-year bond over a period of two years. A different way of saying this is that interest rates for longer maturities are related to expectations of short maturity interest rates.

Why are central banks interested in the expectations hypothesis?

For a central bank, it is important to investigate whether or not the EH is an adequate description of changes in the yield curve. At the core, this issue has to do with how expectations of the central bank policy rate affects market rates and how this can be measured. If the EH is valid, changes in longer maturity market rates are easy to analyse. For example, an increase in the two-year interest rate can be interpreted as an increase in the

[2] See Campbell (1995) for a discussion of the concept.

[3] A more exact definition is built on the assumption that the bond market is free from arbitrage. The fact that the market is free of arbitrage is guaranteed by the existence of a risk neutral probability measure. Under these assumptions, all bonds will have the same expected return as the risk-free interest rate (assuming equal holding periods). The expectations hypothesis is valid under the assumptions of no-arbitrage and the existence of a risk neutral probability measure. If this is the case, then longer maturity interest rates are determined only by expectations of short maturity interest rates.

expected one-year interest rate one year ahead, if the current one-year interest rate is unchanged:

$$\underbrace{i(2,t)}_{increased} = \frac{1}{2}\left\{ \underbrace{i(1,t)}_{unchanged} + \underbrace{E_t i(1,t+1)}_{increased} \right\}$$

If the EH is valid, it is also easy to evaluate whether the central bank forecast of the policy rate (the interest rate path) is in line with market expectations or if there is a discrepancy. Questions about the 'credibility' of the interest rate path are easy to analyse. Also, it is sufficient to have just one interest rate in the macro-models used for policy analysis in order to have an adequate description of the transmission mechanism of monetary policy. However, if the EH does not hold, there are reasons to expand on this analysis.

There are signs of time-varying (risk) premiums in financial market data

In this section, we summarize some of the international empirical evidence on the EH. **The empirical support for the hypothesis is weak**. When we replicate one of the classic tests of the EH on Swedish data, we find the support for the hypothesis to be equally weak in Sweden.

International research on the EH

In one of the classic studies of the EH, Campbell and Shiller (1991) find the hypothesis to be rejected based on US data. It is important to note that these classic tests of the EH actually tests two hypotheses at the same time: the validity of the EH and the rationality of expectations in the money and bond markets. International research on the EH is indeed vast and we have no intention of covering the whole field. It is sufficient to say that the hypothesis is usually rejected, but the reason for this is more of a debate in the scientific community.

Guidolin and Thornton (2008) have rejected the EH but argue that the reason for this is the lack of rationality of expectations of future interest rates in bond markets.[4] Along those lines, Carriero, Favero and Kaminska (2006) argue that when expectations are modelled as a rational (unbiased and efficient) forecast of future interest rates, the EH tends not to be rejected. A more common explanation behind the failure of the EH is the existence of time-varying (risk) premiums. Cochrane and Piazzesi (2005) study the EH based on US data and reject the hypothesis. The authors find clear evidence of time-varying premiums in the yield curve and evidence that the premiums are also possible to forecast. Another, often cited, study is Dai and Singleton (2002), who find clear evidence of time-varying premiums. According to the authors, this explains why the EH is rejected. It also explains why, once the premiums are controlled for in the data, the EH cannot be rejected.

Studies of the EH based on Swedish data are much more sparse, but a relatively new study of the hypothesis that also includes Swedish interest rate data is described by Beechey, Hjalmarsson and Österholm (2009). The authors look at the hypothesis in a number of countries using econometric techniques that take into consideration the fact that interest rates have a high degree of persistence. The authors reject the EH for all countries and argue that this rejection is consistent with the existence of time-varying premiums.[5] In the appendix, we show one of the classic tests of the EH for Swedish data and we illustrate that the hypothesis is rejected for most of the maturities.[6]

To conclude, using the EH to interpret movements in market interest rates should be done with caution. In some of the international research on the yield curve, the EH is retained, but the analysis is usually extended to allow for the existence of time-varying

[4] See the discussion of rationality in Beechey and Österholm (2012). Also see Alsterlind (2017), where we find similar results, namely that most (all) forecasts are biased and would fail tests for unbiasedness and efficiency, at least for longer horizons.
[5] This is not evidence of the existence of time-varying premiums, just of the fact that the empirical results are consistent with their presence.
[6] For maturities up to two years, the test shows that the EH is not easily rejected in this particular case.

premiums. This type of analysis of the yield curve is described in Dai and Singleton (2002) and in De Rezende (2017a). In short, this analysis expands the relationship between the two-year interest rate, the one-year interest rate and the one-year interest rate expected in one year's time to include an additional component:

$$i(2,t) = \frac{1}{2}\{i(1,t) + E_t i(1,t+1)\} + tp_t$$

where tp_t denotes the premium. The main issue is that neither the expected one-year interest rate nor the premium can be observed. How this influences the analysis and interpretation of the yield curve is something we devote the remainder of this memo to.

How we measure expectations affects how we interpret movements in forward rates

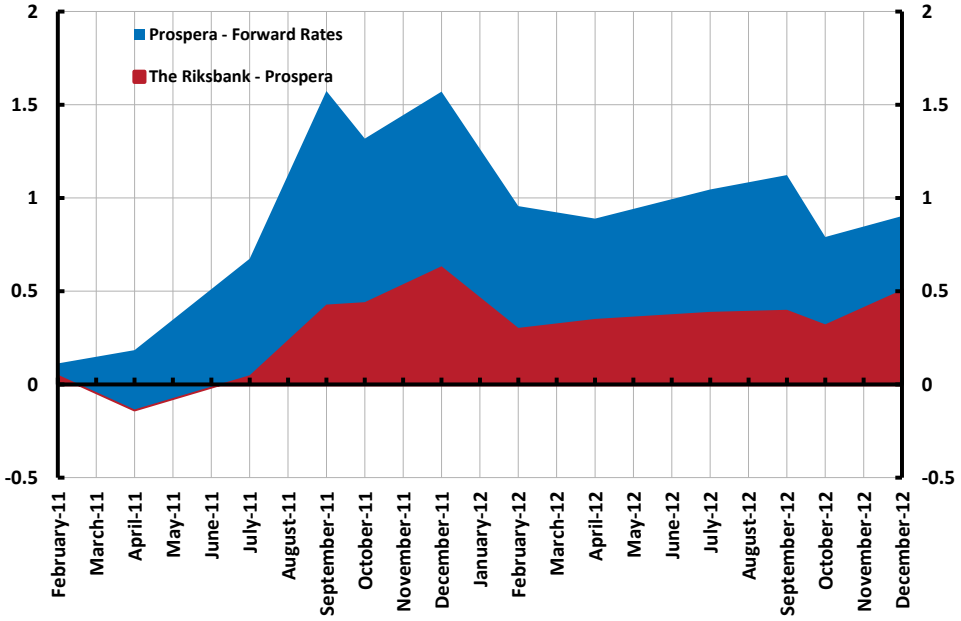
If the EH is not valid, we have **no simple way to measure market expectations of future interest rates**. This means we have to analyse different measures, and base our conclusions on several imperfect measures. One such measure is to use surveys of interest rate expectations, a measure that also has a relatively good forecasting performance of the future policy rate. But with several imperfect measures (possibly several surveys), are there some measures that we trust more than others? This question is not easy to answer and we have reason to return to it further on. What is clear is that the choice of measure for expectations will influence how we interpret movements in market interest rates, something we will discuss in the section below.

Forecasts of the policy rate in Sweden during 2011-2012

In this section, we take a small digression and look at the forecasts of the policy rate in Sweden during the years 2011-2012. In the debate on monetary policy in Sweden, there has been a focus on differences between market interest rates (in this case: forward interest rates) and the forecasts published by the Riksbank. As we argue, the use of market interest rates as a pure measure of market expectations has weak support in data, both in Sweden and internationally. Other measures of expectations exist and should also be analysed, especially if one suspects that substantial movements in premiums have occurred. One such alternative measure of market expectations of future interest rates is the survey performed by Prospera.[7] As we have shown elsewhere, see Alsterlind (2017), such survey expectations provide relatively good forecasts of the policy rate.[8]

[7] See http://www.prospera.se/inflation-expectations/
[8] Alsterlind (2017).

Figure 1: The difference between the Riksbank repo rate forecasts and surveys and the difference between surveys and market interest rates, 2011-2012, at the two-year horizon
Percentage points



Source: The Riksbank, Prospera and the author's own calculations

In Figure 1, the sum of the red and blue areas show the difference between the Riksbank forecast of the repo rate and the market interest rates (forward rates in this case) at the two-year horizon. We can see that in September and December 2011, the difference was greatest, slightly more than 1.5 percentage points. The red and blue area decomposes this difference into two parts. Looking at just the red area in Figure 1, we can see that the difference between the Riksbank forecast and the expected future interest rate according to the survey by Prospera was considerably smaller. On those two occasions (September and December 2011), the difference was 0.4 and 0.6 percentage points respectively. **The difference between the Riksbank forecast and the Prospera survey was about 0.3 percentage points on average during the period**. Thus, it is possible that the main part of the difference between the market interest rates and the Riksbank forecasts were due to other factors rather than differences in expectations. It is important to stress that both measures of market expectations were lower than the Riksbank forecast. However, there is a clear if the difference between the Riksbank and the market expectations of future interest rates was 1.5 percentage points or if it was 0.5 percentage points.

When analysing the difference between market expectations and the Riksbank forecast, the question of what measure is the best representation of the true expectations is central. One explanation for the relatively large discrepancy between market interest rates and the survey is that that the money market traders and the respondents to the Prospera survey are not necessarily the same people. So far, we have also discussed surveys like Prospera as something homogenous. This is not the case and survey measures of expectations can have clear discrepancies among various respondents.

There can also be other reasons for the discrepancy, such as shifts in the premiums. There is some evidence that the premium on low risk assets could have been negative at that time. Central bank policy measures (quantitative easing) and a general 'flight to quality', as a consequence of the euro crisis, could have pressed premiums on low-risk assets to extremely depressed levels, and even into negative territories. Market interest rates would have been affected by this and a negative (or lower than usual) premium and could then have contributed to a relatively better forecast performance.[9] Of course, this does not mean that

[9] When we discuss market interest rates in this context, we are talking about interest rates on government bonds (or close substitutes), or market rates where the credit risk premiums have been subtracted. Subtracting credit risk premiums from interbank interest rates

monetary policy can totally ignore movements in premiums. However, one should be careful to distinguish between movements in expectations and movements in premiums.

One conclusion is that one can never be sure what measure is the best representation of true expectations and a central bank has to recognise the vast uncertainty in the matter. In the following section, we will discuss how to form a view despite this uncertainty.

Issues measuring expectations and their implication for the analysis of the yield curve

If the EH alone cannot be used to analyse market interest rates, the analysis has to be refined to distinguish between expectations and premiums. However, such an analysis is inherently uncertain and one has to be prepared to constantly update one's view as new evidence presents itself. In this section, we will discuss a method where we could carry out that analysis in a structured way.

Models of the yield curve and their implication for expectations and the premium

As we have already pointed out, there is vast international research showing that the EH is usually rejected. This means that it is unclear to central banks and others how to measure expectations. However, there is a framework of analysis where the separation of expectations and premiums is treated in a structured way. This framework is built on term structure models that are being used by many central banks and that have a foundation in theoretical finance, see Duffie and Kan (1996).[10]

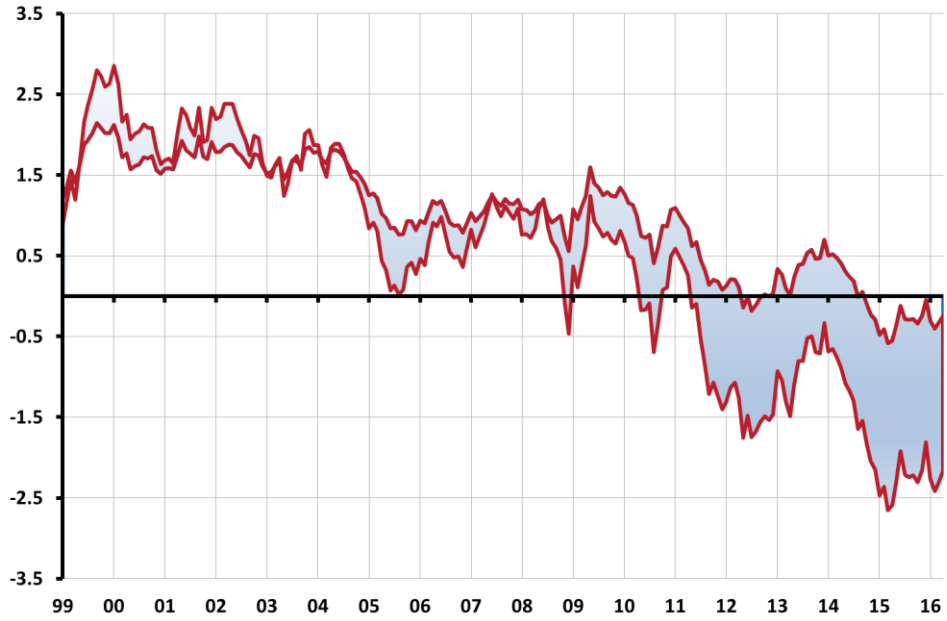
Dai and Singleton (2002) show that this class of models encompasses the EH but at the same time allows for an explanation of why the hypothesis is found to be rejected in empirical tests. By specifying a time-varying premium, a model that is consistent with the EH, but that also explains some of the empirical features of the yield curve can be constructed. As with all models, the result depends on the assumptions. In the appendix of Alsterlind (2017), we show that slight variation in the assumptions can lead to huge differences in how expectations and premiums are separated. In Figure 2, we show how different term structure models measure the premium, and thus also the expectations.

during the time of the euro crisis severely understates the actual interest rates levels prevailing at that time. If one is interested in how "actual" interest rates compare to central bank forecasts, it is not obvious that one should deduct neither the "term" premiums nor the "credit risk" premiums from the forward rates.

[10] See also the appendix in Alsterlind (2017).

Figure 2: Range of the premium of 5 years' maturity in Sweden according to different models

Percent



Source: The Riksbank

Remark: The shaded area show the measures of the premium from different models.

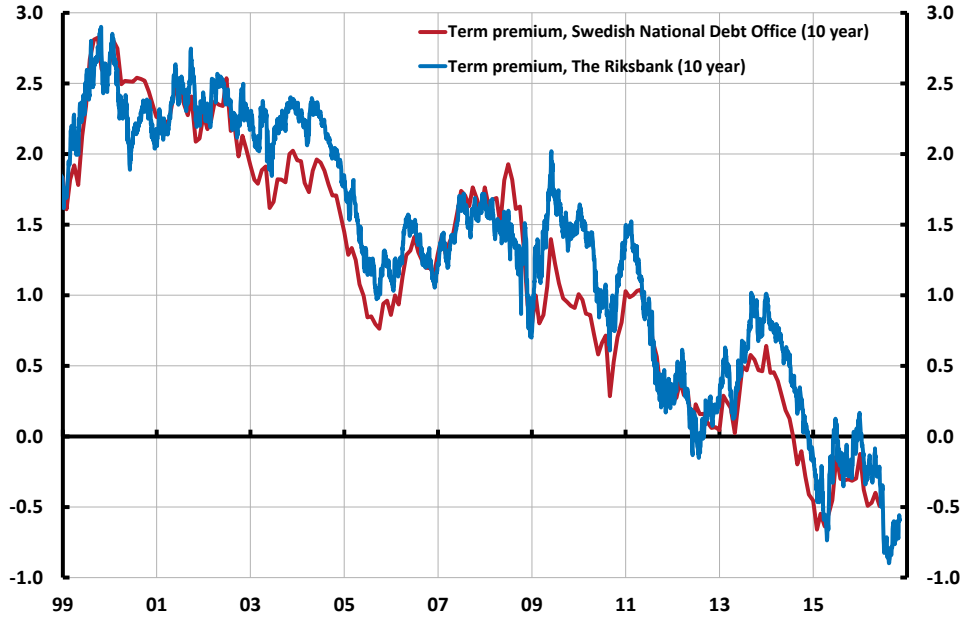
As illustrated in Figure 2, there are clear differences in how the models estimate the premium, although the variation over time is similar. As we discussed earlier, during the euro crisis in 2011-2012, the measure of the premium was depressed and in most models the premium turned negative. Since 2014 and onwards, expectations of quantitative easing by both the Riksbank and by the ECB have pushed the premium even further into negative territory.

The Swedish National Debt Office (SNDO) also assesses the premium as a part of its active debt management.[11] In Figure 3, we show that the SNDO's measure of the premium correlates with the average of the different models. The SNDO estimate the premium for swap rates, which can be somewhat higher compared to government bonds, at least in theory. According to Figure 2, however, the two different measures are rather similar.

[11] See Swedish National Debt Office (2016).

Figure 3: Premiums with 10 years' maturity according to the Riksbank (government bonds) and SNDO (swap rates)

Percent



Source: The Riksbank and SNDO

Remark: The Riksbank measure of premium is based on the average measure from different models, shown in Figure 2.
The SNDO measure of the premium is calculated for swap rates. In theory, those premiums can be considered to be somewhat higher on average than premiums derived from government bond yields, especially as swap rates can contain counterparty risks and on occasion also liquidity risks. One such occasion could be during the euro crisis when the interbank market in the euro area was under pressure.

How can we better measure market interest rate expectations and the premium?

A structured way of combining information of interest rate expectations from different sources is to use a model where we jointly can analyse expectations and premiums. De Rezende (2017b) describes different ways a central bank (and others) can use models to better analyse the yield curve. In that class of models, the EH is a special case and there are several testable implications that let us distinguish between expectations and premiums.[12] Models can also be used to efficiently combine different types of information to better identify expectations and premiums in financial market data. In the appendix of Alsterlind (2017), we show an example of combining models and survey information of markets interest rate expectations to give us a better understanding of the true underlying expectations in financial market data.

Concluding discussion

In this memo, we have discussed that the EH is a common starting-point when analysing how longer interest rates relate to expectations of shorter maturity interest rates. The hypothesis is simple and has a powerful intuition; that expectations of future interest rates are important for longer maturity interest rates, and are generally of importance to today's financial market prices. However, the EH has weak empirical support according to international research. This has been well known for a long time and in this memo, we suggest that there is little reason to believe that the Swedish yield curve is different.

If one cannot interpret market interest rates as pure measures of expectations, one has to use all available information when analysing the yield curve. When there are large differences between different measures of market expectations of future interest rates, one should be

[12] One testable implication is that the parameters in the market price of risk should be zero for the EH to hold. Also, see Dai and Singleton (2002) for ways of selecting good models for the yield curve.

careful not to put too much emphasis on just one measure. **A more careful discussion of what are movements in expectations and what are movements in premiums is warranted**.

Here, we suggest some methods that can be used to distinguish between expectations and premiums in market interest rates. These methods encompass the EH but at the same time allow for time variation in the premiums. This way, one has a method that gives a better description of the movements in the yield curve. However, those methods should not be seen as a universal remedy, but rather as a tool to illustrate and quantify the uncertainty of the analysis. In the end, one has to use judgements, and be prepared to alter those judgements as new information becomes available.

Using the EH to analyse how monetary policy affects market interest rates can easily lead to the wrong conclusions if policy affects expectations in one direction but the effects on the premiums are the opposite. In the end, this is an empirical question where the effects of policy on the yield curve have to be analysed carefully. The fact that monetary policy could affect premiums has aroused new interest as quantitative easing has become more common as a monetary policy tool.

For monetary policy purposes, one might hold the view that it is not necessary to distinguish between expectations and premiums. A reasonable view is that monetary policy affects both, and a change in market interest rates affects economic decisions by households and companies. However, it is quite possible that changes in expectations affect the economy in a different way than changes in premiums. This gives another reason to try and separate shifts in expectations from shifts in premiums.

Research literature

Alsterlind, Jan (2017), "Common features in short maturity interest rate forecasts", Staff memo, Sveriges Riksbank, June.

Armelius Hanna, Paolo Bonomolo, Magnus Lindskog, Julia Rådahl, Ingvar Strid and Karl Walentin (2014), "Lower neutral interest rate in Sweden?", Economic commentaries, Sveriges Riksbank, No. 8.

Beechey, Meredith, Erik Hjalmarsson and Pär Österholm (2009), "Testing the expectations hypothesis when interest rates are near integrated", *Journal of Banking & Finance*, No 33, Pages 934–943.

Beechey, Meredith and Pär Österholm (2012), "Policy Interest-Rate Expectations in Sweden: A Forecast Evaluation", *Applied Economics Letters*, Vol. 21(14), September, Pages 984-991.

Campbell, John Y. and Robert J. Shiller (1991), "Yield Spreads and Interest Rate Movements: A Bird's Eye View", *Review of Economic Studies*, Vol. 58, Issue 3, Pages 495-514.

Campbell, John Y. (1995), "Some Lessons from the Yield Curve", *Journal of Economic Perspectives*, Vol. 9, No. 3, Summer, Pages 129-152.

Carriero, Andrea, Carlo A. Favero and Iryna Kaminska (2006), "Financial factors, macroeconomic information and the Expectations Theory of the term structure of interest rates", *Journal of Econometrics*, Vol. 131, Issues 1–2, Pages 339–358.

Cochrane, John, H. and Monika Piazzesi (2005), "Bond Risk Premia", *American Economic Review*, Volume 94, Issue 1, Pages 138-160.

Dai, Qiang and Kenneth J. Singleton (2002), "Expectation puzzles, time-varying risk premia, and affine models of the term structure", *Journal of Financial Economics*, Vol. 63, Issue 3, March 2002, Pages 415–441.

De Rezende, Rafael, B. (2017a), "The interest rate effects of government bond purchases away from the lower bound", Forthcoming in *Journal of International Money and Finance*.

De Rezende, Rafael, B. (2017b), "How can term structure models be used by central banks?", *Economic Review*, Sveriges Riksbank, 2007:1.

Duffie, Darrell and Rui Kan (1996), "A Yield-Factor Model of Interest Rates", *Mathematical Finance*, Vol. 6, No. 4, Pages 379-406.

Dziwura, Joseph, R., and Eric, M., Green (1996), "Interest rate Expectations and the Shape of the Yield Curve", *Federal Reserve Bank of New York Research Paper*, No. 9631.

Fama, Eugene F (1984), "The Information on the Term Structure", *Journal of Financial Economics*, No 13, Pages 509-528.

Goodhart, Charles A. E. and Wen Bin Lim (2011), "Interest Rate Forecasts: A Pathology", *International Journal of Central Bankin*g, Vol 7, No 2, June, Pages 135-171.

Guidolin, Massimo and Daniel L. Thornton (2008), "Predictions of Short-Term Rates and the Expectations Hypothesis of the Term Structure of Interest Rates" ECB Working paper series, No. 977.

Lukasz Rachel and Thomas D Smith (2015), "Secular drivers of the global real interest rate", Bank of England Staff Working Paper, No. 571.

Swedish National Debt Office (2016), "Central government debt management – proposed guidelines 2017–2020".

Appendix

A test of the expectations hypothesis of interest rates

Campbell and Shiller (1991) provide a test that is frequently used to investigate the validity of the expectations hypothesis (EH) by estimating the regression:

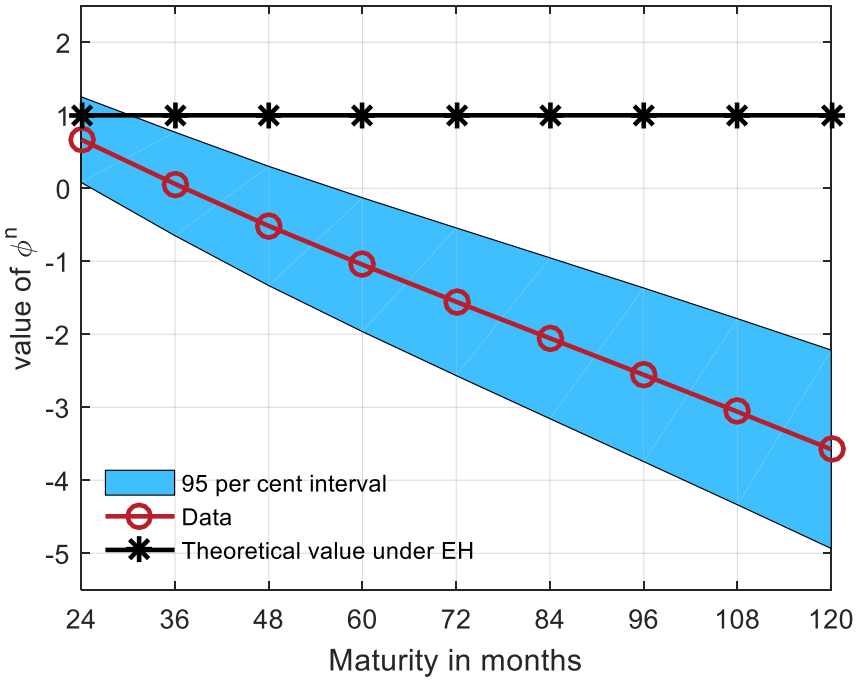
$$(i_{t+1}^{n-1} - i_t^n) = \theta^n + \phi^n (i_t^n - i_t^1)/(n-1) + \varepsilon_t \qquad \text{(A1)}$$

where i_t^n is the interest rate of an n-period bond, and i_t^1 is the interest rate of a one-period bond. The general idea is that if the EH is valid, longer maturity interest rates contain information on the future return of a n −period bond, over the holding period. Implicitly, it is also assumed that the market has an unbiased forecast on future interest rates. The testable implication of equation (A1) is that $\phi^n = 1$ for all n-period bonds.

A test of the EH on Swedish data

In Figure A1, we have replicated the classical test of the EH as described by Campbell and Shiller (1991) on Swedish data from 1994 to 2016. In Figure A1, the black line is the theoretical values of the coefficient ϕ^n if the EH is valid, and the red line is the corresponding empirical values of ϕ^n, with a 95 percent confidence interval.

Figure A1: Test of the expectations hypothesis on Swedish data, 1994-2016



Source: The author's own calculations

Remark: Continuously defined spot rates calculated from T-bills and government bonds using the extended Nelson and Siegel-method.

We can conclude that there is a big difference between the theoretical and actual values of ϕ^n. Considering the confidence intervals for the empirical values of ϕ^n, we conclude that we can reject the theoretical values and thus reject the EH. The results for Sweden are in line with international experience and numerically, they are close to the results in Dai and Singleton (2002) based on US data.[13]

[13] Also, compared to Campbell and Shiller (1991), the numerical results are similar.

A formal rejection of the EH is what is usually found in the international literature and is indicative of time variation in the term premium. In other words, the rejection of the EH is an indication that changes in interest rates of longer maturities are not only affected by expectations but also by changes in the premium. The frequent use of the EH in interpreting movements in the yield curve is perhaps surprising considering its weak support. However, the simple intuition is powerful and can at times be useful, notwithstanding the lack of empirical support.

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