Did the U.S. Really Grow Out of Its World War II Debt?[*]

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Abstract

This paper examines the effects of primary budget surpluses, surprise inflation, and pegged interest rates before the Fed-Treasury Accord of 1951 on the U.S. public debt/GDP ratio. We find that with primary budget balance and without the distortions in real interest rates caused by surprise inflation and the pre-Accord peg, debt/GDP would have declined only from 106% in 1946 to 74% in 1974, not to 23% as in actual history. Our findings imply that, over the last 76 years, only a small amount of debt reduction has been achieved through growth rates that exceed undistorted interest rates.

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1 Introduction

Does a high level of national debt impose a burden on future generations who must pay it off? In recent years, economists such as Blanchard (2019) and Furman and Summers (2020) have suggested that the answer may be no, because $r < g$: the real interest rate on debt is usually below the growth rate of the economy. Under that condition, the government can roll over the debt and accumulating interest without raising taxes, and the debt/GDP ratio will fall over time. Because of this possibility, a growing number of economists agree with Blanchard that "public debt may have no fiscal cost." This idea has decreased concern about the high current level of U.S. debt.

Thinking on this issue has been influenced by a salient historical experience: the decline in the U.S. debt/GDP ratio after World War II. Paying for the war increased this ratio from 42% in fiscal year 1941 to 106% in 1946, but then it started to fall and reached a trough of 23% in fiscal year 1974. As Elmendorf and Mankiw (1999) report, "an important factor behind the dramatic drop between 1945 and 1975 is that the growth rate of GDP exceeded the interest rate on government debt for most of that period." Krugman (2012) says that the "debt from World War II was never repaid and just became increasingly irrelevant as the U.S. economy grew." This interpretation of history lends credence to the idea that a high level of debt should not cause great concern.[1]

However, other researchers have suggested reasons to question this interpretation. First, as discussed by authors such as Hall and Sargent (2011) and Eichengreen and Esteves (2022), the U.S. actually paid off part of the World War II debt by running primary surpluses—by levying taxes in excess of current government spending—over much of the period when the

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[1]This interpretation appears in many public discussions of debt. For example, the Council on Foreign Relations (McBride et al. 2023) says: "Over the next thirty years [after World War II], sustained economic growth gradually reduced debt as a percentage of the economy, despite expensive wars in Korea and Vietnam and the establishment of major entitlement programs." Writing in *The Atlantic*, Phillips (2012) says "The US never really tried to pay down much of the debt it incurred during World War II. Still the debt shrank in significance as the US economy grew."

debt/GDP ratio was falling. Second, as discussed by authors such as Reinhart and Sbrancia (2015), interest rates were held down relative to economic growth through policies that are not likely to be feasible and/or desirable in the future. These policies included episodes of financial repression, most clearly the Fed's pegging of interest rates at low levels from 1942 to 1951, which was aimed at decreasing the cost of the war. In addition, ex-post real interest rates were reduced by unexpected rises in inflation in the aftermath of the war and later in the 1960s and 1970s. Because of these factors, the postwar experience does not necessarily suggest that the U.S. economy naturally grows out of debt.

This paper seeks to explain the path of the debt/GDP ratio since its 1946 peak of 106%. We estimate the effects on this path of the government's primary surpluses, the interest rate peg before 1951, and surprise inflation. We then derive a counterfactual path that the debt/GDP ratio would have followed in the absence of these factors. This counterfactual shows how much the ratio was reduced by growth rates in excess of undistorted real interest rates—rates that are not reduced by either a peg or surprise inflation.

We find that this counterfactual scenario differs greatly from actual history. Without primary surpluses and interest rate distortions, the debt/GDP ratio falls only from 106% in 1946 to 74% in 1974, rather than falling to 23% as in reality. Over the three decades after World War II, the natural erosion of debt from economic growth was considerably smaller than is often suggested.

We also extend our counterfactual to the present, with even more negative findings about growing out of debt. The counterfactual debt/GDP ratio starts rising again in 1980, and in 2022 it is 84%: the earlier decline in the ratio is partially reversed, leaving it only 22 percentage points below its 1946 level. The rise in the ratio reflects the fact that the economy's growth rate has averaged *less* than the undistorted real interest rate on debt since 1980.

Our method for constructing counterfactual debt paths builds on previous work, but uses a richer set of information to make our quantitative results as accurate as possible. A key

step is to measure the fractions of outstanding debt in a given year that were issued in each earlier year—the "reverse maturity structure" of the debt—which we do using granular data on Treasury securities produced by Hall, Payne, and Sargent (2018) before 1960 and by CRSP thereafter. We also construct a term structure of inflation expectations from surveys of short- and long-term expectations, which allows us to estimate the effects of surprise inflation on the real returns on debt issued in different years. Finally, we estimate the effects of the pre-1952 interest rate peg by comparing the pegged rates on debt of various maturities to market rates during the post-peg period of 1952-1960. Our various calculations require some assumptions about unobserved variables, but our results are not greatly changed by varying these assumptions in reasonable ways.

The next section of this paper provides some historical background on the post-World War II period. We then present our methodology for constructing counterfactual paths of the debt/GDP ratio, describe our data sources, and present our results. We also compare our analysis to Hall and Sargent's (2011) well-known work on the debt/GDP ratio since World War II.

2 Factors Influencing the Debt/GDP Ratio

Figure 1 shows the path of the public debt/GDP ratio in the United States from fiscal year 1941 to fiscal year 2022. We see that this ratio grew rapidly during World War II, rising from 42% in 1941 to 106% in 1946. It then fell steadily until it reached 23% in 1974, an experience commonly attributed to economic growth. Since the trough in 1974, the debt/GDP ratio has risen in most years and it reached 102% in 2022.

This section provides some background on three factors, in addition to economic growth, that have influenced the debt/GDP ratio: primary surpluses and deficits, the Fed's interest rate peg from 1942 to 1951, and unexpected inflation. Later sections quantify the effects of these factors.

2.1 Primary Surpluses and Deficits

Figure 2 shows the primary budget surplus as a fraction of GDP from fiscal year 1947, the year after debt/GDP peaked, to the present. We can see that deviations from primary balance contributed to both the fall in the debt/GDP ratio through 1974 and the rise since then.

The sharp fall in government spending after the war produced primary surpluses of 3.6% of GDP in 1947 and 6.3% in 1948. After that, the surplus remained positive in most years, ranging between 3.4% and -1.5% of GDP through 1974. This experience reflected a strong political consensus in favor of budgetary restraint in the 1950s and 1960s. During the entire period from 1947 through 1974, the primary surplus averaged 1.1% of GDP, helping to reduce the debt/GDP ratio.

After 1974, the pattern reversed and habitual primary deficits contributed to a rising debt/GDP ratio. The primary balance was negative at almost all times except in the late 1990s, and the deficit was especially high around the Great Recession of 2008 and the recent COVID pandemic.

2.2 Interest Rates Before the Fed-Treasury Accord

In April 1942, at the request of the Treasury department, the Federal Reserve adopted a policy of pegging interest rates on government bonds at low levels. This policy was intended to contain the cost of financing the war. The Fed capped yields at levels ranging from 0.375% for Treasury bills to 2.5% for 30-year bonds, maintaining these caps by standing ready to buy any quantity of bonds.

The Fed's policy presumably kept interest rates below the neutral level and made it impossible to adjust rates to control inflation. During World War II, inflation was contained through government price controls. Price controls were eliminated in June 1946, and inflation

became unstable: in fiscal years 1947 through 1951, CPI inflation rates ranged from -0.7 percent to 18.3 percent, with an average of 7.1 percent. Fed officials became increasingly unhappy with their inability to control inflation and eventually persuaded the Treasury that the peg should be abandoned, a decision announced by the two agencies in their March 1951 "Accord."[2]

The interest rate peg had a big effect on debt dynamics because it was in effect during the build-up and initial rolling over of the large World War II debt. In addition, a large share of the debt issued with low interest rates had maturities of ten to thirty years, so the influence on the costs of debt service was felt long after the peg ended in 1951. Both during and after the peg, periods of high inflation produced ex-post real interest rates that were deeply negative for many government securities.

The pre-Accord peg is the most clear-cut case of financial repression that held interest rates down in the postwar U.S. Reinhart and Sbrancia (2015) cite other types of financial repression, such as caps on banks' interest rates under Regulation Q. We ignore these other policies because we do not know how to quantify their effects on the interest rates on government debt. To the extent they are important, the effects of the pre-Accord peg that we measure are a lower bound on the total effects of financial repression.

2.3   Surprise Inflation

Unexpected inflation reduces the debt/GDP ratio by pushing ex-post real interest rates below ex-ante real rates. The relevant inflation rate is the growth rate of the GDP deflator. As detailed below, the effect on the debt/GDP ratio in a given period depends on the current level of inflation relative to the level expected at various times in the past when the currently outstanding debt was issued.

Figure 3 shows inflation surprises for fiscal years 1952 through 2022. For each year,

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[2]See Hetzel and Leach (2001) for a detailed narrative of this episode.

the Figure compares the actual level of inflation to the level expected one year earlier and (starting in 1962) the level expected ten years earlier, based on several sources of data on inflation expectations (see details below). We see the well-known rise in inflation in the 1960s and 1970s and the fact that inflation persistently exceeded expected inflation during that period—especially the levels expected ten years before, implying a big erosion of real interest rates on long-term securities. These inflation surprises contributed to the decrease in the debt/GDP ratio through 1974 and moderated the first part of the subsequent increase in the late 1970s.

Starting in the 1980s, the story was reversed: as inflation fell following the Volcker regime shift, actual inflation was usually lower than expected inflation. This pattern pushed ex-post real interest rates above ex-ante rates and contributed to the rising debt/GDP ratio, although we will see that this effect was smaller than that of the earlier surprise inflation, in part because the average maturity of the debt was shorter. The sharp increase in inflation in 2021-2022 meant that unexpected inflation again became a factor reducing the debt/GDP ratio.

3   Constructing Counterfactual Paths of the Debt/GDP Ratio

This section describes how we construct counterfactual paths of the debt/GDP ratio with zero primary surpluses and/or without distortions in real interest rates from the pre-Accord peg and surprise inflation. The differences between these paths capture the different factors driving the actual debt/GDP path. The counterfactual without surpluses or interest rate distortions shows how much the debt/GDP ratio has been reduced through economic growth in excess of undistorted interest rates. We interpret this effect as the natural tendency of the

economy to grow out of debt, which is sometimes called the "negative snowball" or "melting" effect (e.g., Krugman 2019; Aizenman and Ito 2020).

3.1 Overview

We start with a standard equation describing the evolution of the nominal stock of debt:

$$D_t = (1 + i_t)D_{t-1} - P_t, \tag{1}$$

where D_t is the par value of the debt in period t, i_t is the average interest rate on the debt, and P_t is the primary surplus.[3] From period $t - 1$ to t, the debt grows at rate i_t as it is rolled over, and primary surpluses reduce the debt (while primary deficits increase it). In our empirical work, a period is a fiscal year, and debt is measured at the end of the year. The interest rate is measured as total interest payments in fiscal year t divided by D_{t-1}.

In all simulations, we start with the debt at its actual level at the end of fiscal year 1946 and derive its path after that for alternative assumptions about primary surpluses and interest rates. We then derive the path of the debt/GDP ratio, assuming in all cases that the path of nominal GDP is the same as in actual history.

It is straightforward to construct a counterfactual with no primary surpluses: we simply set $P_t = 0$ for all t in equation (1).

Adjusting for the pre-Accord peg and inflation surprises is more complex. In our counterfactual exercise, we replace the actual nominal interest rates i_t with the rates that would have prevailed under two conditions. The first, which is relevant for debt issued before 1952, is that the ex-ante real interest rate is the one that would have prevailed in the absence of the

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[3]We consider the par value of the debt because it is a focus of policy discussions. Hall and Sargent (2011) analyze a version of equation (1) that applies to the market value of debt. Taking that approach would complicate our analysis without making a material difference for our results, because the paths of the par and market values of the debt are very close to each other over long periods (see Figure A.1 in the Appendix).

peg, if the Federal Reserve were operating normally (presumably setting rates to stabilize output and inflation). We estimate these undistorted interest rates from ex-ante real rates observed after the peg period, as detailed below.

The second condition is that the ex-post real interest rate is always equal to the ex-ante real rate. We implement this assumption by adding the unexpected component of inflation (measured using surveys of expectations) to actual nominal interest rates. This adjustment yields the nominal rates that would have prevailed given ex-ante real rates if financial market participants had known the future path of inflation. We can also interpret our counterfactual interest rates as those that would have been observed if all debt were indexed to inflation.[4]

Our simulations of debt/GDP ratios assume that both undistorted real interest rates and real GDP are the same in our counterfactual scenarios as in actual history. Conventional macroeconomics implies that the higher debt levels in the counterfactuals would increase real rates and reduce real GDP by crowding out capital, and both of these effects would magnify the increases in the debt/GDP ratio relative to actual history. In light of these omitted effects, the debt/GDP paths we derive can be interpreted as lower bounds on the paths implied by our counterfactual assumptions.[5]

3.2 Counterfactual Interest Rates and the Reverse Maturity Structure

We need to calculate the average interest rates on debt in our counterfactual with no real rate distortions, which we denote by $\hat{i}_t$. In doing so, a critical nuance is that the debt outstanding

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[4]Andreolli and Rey (2023) conduct a similar counterfactual exercise for Euro Area countries for the period since 1999.

[5]This overview of our analysis ignores one detail: equation (1) describing the evolution of debt does not hold exactly in the data. There is a residual reflecting factors that affect debt besides interest payments and primary surpluses, the most important of which is changes in the cash balances held by the Treasury department. In all our simulations, we hold the residuals in equation (1) constant at their historical levels. The nature of these residuals and their effects on the debt are discussed further in the Appendix.

in a given fiscal year is a mix of securities issued in different past years. We call the mix of years when the current debt was issued the "reverse maturity structure" of the debt (as opposed to the regular maturity structure, which is the mix of future years when the current debt will mature). The reverse maturity structure matters because the outstanding debt may include some securities issued during the pre-Accord peg and some issued later, and because inflation expectations differed at the various times securities were issued.

To see the role of the reverse maturity structure, we introduce some notation. The stock of debt outstanding at the end of fiscal year $t - 1$ is a sum of debt issued during $t - 1$ and earlier years:

$$D_{t-1} = \sum_{\tau=t-M}^{t-1} D_{t-1}^{\tau} \tag{2}$$

where D_{t-1}^{τ} is the debt outstanding at the end of year $t - 1$ that was issued during year τ and $t - M$ is the earliest year when part of the debt was issued. We denote the fraction of outstanding debt at $t - 1$ that was issued at τ by w_{t-1}^{τ}:

$$w_{t-1}^{\tau} \equiv D_{t-1}^{\tau}/D_{t-1} \tag{3}$$

The weights w_{t-1}^{τ} capture the reverse maturity structure of the debt.

We let i_t^{τ} equal the actual average interest rate paid at t on the part of the debt issued at τ. The overall average rate i_t is an average of the i_t^{τ}'s weighted by the shares of total debt to which they apply:

$$i_t = \sum_{\tau=t-M}^{t-1} w_{t-1}^{\tau} i_t^{\tau} \tag{4}$$

In any counterfactual, the overall average interest rate $\hat{i}_t$ will depend on how we adjust the weights and interest rates in equation (4). In our main simulations, we assume for simplicity that the w_{t-1}^{τ}'s in each year are the same as in actual history. That is, we assume that the increase in aggregate debt in our counterfactuals does not affect the reverse maturity

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structure of the debt. The Appendix (Section A.4) takes a different approach: we assume that the reverse maturity structure evolves based on the amount of additional debt issued each year, given plausible assumptions about the maturity structure of this additional debt. We do not focus on this version of our analysis because, while arguably more precise than the one presented here, it is much more complicated and does not make a material difference for our results.

In counterfactuals with undistorted interest rates, we adjust each i_t^τ to eliminate the effects of the peg and surprise inflation and denote the result by $\hat{i}_t^\tau$. Let x_t^τ be the adjustment $\hat{i}_t^\tau - i_t^\tau$. Given fixed weights w_{t-1}^τ, the counterfactual aggregate interest rate $\hat{i}_t$ can be written as:

$$\hat{i}_t = \sum_{\tau=t-M}^{t-1} w_{t-1}^\tau \left(i_t^\tau + x_t^\tau \right) = i_t + \underbrace{\sum_{\tau=t-M}^{t-1} w_{t-1}^\tau x_t^\tau}_{\equiv x_t} \tag{5}$$

The path of $\hat{i}_t$ can be derived from the actual aggregate interest rates i_t, the weights w_{t-1}^τ, and the adjustments x_t^τ.

3.3 The Adjustments to Interest Rates

An adjustment x_t^τ applies to the interest rate paid on debt issued at τ, so it depends on whether the date τ is before, during, or after the peg. The expressions for x_t^τ are:

$$x_t^\tau = \begin{cases} 0 & \text{for } \tau \leq 1942 \\ r_t^{\star\,\tau} - (i_t^\tau - \pi_t) & \text{for } 1943 \leq \tau \leq 1951 \\ \pi_t - \mathbb{E}_\tau[\pi_t] & \text{for } \tau \geq 1952 \end{cases} \tag{6}$$

where $r_t^{\star\,\tau}$ is the undistorted real interest rate on the outstanding debt at t that was issued at τ and $\mathbb{E}_\tau[\pi_t]$ is the expectation at τ of the inflation rate at t.

Let us review these adjustments in reverse chronological order. For $\tau \geq 1952$, x_t^τ is an adjustment to an interest rate paid at t on debt that was issued after the Fed-Treasury Accord. In this case, we assume that the ex ante real interest rate is the undistorted rate and adjust the ex-post rate to eliminate the effect of unexpected inflation. The relevant inflation surprise is the difference between the actual inflation rate in year t and the expectation of that rate at τ. The next section describes how we derive estimates of this expectation from surveys.[6]

The second line is the adjustment to interest rates in year t on debt issued during the peg period, which we date from fiscal year 1943 through fiscal year 1951.[7] This adjustment is the difference between the ex-post real interest rate and the undistorted real rate (which no longer equals the actual ex-ante rate). The ex-post real rate is the average nominal interest rate on the outstanding debt that was issued at τ minus the inflation rate at t. The undistorted real interest rates $r_t^{\star \tau}$ are not observed, so we must make educated guesses about their levels. As detailed in the next section, we do so using ex-ante real rates in the years after the peg.

Finally, the first line of equation (6) indicates that we do not make any adjustment to the interest rates on debt issued before the start of the peg. These interest rates are relevant because part of that debt was still outstanding in 1946, when we start our simulations. In principle, we should adjust interest rates set before the peg for surprise inflation, but we lack data on inflation expectations before 1943. As a robustness check, the Appendix considers interest rate adjustments based on a range of reasonable conjectures about pre-

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[6]Some readers have suggested that the Fed's quantitative easing policy after 2008 caused interest rate distortions. We view this policy as an effort to move rates toward their equilibrium levels in the presence of the zero bound on the policy rate, not as a distortion of rates. In any case, if we adjust interest rates to eliminate the effects of QE, the levels of debt/GDP in our counterfactuals become larger after 2008, which strengthens our conclusions. See the Appendix (Section A.5) for details.

[7]The peg was adopted in April 1942 and ended in March 1951. Our dating of the peg period as fiscal years 1943-1951, which run from July 1942 through June 1951, is an approximation that is necessary because our data are available by fiscal year.

1943 expectations. These adjustments modestly increase the amount of debt reduction that we attribute to surprise inflation, because the high inflation of the late 1940s and early 1950s was probably not expected before 1943.

3.4   Two Complications

The derivation of equation (6) ignores two details about the types of securities issued by the U.S. Treasury. Here we briefly describe these details and how we address them. The Appendix presents a modification of equation (6) that accounts for these issues, and we use that equation in our simulations.

Treasury Bills. A substantial fraction of the debt consists of Treasury bills with maturities of less than a year, most commonly 90 days. This poses a problem because equation (6) assumes that all debt outstanding at the end of year $t-1$ is eroded by surprise inflation over year t, but most of the outstanding Treasury bills will be rolled over during the year at interest rates that adjust to inflation news. To be conservative in assessing the effects of surprise inflation, we assume it has no effect on the real returns on Treasury bills and modify equation (6) accordingly.[8]

Inflation Indexed Debt. Starting in 1997, part of the debt consists of Treasury Inflation-Protected Securities, whose nominal interest rates adjust to ensure that ex-post real interest rates equal ex-ante rates. Inflation surprises do not erode the value of these securities, so we adjust equation (6) accordingly.

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[8]Some long-term bonds also mature within year t, so their returns are not eroded by the full inflation surprise over t. This fact implies some overstatement of the effects of inflation surprises in equation (6), but this bias is counterbalanced by one in the other direction: securities are issued throughout each year and we measure expected inflation at the end of the year, which probably understates inflation surprises relative to expectations when securities were issued. We doubt that these factors are important, but future research could address them with higher-frequency data.

4 Data and Measurement

This section describes our data sources and the measurement of the variables in our counterfactual simulations. These variables include a number of fiscal variables, inflation expectations at various horizons, and undistorted real interest rates during the peg period. We give an overview of our approach and leave a number of details to the Appendix.

4.1 Timing

The unit of time in our analysis is a fiscal year, because much of our data are reported by fiscal year. In the early part of our sample, fiscal year t runs from July of calendar year $t-1$ through the following June. Starting with fiscal year 1977, the timing shifts and fiscal years run from October of calendar year $t-1$ through the following September. This shift creates a "transitional quarter" (the third quarter of calendar year 1976) that is treated separately in the government's fiscal accounts and which requires some modifications of our procedures around that time (see Appendix).

4.2 Fiscal Variables

We use fiscal data from the Office of Management and Budget (OMB) Historical Database, the Hall, Payne, and Sargent (2018) database on government securities, the CRSP Monthly U.S. Treasury Database, and the Treasury Bulletin.

Aggregate Debt. Our measure of D_t, the total debt in fiscal year t, is the level of debt held by the public at the end of the year, from the OMB historical database. Debt held by the public includes debt held by the private sector and the Federal Reserve, but excludes intragovernmental holdings such as debt held by the Social Security Trust Fund. This variable is the most common measure of public debt in the literature.

To measure debt/GDP ratios, we use the series for nominal GDP by fiscal year from the OMB database. As discussed above, our simulations hold the path of GDP constant when we consider counterfactual paths of debt.

The Reverse Maturity Structure. We construct the reverse maturity structure of the debt from the Hall et al. (2018) database for the period from 1942 through 1960, and from the CRSP Monthly U.S. Treasury Database for 1961 through 2022. For every month, these databases provide an accounting of almost every issue of a Treasury security that is currently outstanding, including its quantity and issue date. We use data for the final month of each fiscal year to construct D_t^τ, the amount of debt outstanding at the end of year t that was issued in year τ. Dividing the quantities D_t^τ by the total debt D_t yields the weights w_t^τ that define the reverse maturity structure.

The Appendix provides details of this procedure, including approximations needed because of missing information in the Hall et al. (2018) and CRSP data sets. Perhaps the most significant issue is that the post-1960 data from CRSP do not include non-marketable debt such as savings bonds. We assume that the reverse maturity structure of non-marketable debt remains constant after 1960.

Figure 4 summarizes the evolution of the reverse maturity structure by showing the fractions of outstanding debt with maturities in various ranges. Over the first part of our sample, during the pre-Accord peg, the share of debt with reverse maturities above five years rose due to debt issued more than five years earlier to finance World War II. This longer-term debt share peaked at 48 percent in 1951 and then fell, and from 1975 through 2022 it fluctuated between 10 and 25 percent. The average reverse maturity of all outstanding debt fell from 4.4 years in 1951 to 2.2 years in 2022.

Aggregate Interest Rates. Following previous researchers, we define the aggregate inter-

est rate on debt in fiscal year t, i_t, as total interest payments during the year divided by total debt outstanding at the end of year $t-1$. This definition is the one for which the accounting identity in equation (1) holds. For consistency with our measure of D_t, which excludes intragovernmental holdings of Treasury debt, total interest payments exclude payments to government entities.

Our data on interest payments come from the OMB Historical Data. Starting in 1962, we derive the appropriate series by subtracting intragovernmental payments from gross interest payments. Before 1962, we lack data on intragovernmental payments. The OMB reports "net interest", but this series subtracts not only intragovernmental payments but also interest received by the government (e.g., through credit programs such as student loans), which is not appropriate. In the Appendix, we examine the relation between the net interest series and our desired series for interest payments on the debt, and find that the latter is approximately ten percent higher in years when we can measure both. Therefore, before 1962 we measure total interest payments by multiplying net interest by 1.1. Other reasonable approaches yield similar results.

The Primary Balance. The primary balance is also calculated from OMB data. It is computed as the sum of the total fiscal surplus (which is usually negative) and total interest payments, with total interest calculated as described above.

4.3 Inflation and Inflation Expectations

Here we describe our measurement of actual and expected inflation, which determine the inflation surprises that enter our calculations. We use data on one-year and ten-year inflation expectations to estimate the entire term structure of expectations.

The Inflation Rate. When studying the debt/GDP ratio, the relevant price index is the GDP deflator. We measure the inflation rate in fiscal year t, π_t, as the growth rate of the deflator (not seasonally adjusted) from the fourth quarter of year $t-1$ to the fourth quarter of t. (The Appendix describes complications around the Transitional Quarter in 1976.)

One-Year Expectations. We let $\mathbb{E}_t[\pi_{t+1}]$ denote the expectation at the end of year t of the inflation rate in $t+1$. We measure this expectation using two different surveys for two parts of our sample.

Starting with fiscal year $t = 1970$, we use forecasts reported in the Survey of Professional Forecasters in the last quarter of the fiscal year (either the second or third quarter of the calendar year). We use the median forecast of the GDP deflator growth rate over the following four quarters, that is, over fiscal year $t+1$.

For fiscal years before 1970, we lack data on expectations of the GDP deflator, so we create a proxy. We start with expectations of the CPI from the semiannual Livingston Survey of business economists. We use forecasts published in June—before 1970, the last month of the fiscal year—of the CPI in the following June. We derive an expected CPI inflation rate over the coming fiscal year from the forecast of the CPI level.

To derive expected inflation in the GDP deflator, we assume that the expectation error $\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]$ is the same for inflation measured by the deflator and inflation measured by the CPI. This assumption is close to true during periods when we have survey expectations of both variables (see Appendix). With our assumption, we can measure the expectation of GDP deflator inflation $\mathbb{E}_t[\pi_{t+1}]$ as actual deflator inflation π_{t+1} minus the expectation error for the CPI.

Ten-Year Expectations. Our analysis also uses an expectation of the average inflation rate over the next ten years, which we denote by $\mathbb{E}_t[\pi^{10}]$. For fiscal years back to $t = 1968$,

we measure this variable with a series for expected ten-year inflation from the Fed's data set for its FRB/US Model. (We use the observations for the last quarter of each fiscal year.) These data are expectations of inflation in the PCE deflator, but the paths of the PCE and GDP deflators are usually close (see Appendix). The Fed produces its series by combining forecasts from the Survey of Professional Forecasters and Consensus Economics with econometric estimates of expected inflation when survey measures are not available.

We have not found data on ten-year expected inflation before 1968, and our calculations require these expectations back to 1952. We construct estimates of the missing data using our series for one-year expectations $\mathbb{E}_t[\pi_{t+1}]$. Specifically, for the period from 1968 to 1997, we find that the level of ten-year expectations is well-explained statistically by the level and change in a smoothed version of one-year expectations. We use this estimated relation to construct fitted values for long-term expectations before 1968. See the Appendix for details.

Figure 5 shows our final series for one-year inflation expectations $\mathbb{E}_t[\pi_{t+1}]$ and ten-year expectations $\mathbb{E}_t[\pi^{10}]$. Note that these forward-looking expectations differ from the series shown above in Figure 3, which are expectations of current inflation in past years.

The Term Structure of Inflation Expectations. To adjust interest rates in our counterfactuals, we need expectations of inflation at all horizons. We derive estimates of these expectations by making assumptions about the shape of the term structure of expected inflation. Specifically, we assume that the term structure $\mathbb{E}_t[\pi_{t+1}]$, $\mathbb{E}_t[\pi_{t+2}]$, ... is linear from $t + 1$ through $t + 5$ and then perfectly flat. Along with our series for $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[\pi^{10}]$ (which is the average of $\mathbb{E}_t[\pi_{t+x}]$ from $x = 1$ to $x = 10$), our shape assumptions determine the entire term structure. Once again, the Appendix discusses the details of our procedure, its rationale, and robustness to other reasonable assumptions.

Figure 6 shows the term structure of expected inflation for each year in our sample. In some years, the entire term structure is flat, but long-term expectations lag behind short-term

expectations when the latter are trending up or down (which occurs when actual inflation trends up or down).

4.4   The Peg Period

For debt issued during the peg period before 1952, our interest rate adjustments require series for i_t^τ, the actual nominal interest rates on the debt, and for $r_t^{\star\,\tau}$, the undistorted real interest rates. We derive these variables as follows.

Actual Interest Rates Under the Peg. For each issue of a Treasury security, the Hall et al. (2018) database reports the issue date, quantity, maturity, and usually the coupon rate, which is the relevant interest rate. When the coupon rate is missing, we use the interest rates by maturity under the peg reported by Friedman and Schwartz (1963).

We construct i_t^τ, the average interest rate on the outstanding securities at t that were issued at τ, by averaging across the interest rates on the individual securities.

Counterfactual Real Rates. We do not have direct evidence on the ex-ante real interest rates that would have prevailed on securities issued during the pre-Accord period if the Fed had not pegged rates. As a baseline measure, we simply assume that the rate for any security of a given maturity (at issuance) would have been equal to the average of the ex-ante real rates on securities with that maturity issued over the decade after the peg ended, from fiscal years 1952 through 1961. The counterfactual real rate $r_t^{\star\,\tau}$ is the average of the assumed rates on securities of different maturities weighted by the term structure of securities issued at τ and outstanding at t. The Appendix considers alternative assumptions that produce higher or lower rates and finds that our results are fairly robust across reasonable cases.

To calculate ex-ante real rates by maturity for 1952-1961, we use data on nominal interest

rates on debt issued during that period from the Global Financial Database. (The interest rate data in Hall et al. (2018) do not cover the period.) We obtain ex-ante real rates using the term structure of inflation expectations, derived as described above. We average the ex-ante real rates at each maturity over all securities issued over 1952-1961.

The resulting term structure of undistorted real interest rates includes 1.7% at the one-year horizon, 2.2% at five years, 2.5% at ten years, and 2.7% at thirty years.

5   Results

Here we present our central results, which are simulations of the path of the debt/GDP ratio. All simulations begin in 1946 with the ratio at its actual level of 106%. We compare the actual path of debt/GDP after 1946 to three counterfactual scenarios. In one, the "primary balance scenario," we set the primary surplus to zero in all years (but leave interest rates unchanged at their historical levels). In another, the "adjusted interest rate scenario," we apply the adjustments x_t^τ to eliminate the effects of both surprise inflation and the pre-Accord peg (but leave primary surpluses at their historical levels). Finally, in a "combined scenario" we assume primary balance and also adjust interest rates. The path of debt/GDP in the combined scenario is determined by $r^\star - g$, the difference between the undistorted real interest rate and the growth rate of output.

Figure 7 presents the alternative paths of debt/GDP. In interpreting these results, we divide the period since 1946 into two parts: 1946-1974, the period when the actual debt/GDP ratio declined to its trough of 23%; and 1975-2022, when the ratio rose to 97%. For each counterfactual, Table 1 reports the total changes in debt/GDP over the two periods, which we examine in turn.

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5.1   The Postwar Erosion of Debt, 1946-1974

The actual debt/GDP ratio declined steeply during the 1946-1974 period. Our counterfactual ratios also decline, but more slowly. As a result, while the actual debt/GDP ratio reached 23% in 1974, the counterfactual ratios in 1974 are substantially higher: 40% in the primary balance scenario, 51% in the adjusted rate scenario, and 74% in the combined scenario.

To appreciate these results, recall that the actual debt/GDP ratio fell by 83 percentage points from 1946 to 1974 (from 106 to 23 percent). In the combined scenario, the ratio falls by only 32 points (from 106 to 74 percent). Therefore, of the actual 83-point fall, 51 points are explained by the combination of primary surpluses and interest rate distortions. By comparing the different counterfactuals, we can divide this 51 points into 17 points explained by primary surpluses alone, 28 points explained by interest rate distortions alone, and 6 points from the interaction of the two factors. The interaction arises because adjusting the primary balance raises the level of debt, and higher debt magnifies the effects of adjusting interest rates.

The adjusted interest rate scenario eliminates distortions from both surprise inflation and the suppression of ex-ante real rates under the peg. It would be interesting to separate the effects of these two distortions, but that would be difficult because it requires measures of expected inflation during the peg period from 1942 to 1951. There are no data on long-term inflation expectations before 1951 or short-term expectations before 1947 (the start of the Livingston survey), and it is difficult even to make educated guesses. (What was long-term expected inflation in the middle of World War II and its price controls?)

In Figure 8, we examine more closely the interest rate distortions that helped to reduce the debt/GDP ratio. The Figure shows the series for x_t, the adjustment of the aggregate interest rate in our counterfactuals, for the period 1947-1974. We see large adjustments at the start of the period—13 percentage points in 1947 and 8 points 1951—when surges in

inflation with pegged nominal interest rates produced deeply negative real rates. Because of these episodes, the actual debt/GDP ratio in 1951 had already diverged by more than 20 percentage points from its level in our rate-adjusted scenario. After that, the x_t adjustment is mostly small until the late 1960s, when unexpected inflation starts pushing it up. The adjustment is over 4 percentage points in 1974. It then stays high through the rest of the 1970s, somewhat dampening the rise in the debt/GDP ratio that we discuss next.

5.2 The Debt Buildup, 1975-2022

As shown in Figure 7, the actual debt/GDP ratio started to rise in 1975 and continued to rise except for a dip in the late 1990s. In 2022 it stood at 97%, not far from its level in 1946. The biggest factor behind the increase was a shift from primary surpluses to primary deficits. Persistent deficits emerged as a result of tax cuts at several points, most notably the Reagan tax cuts of the early 1980s, and the deficit ballooned in the wake of the 2008 financial crisis and the 2020 pandemic.

For our purposes, the most important part of Figure 7 is the combined counterfactual with primary balance and undistorted real interest rates. In this scenario, the debt/GDP ratio falls from its 1974 level of 74% to 70% in 1979, but then starts to rise. In 2022, the debt/GDP ratio is 84%.

Recall that the evolution of debt/GDP in the combined counterfactual depends on $r^\star - g$, the difference between the undistorted real interest rate and the growth rate. The increase in debt/GDP from 1979 to 2022 reflects the fact that on average $r^\star > g$ during that period, a reversal of $r^\star < g$ over 1947-1979. This shift resulted from both a rise in the average $r^\star$, from 2.3% over 1947-1979 to 2.8% over 1980-2022, and a fall in the average g, from 3.5% to 2.6%.

All in all, our findings cast doubt on the common narrative that the U.S. "grew its

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way" out of its World War II debt. Over the 76 years from 1946 to 2022, economic growth without primary surpluses or interest-rate distortions would have reduced the debt/GDP ratio by only 22 percentage points, from 106% to 84%. This experience suggests that we should not count on a major contribution from economic growth to resolving the problem of a high debt level.

A nuance of our results is that the post-1979 rise in the debt/GDP ratio is even larger in the primary-balance counterfactual, which maintains real interest rates at their actual ex-post levels, than in the combined counterfactual (19 percentage points, from 34% to 53%, compared to 14 points). This result reflects the fact that inflation surprises since 1979 have been negative on average, so they have increased ex-post real rates and debt/GDP. The rise in debt/GDP in the primary-balance counterfactual indicates that the actual real interest rate r has exceeded the growth rate g since 1979.

This finding might appear inconsistent with the analysis in Blanchard's 2019 Presidential Address. In arguing that debt dynamics may be benign, Blanchard reports that r has been less than g over almost all of the post-World-War-II era, including the period since 1979. Our findings differ from Blanchard's because of two differences in the measurement of interest rates. First, we use the government's interest payments on outstanding debt, while Blanchard uses market yields on debt, which have been lower than the rates paid by the government because interest rates have trended downward. Second, we use pre-tax interest rates, and Blanchard uses after-tax rates. The Appendix details these differences and argues that our measurement of interest rates is appropriate for our purposes.

6 Comparison to Hall and Sargent (2011)

Hall and Sargent's well-known paper also explores the factors behind changes in the debt-GDP ratio since World War II. There are two main differences between their analysis and

ours. The first, which we want to stress, is that they estimate the role of $r - g$ but do not ask how that difference is influenced by the interest-rate distortions that we consider. The second is a more subtle distinction between our counterfactual debt paths and Hall and Sargent's decomposition of actual debt changes. To make these points clear, we first present an analysis that is similar to Hall and Sargent's and then compare it to our approach.

6.1 Hall and Sargent's Decomposition

Here we perform an exercise in the spirit of Hall and Sargent. (See also the Eichengreen et al. (2021) study of international data.) We analyze the change in debt from 1946 to 1974, when the debt/GDP ratio fell to its trough of 23 percent. Starting with the debt dynamics equation (1), we divide all variables by nominal GDP in year t, Y_t, which yields:

$$d_t = \frac{1 + i_t}{1 + z_t} d_{t-1} - p_t \tag{7}$$

where $d_t = \frac{D_t}{Y_t}$, $p_t = \frac{P_t}{Y_t}$, and z_t is the growth rate of nominal GDP. Rearranging this equation yields the change in the debt/GDP ratio over one period:

$$d_t - d_{t-1} = \frac{i_t - z_t}{1 + z_t} d_{t-1} - p_t \tag{8}$$

Finally, cumulating over time yields the total change in debt/GDP over 1946-1974:

$$d_{1974} - d_{1946} = \underbrace{\sum_{t=1947}^{1974} \frac{i_t - z_t}{1 + z_t} d_{t-1}}_{(a)} - \underbrace{\sum_{t=1947}^{1974} p_t}_{(b)} \tag{9}$$

Equation (9) is a simplified version of Hall and Sargent's equation (11). Note that $i - z$ (the nominal interest rate minus the nominal growth rate) is the same as $r - g$ (the real interest rate minus the real growth rate). Hall and Sargent interpret the two terms on the

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right side of equation (9) as the contributions of $r - g$ (term a) and primary surpluses (term b) to the decline in debt/GDP to its 1974 trough.

We report the two terms in equation (9) in Table 2A. The contributions of $r - g$ and primary surpluses to the fall in D/Y are -48 percentage points and -30 percentage points, respectively. These numbers are roughly consistent with Hall and Sargent's results for the period 1945-1974, despite some differences in the details of calculations. (Hall and Sargent report contributions of -48 points from $r - g$ and -35 points from primary surpluses.)[9]

6.2 Accounting for Interest-Rate Distortions

Using the earlier analysis in this paper, we can go a step beyond equation (9) and divide $r - g$ into two parts: $r^* - g$ and $-x$, where r^* is again the undistorted real interest rate and $-x = r - r^*$ is the distortion from surprise inflation and the pre-Accord peg. (Recall that $+x$ is the adjustment to interest rates that eliminates the distortion). We can decompose the decline in D/Y into contributions from these two terms and from primary surpluses:

$$d_{1974} - d_{1946} = \underbrace{\underbrace{\sum_{t=1947}^{1974} \frac{r_t^* - g_t}{1 + z_t} d_{t-1}}_{(a_1)} - \underbrace{\sum_{t=1947}^{1974} \frac{x_t}{1 + z_t} d_{t-1}}_{(a_2)}}_{\equiv (a)} - \underbrace{\sum_{t=1947}^{1974} p_t}_{(b)} \tag{10}$$

Table 2B shows the three terms in equation (10). The new result is that, of the 48 percentage point contribution of $r-g$ to debt reduction (term a), only 12 points are attributed to $r^* - g$ (term a_1). The other 36 points reflect distortions in interest rates (term a_2). Put differently, for the period 1946-1974, about three quarters of the fall in the debt-GDP ratio

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[9]Minor differences between Hall and Sargent's calculations and ours include slightly different time periods (calendar years 1945-1974 vs. fiscal years 1946-1974) and Hall and Sargent's focus on the market value of debt rather than the par value. Hall and Sargent also decompose the $r - g$ term in equation (9) into contributions from i, π, and g (without distinguishing between expected and surprise π). They find that the g component reduced D/Y by 32 percentage points, which they interpret as "a lot." We follow more recent work in focusing on $r - g$ rather than g by itself.

traditionally attributed to $r - g$ comes from real rate distortions. These results confirm our earlier conclusion that debt/GDP would have fallen much less without the distortions and primary surpluses.[10]

6.3 Comparison to Counterfactual Histories

The decompositions of debt changes in both equation (9) and equation (10) differ from this paper's counterfactual histories in subtle but quantitatively significant ways. To illustrate this point, consider the question of how much of the fall in debt/GDP over 1946-1974 to attribute to primary surpluses. In Hall and Sargent (2011) and equations (9) and (10), the answer is given by term (b), the sum of surpluses, which is 30 percentage points. The alternative that we develop above is the difference between the fall in debt/GDP in actual history and in the primary balance scenario, which is considerably lower at 17 points. Why this difference?

To answer this question, start again with equation (7), which shows the evolution over time of the actual debt/GDP ratio d_t. The evolution of debt/GDP in the primary balance scenario is given by (7) with two modifications: the surplus p_t is set to zero in all years, and the actual values of debt d_t are replaced by the values in the primary balance scenario, which we denote $\hat{d}_t$. This modified version of (7) leads to:

$$\hat{d}_{1974} - d_{1946} = \sum_{t=1947}^{1974} \frac{i_t - z_t}{1 + z_t} \hat{d}_{t-1} \tag{11}$$

which gives the counterfactual fall in debt/GDP through 1974. (Note that the initial level d_{1946} is the same in the counterfactual and actual history.) Combining equations (9) and

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[10]The terms in Table 2 sum to 78 percentage points, which is less than the total decrease in D/Y over 1946-1974 (83 percentage points). This difference reflects the residual in the debt dynamics equation, which accounts for 5 percentage points and which we discuss in footnote 5 and the Appendix.

(11), the difference between 1974 debt/GDP in the counterfactual and actual history is:

$$\hat{d}_{1974} - d_{1974} = \sum_{t=1947}^{1974} \frac{i_t - z_t}{1 + z_t}\left(\hat{d}_{t-1} - d_{t-1}\right) + \sum_{t=1947}^{1974} p_t \qquad (12)$$

We see that this expression includes the sum of surpluses but there is also another sum. The terms in this sum are negative in most years because $\hat{d}_t > d_t$ (debt/GDP is higher in the counterfactual than in actual history) and $i_t - z_t = r_t - g_t$ is negative for most years in 1947-1974. Mathematically, this explains why $\hat{d}_{1974} - d_{1974}$ (17 percentage points) is less than the sum of surpluses (30 points).

The economics behind equation (11) involve the erosion of debt resulting from $r < g$. The higher levels of debt in the primary balance counterfactual magnify this effect, which moderates the rise in debt. That is, the elimination of primary surpluses in the counterfactual creates additional debt, but part of this debt is eroded by $r < g$, so the total increase in debt is less than the sum of surpluses that are eliminated.

For similar reasons, the contribution of interest rate distortions x_t as measured in equation (10) is different from the effects of distortions as measured by our counterfactuals. We conclude that, while the decompositions in (9) and (10) are useful for comparing our results to other work, the counterfactuals in Figure 7 and Table 1 are more informative in showing how debt/GDP would have evolved in alternative scenarios.

7   Conclusions

This paper studies the factors behind the behavior of the U.S. debt/GDP ratio since 1946, both the large decline in the ratio from 1946 to 1974 and the large increase since then. We examine the roles of primary surpluses and deficits; distortions of real interest rates from surprise inflation and from pegged nominal rates before the 1951 Fed-Treasury Accord;

and the difference between the undistorted real interest rate and the growth rate of output $(r^\star - g)$.

For the period up to 1974, we find that the fall in the debt/GDP ratio is explained mostly by primary surpluses and interest-rate distortions. Absent those factors, with the path of the ratio determined entirely by $r^\star - g$, the ratio of 106% in 1946 would have fallen only to 74% in 1974 rather than the actual trough of 23%.

For the debt increase since 1974, the most important factor is large primary deficits. Another factor, however, is a switch in the sign of $r^\star - g$: on average, the undistorted real interest rate has exceeded the growth rate. As a result, with primary balance and undistorted interest rates, the debt/GDP ratio would have grown from 74% in 1974 to 84% in 2022, not too far from its 1946 level.

At the end of fiscal year 2022, the actual debt/GDP ratio had risen to 97%, close to its peak of 106% in 1946. What are the prospects for debt going forward? Can we hope for a steep fall in the debt/GDP ratio like the one after World War II?

It is unlikely that debt/GDP will be reduced significantly by the kinds of interest-rate distortions that reduced the ratio after 1946. Presumably, U.S. policymakers are not considering the kind of interest-rate peg (with price controls to contain the inflationary effects) that was imposed during World War II. And despite the recent surge in inflation, the Federal Reserve appears committed to pushing inflation back down and keeping it low, which would preclude debt erosion through surprise inflation. Additionally, any inflation surprises that occur will have smaller effects than they did in the past because the average maturity of the debt is shorter (Aizenman and Marion 2011, Hilscher et al. 2021).

In theory debt could be reduced rapidly through primary surpluses, but that appears unlikely. Under current policy, the Congressional Budget Office predicts large primary *deficits* over the next three decades, and political leaders are not proposing the kind of fiscal adjustment that could change this outcome.

Will economic growth help resolve the debt problem? On average since World War II, a negative $r^\star - g$ has contributed to debt reduction, but this effect has been weak: it has reduced debt/GDP by only 22 percentage points over 76 years. This history suggests that, if future primary surpluses were zero, the debt would not explode but it would also not fall by much in the next few decades. And with the primary deficits that are forecasted, we are likely to see debt/GDP climb to higher and higher levels.[11]

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[11]Jiang et al. (2022) use CBO budget projections to analyze the prospects for debt.

References

AIZENMAN, J. AND H. ITO (2020): "U.S. Macro Policies and Global Economic Challenges," Working Paper 28232, National Bureau of Economic Research.

AIZENMAN, J. AND N. MARION (2011): "Using Inflation to Erode the U.S. Public Debt," *Journal of Macroeconomics*, 33, 524–541.

ANDREOLLI, M. AND H. REY (2023): "The Fiscal Consequences of Missing an Inflation Target," Working Paper 30819, National Bureau of Economic Research.

ARUOBA, S. B. (2020): "Term Structures of Inflation Expectations and Real Interest Rates," *Journal of Business & Economic Statistics*, 38, 542–553.

BALL, L. M., J. E. GAGNON, P. HONOHAN, AND S. KROGSTRUP (2016): *What Else Can Central Banks Do?*, International Center for Monetary and Banking Studies Geneva.

BARSKY, R. B. (1987): "The Fisher Hypothesis and the Forecastability and Persistence of Inflation," *Journal of Monetary Economics*, 19, 3–24.

BLANCHARD, O. (2019): "Public Debt and Low Interest Rates," *American Economic Review*, 109, 1197–1229.

CARLSON, J. (1977): "A Study of Price Forecasts," in *Annals of Economic and Social Measurement, Volume 6, number 1*, National Bureau of Economic Research, Inc, 27–56.

COCHRANE, J. H. (2021): "The Fiscal Theory of the Price Level: An Introduction and Overview," *Journal of Economic Perspectives*.

EICHENGREEN, B. AND R. ESTEVES (2022): "Up and Away? Inflation and Debt Consolidation in Historical Perspective," *Oxford Open Economics*, 1.

EICHENGREEN, B. J., A. EL-GANAINY, R. ESTEVES, AND K. J. MITCHENER (2021): *In Defense of Public Debt*, Oxford University Press.

ELMENDORF, D. W. AND N. G. MANKIW (1999): "Government Debt," *Handbook of Macroeconomics*, 1, 1615–1669.

FRIEDMAN, M. AND A. J. SCHWARTZ (1963): *A Monetary History of the United States, 1867-1960*, Princeton University Press.

FURMAN, J. AND L. SUMMERS (2020): "A Reconsideration of Fiscal Policy in the Era of Low Interest Rates," Unpublished manuscript, Brookings Institution.

HALL, G., J. PAYNE, AND T. J. SARGENT (2018): "US Federal Debt 1776-1960: Quantities and Prices," Working Paper 18-25, New York University, Leonard N. Stern School of Business, Department of Economics.

HALL, G. J. AND T. J. SARGENT (2011): "Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics," *American Economic Journal: Macroeconomics*, 3, 192–214.

——— (2022): "Three World Wars: Fiscal–Monetary Consequences," *Proceedings of the National Academy of Sciences*, 119.

HAUBRICH, J., G. PENNACCHI, AND P. RITCHKEN (2012): "Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps," *The Review of Financial Studies*, 25, 1588–1629.

HETZEL, R. L. AND R. F. LEACH (2001): "The Treasury-Fed Accord : A New Narrative Account," *Economic Quarterly*, 33–55.

HILSCHER, J., A. RAVIV, AND R. REIS (2021): "Inflating Away the Public Debt? An Empirical Assessment," *The Review of Financial Studies*.

KOZICKI, S. AND P. TINSLEY (2001): "Term Structure Views of Monetary Policy Under Alternative Models of Agent Expectations," *Journal of Economic Dynamics and Control*, 25, 149–184.

KRUGMAN, P. (2019): "Melting Snowballs and the Winter of Debt," *The New York Times*, https://www.nytimes.com/2019/01/09/opinion/melting-snowballs-and-the-winter-of-debt.html.

MCBRIDE, J., N. BERMAN, AND S. ANSHU (2023): "The U.S. National Debt Dilemma," *Council on Foreign Relations*, https://www.cfr.org/backgrounder/us-national-debt-dilemma.

PHILLIPS, M. (2012): "The Long Story of U.S. Debt, From 1790 to 2011, in 1 Little Chart," *The Atlantic*, https://www.theatlantic.com/business/archive/2012/11/the-long-story-of-us-debt-from-1790-to-2011-in-1-little-chart/265185/.

REINHART, C. M. AND M. B. SBRANCIA (2015): "The Liquidation of Government Debt," *Economic Policy*, 30, 291–333.

Tables

Table 1 DEBT/GDP RATIO (%) - ACTUAL AND COUNTERFACTUALS

Year	Actual	Counterfactuals		
		Primary Balance	Adjusted Interest Rate	Combined
1946	106.1	106.1	106.1	106.1
1974	23.2	39.9	51.0	73.9
2022	97.0	53.3	122.9	84.2

NOTE. The table shows the values of the debt/GDP ratio in actual history and our counterfactuals in 1946, 1974, and 2022. Source: OMB, authors' calculations.

Table 2 CONTRIBUTIONS TO CHANGE IN DEBT/GDP RATIO (%), 1946-1974

Table 2A

Δ(Debt/GDP)	Contribution of:	
	$r - g$	surpluses
-82.9	-48.1	-29.6

Table 2B

Δ(Debt/GDP)	Contribution of:		
	$r^* - g$	$r - r^*$	surpluses
-82.9	-11.7	-36.3	-29.6

NOTE. Table 2A shows the contributions to the change in Debt/GDP of $r - g$ (term a in equation 9) and primary surpluses (term b). Table 2B divides the contribution of $r - g$ into contributions of $r^* - g$ (term a_1 in equation 10) and the interest rate distortion $r - r^*$ (term a_2). The terms in Table 2 sum to 78 percentage points, which is less than the total decrease in Debt/GDP of 83 points. This difference reflects the residual in the debt dynamics equation, which we discuss in footnote 5, and which contributes 5 percentage points to the drop in Debt/GDP. Source: Authors' calculations.

Figures

Figure 1 FEDERAL DEBT HELD BY THE PUBLIC AS A PERCENT OF GDP
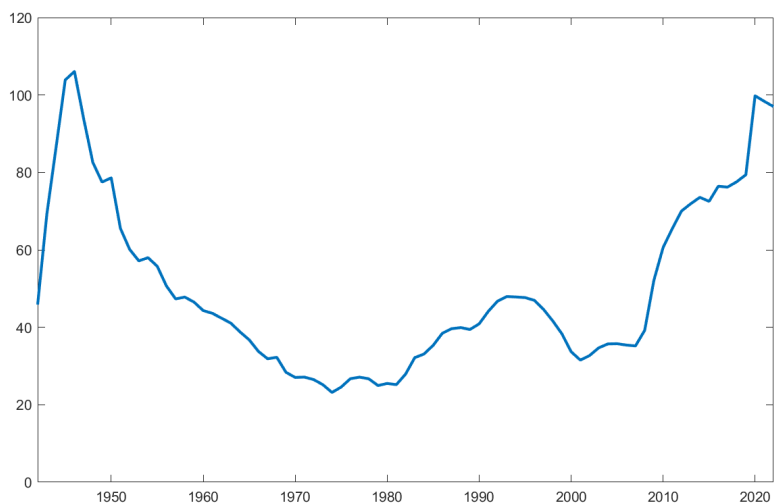
NOTE. The line represents the ratio of the federal debt held by the public to GDP. Source: OMB.

**Figure 2** PRIMARY SURPLUS AS A PERCENT OF GDP
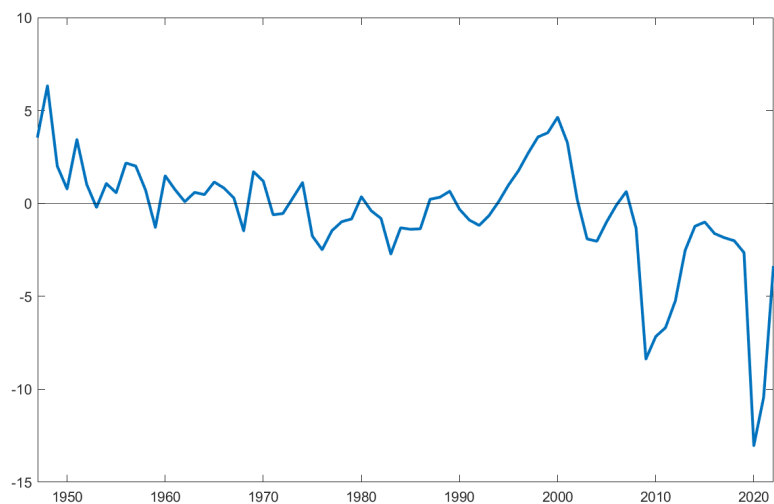
NOTE. The line represents the ratio of the primary budget surplus to GDP. The primary surplus is computed as the sum of the total fiscal surplus and interest payments on debt held by the public. Source: OMB, authors' calculations.

Figure 3 Actual and Expected Inflation
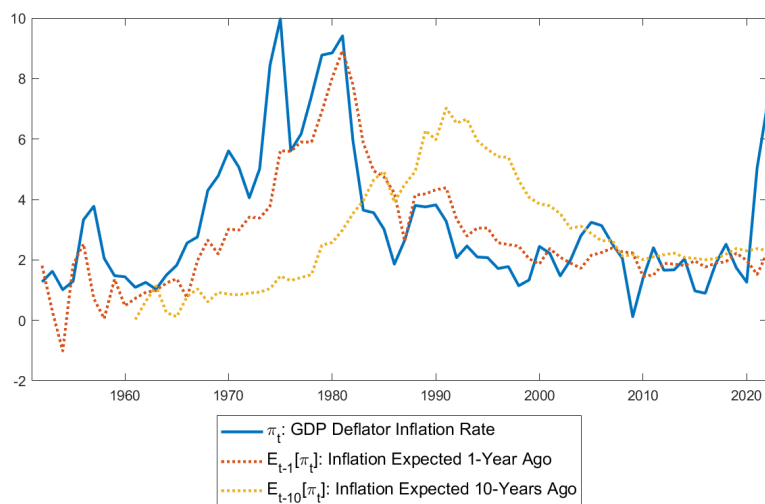
NOTE. The lines represent the GDP deflator inflation rate and forecasts made one year and ten years in the past. Source: Authors' calculations.

Figure 4 Reverse Maturity Structure of Public Debt
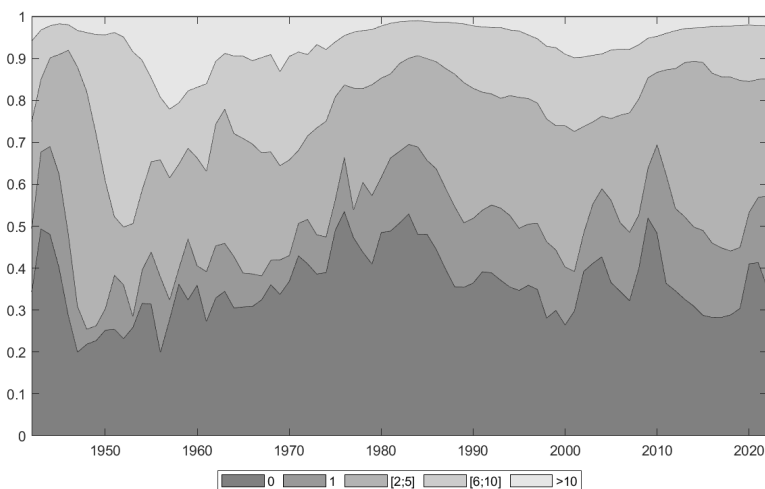
NOTE. This chart shows the reverse maturity structure of outstanding debt held by the public. The different shades represent the share of the debt at the end of the fiscal year which was issued in the same year, the previous year, 2 to 5 years earlier, 6 to 10 years earlier, and more than 10 years earlier. Lighter shades indicate longer reverse maturities. Source: Authors' calculations.

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Figure 5 Short- and Long-Term Inflation Expectations
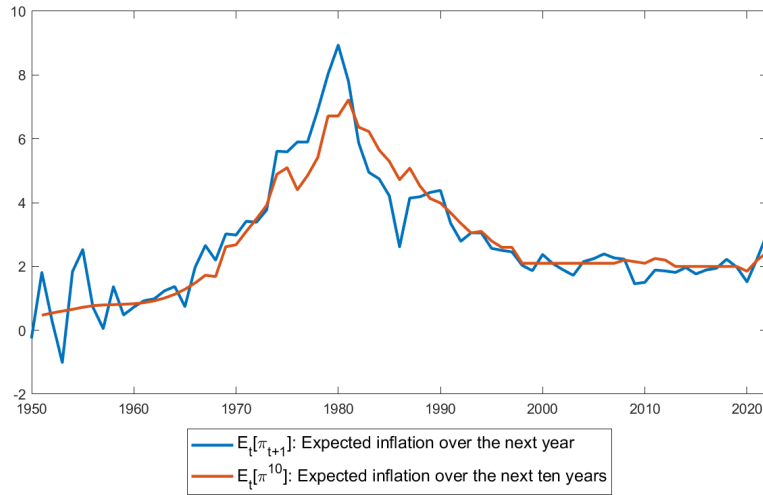
Figure 6 Term Structure of Inflation Expectations
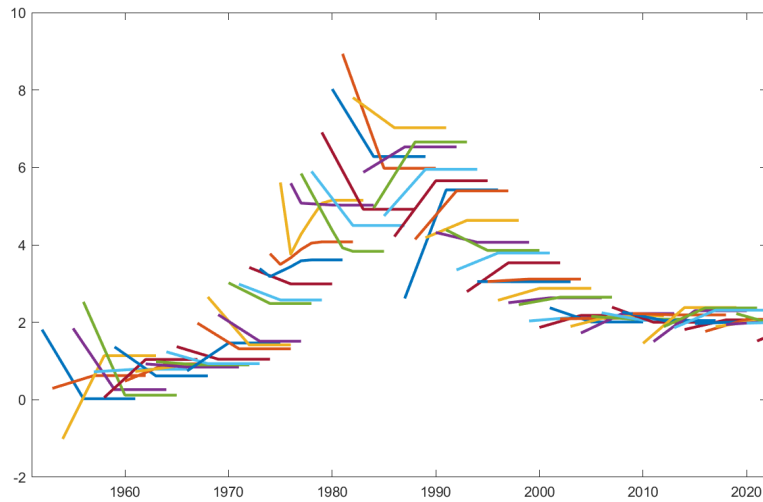
Figure 7 DEBT/GDP PATHS - COUNTERFACTUAL SCENARIOS
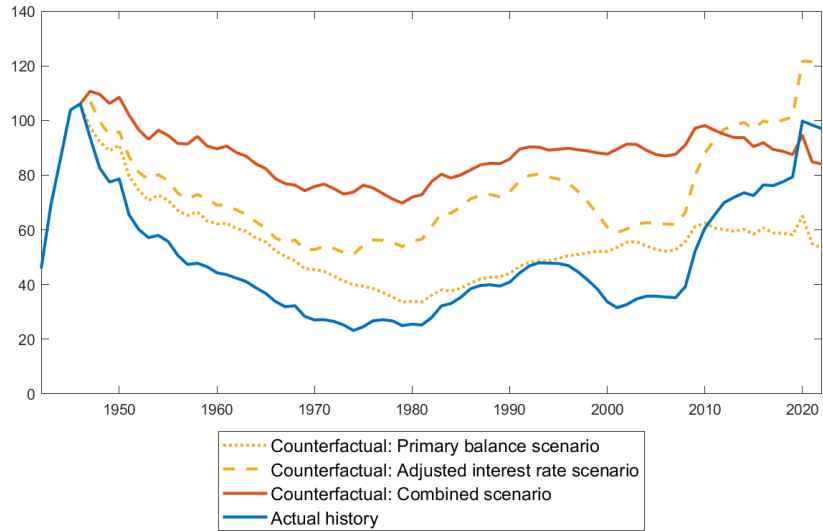
NOTE. The lines represent the path of the debt-GDP ratio in actual history and our different counterfactual scenarios. Source: Authors' calculations.

Figure 8 AGGREGATE INTEREST RATE ADJUSTMENTS, 1947-1974
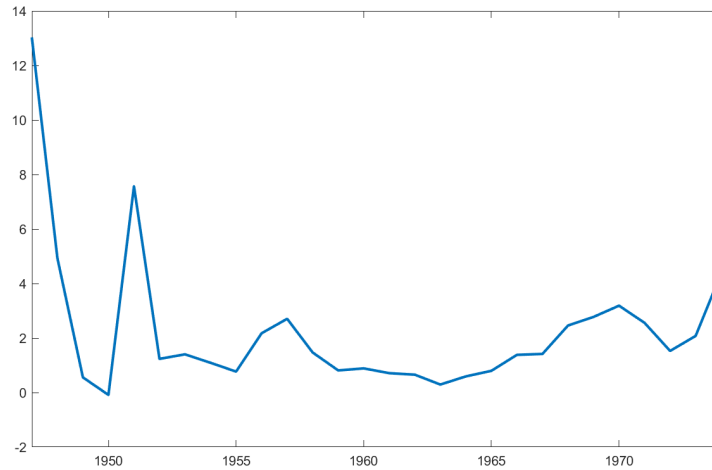
NOTE. The line represents x_t, the difference between the aggregate interest rate in our adjusted rate scenario and the actual interest rate. Source: Authors' calculations.

A Appendix

This Appendix describes various details of our procedures and our data sources.

A.1 Exact Equations for the Evolution of Debt

Aggregate Debt Dynamics

The analysis in the text assumes that equation (1), which relates the evolution of the debt to interest rates and the primary balance, holds exactly. In fact, there is a residual in this relationship:

$$D_t = (1 + i_t)D_{t-1} - P_t + \epsilon_t. \tag{A.1}$$

The residual ϵ_t arises from a number of factors that add to or subtract from the debt besides interest on the debt and the primary balance, including changes in the level of operating cash held by the Treasury and interest paid to the government on assets such as student loans. We measure ϵ_t using (A.1) and our series for D_t, i_t, and P_t. For years before 1962, the residual includes modest errors in our measures of interest rates arising from the need to approximate interest net of intragovernmental payments. In our counterfactuals, when we adjust i_t and P_t, we hold the path of ϵ_t constant. Part 6 of this Appendix reports the series for ϵ_t and analyzes its role in debt dynamics.

Accounting for Treasury Bills and TIPS

For most debt issued after the Fed-Treasury Accord, our counterfactuals adjust the interest rate in year t on debt issued at τ by the inflation surprise $\pi_t - \mathbb{E}_\tau[\pi_t]$. However, as discussed in the main text, we assume that inflation surprises do not affect the real returns on Treasury bills or TIPS (inflation-indexed debt). That means we must modify the interest-rate adjustments in equation (6) so that adjustments for surprise inflation apply only to the fraction of the debt that is not T-bills or TIPS. The equation becomes:

$$x_t^\tau = \begin{cases} 0 & \text{for } \tau \leq 1942 \\ r_t^{\star\,\tau} - (i_t^\tau - \pi_t) & \text{for } 1943 \leq \tau \leq 1950 \\ r_t^{\star\,\tau} - (i_t^\tau - \pi_t) & \text{for } \tau = 1951 \text{ and } t > \tau + 1 \\ (1 - s_{t-1})\left(r_t^{\star\,\tau} - \left(\tilde{i}_t^\tau - \pi_t\right)\right) & \text{for } \tau = 1951 \text{ and } t = \tau + 1 \\ \pi_t - \mathbb{E}_\tau[\pi_t] & \text{for } 1952 \leq \tau \leq 1996 \text{ and } t > \tau + 1 \\ (1 - s_{t-1})(\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } 1952 \leq \tau \leq 1996 \text{ and } t = \tau + 1 \\ \left(1 - z_{t-1}^\tau\right)(\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } \tau \geq 1997 \text{ and } t > \tau + 1 \\ \left(1 - s_{t-1} - z_{t-1}^\tau\right)(\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } \tau \geq 1997 \text{ and } t = \tau + 1 \end{cases} \tag{A.2}$$

where s_{t-1} is the fraction of debt outstanding at the end of year $t-1$ and issued during $t-1$

that is T-bills, z_{t-1}^{τ} is the fraction of the debt outstanding at the end of $t-1$ and issued during τ that is TIPS, and $\tilde{i}_t^{\tau}$ is the average interest rate paid at t on securities other than T-bills that are outstanding at the end of $t-1$ and issued during τ.

In this equation, for $\tau \geq 1952$ we reduce each interest-rate adjustment by the fraction of debt that is T-bills or TIPS. The adjustment for T-bills is only relevant for $\tau = t-1$, because all T-bills outstanding at the end of $t-1$ were issued during $t-1$, and the adjustment for TIPS is only relevant starting in 1997, when TIPS were introduced. For most of the peg period, we do not make any adjustment for the fraction of debt that is T-bills, because T-bills were rolled over at the pegged interest rate. An exception is the last year of the peg, fiscal year 1951, because T-bills outstanding at the end of 1951 were rolled over in 1952, after the peg ended.

The Transitional Quarter

In the main text, we assume that a period is a fiscal year. A nuisance feature of the data is the Transitional Quarter (TQ), the third quarter of calendar year 1976. This quarter is special because the government changed the start of its fiscal year from July to October for fiscal year 1977. In our simulations, we treat the TQ as a period between fiscal years 1976 and 1977. The debt dynamics equation (A.1) holds for all periods including the TQ with i_t and P_t in the TQ measured as the non-annualized interest rate and primary balance. The adjustments x_t^{τ} in equation (A.2) also hold with π_t in the TQ measured as the non-annualized inflation rate. The existence of the TQ complicates the measurement of inflation expectations in some periods, as described below.

A.2 Measurement of Fiscal Variables

Debt and Primary Balance

Our series for debt held by the public at the end of each fiscal year comes from the Office of Management and Budget (OMB) database. We compute debt/GDP ratios using the OMB's series for nominal GDP by fiscal year.

We measure debt at its par value, whereas some analyses of debt measure it at its market value (e.g., work on the fiscal theory of the price level by Cochrane 2021). However, this distinction is not important for our purposes. Unexpected changes in interest rates cause market and par values to diverge in the short run, but the two measures of debt move together closely at the horizons relevant for our analysis; see Figure A.1. The closeness of the two debt paths reflects the fact that a revaluation of a security is offset over time by the corresponding change in its market yield.

We also use the OMB data to compute the primary balance P_t as the sum of the total fiscal surplus (which is usually negative) and total interest payments, with total interest calculated as described below.

Aggregate Interest Rates

The aggregate interest rate i_t is defined as total interest payments during period t divided by the stock of debt at the end of $t - 1$. The debt is debt held by the public, and interest payments are the payments on that debt: they exclude intragovernmental payments on debt held by entities such as the Social Security Trust Fund.

For $t \geq 1962$, we compute the appropriate series for interest payments as gross interest on the debt minus intragovernmental interest payments, using the OMB historical database.

For $t < 1962$, OMB does not report intragovernmental interest payments. It reports gross interest and "net interest," but the latter understates the interest paid on debt held by the public because interest received by the government, as well as intragovernmental payments, are subtracted from gross interest (a problem noted by Hall and Sargent (2011)). We can gauge the extent of this understatement by comparing net interest to the correct series for interest on debt held by the public for $t \geq 1962$, when both series exist. Figure A.2 shows the interest rates computed by dividing each of these series by debt held by the public at the end of $t - 1$.

For the period from 1962 to 2022, we find that the ratio of our correct measure of interest payments to the net interest reported by OMB is equal on average to 1.1, and is fairly stable over that period. Therefore, we estimate the average interest rate before 1962 by multiplying net interest by 1.1 and dividing by the stock of debt at the end of $t - 1$.

Our interest rate series for $t < 1962$ is an approximation based on incomplete data, but our results are not very sensitive to the exact approximation. If for $t < 1962$ we measured interest payments with net interest from OMB, then the debt/GDP ratio in our combined counterfactual would be 73% in 1974 (compared to 74% in our baseline case) and 83% in 2022 (compared to 84%).

The Reverse Maturity Structure

We construct the reverse maturity structure of the debt from the Hall et al. (2018) database for the period from 1942 through 1960, and from the CRSP Monthly U.S. Treasury Database for 1961 through 2022. For every month, these databases provide an accounting of individual Treasury securities outstanding, including issue dates and quantities. We use the data for the final month of each fiscal year to construct D_t^τ, the amount of debt outstanding at the end of year t that was issued in year τ.

The Hall et al. (2018) data set includes almost every outstanding security. Therefore, for $t - 1 \leq 1960$, the weights w_{t-1}^τ that define the reverse maturity structure can be computed simply as:

$$w_{t-1}^\tau = D_{t-1}^\tau / D_{t-1}, \tag{A.3}$$

where here D_{t-1} is the total stock of debt reported in the Hall et al. (2018) data set (which is extremely close to the stock of debt reported by OMB that we use elsewhere). Our measurement of the weights implies that they sum exactly to one.

The CRSP data set that we use for 1961-2022 has two limitations: it excludes non-

marketable debt and it excludes Treasury bills. We proceed as follows. First, we divide the total debt D_t into Treasury bills, marketable debt excluding Treasury bills, and non-marketable debt. We use data from the Monthly Statement of the Public Debt (MSPD) database for Treasury bills and Hall and Sargent (2022) for aggregate marketable debt and non-marketable debt.

We derive a reverse maturity structure for all marketable debt using the quantity of T-bills and the data on other marketable debt from CRSP. The weights $w_{t-1}^{\tau,m}$ for $t-1 > 1960$ are defined by:

$$w_{t-1}^{\tau,m} = D_{t-1}^{\tau,m}/D_{t-1}^m, \tag{A.4}$$

where $D_{t-1}^{\tau,m}$ is the stock of marketable debt at the end of $t-1$ that was issued at τ and D_{t-1}^m is total marketable debt at the end of $t-1$ (the sum of $D_{t-1}^{\tau,m}$ for all τ). For $t-1 > \tau$, $D_{t-1}^{\tau,m}$ is the sum of all securities in the CRSP data set that were issued at τ and are outstanding at the end of $t-1$. For $t-1 = \tau$, $D_{t-1}^{\tau,m}$ is the sum of two components: the CRSP securities that were issued during $t-1$ and are outstanding at the end of $t-1$, and the stock of Treasury bills outstanding at the end of $t-1$, which we assume were also issued during $t-1$. We checked that the sum D_{t-1}^m is extremely close to the stock of marketable debt that we compute from the MSPD database and Hall's website.

Based on the Hall et al. (2018) data set, which includes both marketable and non-marketable debt before 1961, we know that non-marketable securities tend to have longer maturities. (See Figures A.3 and A.4 for the reverse maturity structure of marketable and non-marketable debt.) Lacking granular data on non-marketable debt, we simply assume that the reverse maturity structure of that part of the debt is the same in all years after 1960 as it is in 1960:

$$w_{t-1}^{\tau,nm} = w_{1960}^{\tau,nm} \; \forall t > 1961 \text{ and } \forall \tau \leq t-1, \tag{A.5}$$

where $w_{1960}^{\tau,nm}$ for all τ's is the reverse maturity structure for non-marketable debt in 1960, which we obtain from the Hall et al. (2018) database. Assuming that the reverse maturity structure of non-marketable debt does not change after 1960 introduces some error in our calculations, but we believe the impact is modest because after 1960 non-marketable debt was a fairly small part of total debt: as shown in Figure A.5, it declined from 23% of total debt in 1960 to 3% percent in 2022.

Given the reverse maturity structures of marketable and non-marketable debt after 1960, we construct the w_{t-1}^{τ}'s defining the reverse maturity structure of total debt as the average of $w_{t-1}^{\tau,m}$ and $w_{t-1}^{\tau,nm}$ weighted by the shares of the two types of debt:

$$w_{t-1}^{\tau} = w_{t-1}^{\tau,m} m_{t-1} + w_{t-1}^{\tau,nm}(1 - m_{t-1}) \tag{A.6}$$

where m_{t-1} is the share of marketable debt in total debt outstanding at the end of $t-1$. We compute the weights m_{t-1} from Hall's data on aggregate outstanding marketable and non-marketable debt held by the public. We checked that outstanding marketable debt is extremely close to the sum of outstanding debt reported by CRSP and outstanding Treasury

bills reported in the Treasury Bulletins.

TIPS and Treasury Bills

We compute s_{t-1}, the share of Treasury bills in outstanding debt that was issued in year $t-1$, as:

$$s_{t-1} = \frac{D_{t-1}^{bills}}{D_{t-1}^{t-1}} \tag{A.7}$$

where D_{t-1}^{bills} is the stock of Treasury bills outstanding at the end of $t-1$ from the MSPD. We compute z_{t-1}^{τ}, the share of TIPS securities in outstanding debt that was issued in year τ, as:

$$z_{t-1}^{\tau} = \frac{D_{t-1}^{\tau,tips}}{D_{t-1}^{\tau}} \tag{A.8}$$

where $D_{t-1}^{\tau,tips}$ is the stock of TIPS outstanding at the end of $t-1$ that were issued in τ. In calculating z_{t-1}^{τ}, we use the securities included in CRSP (and T-bills for $t-1 = \tau$) in both the numerator and denominator. The denominator also includes non-marketable debt.

A.3 Measuring Inflation and Inflation Expectations

Actual Inflation

We measure the inflation rate in fiscal year t as the growth rate in the GDP deflator from the last quarter of year $t-1$ to the last quarter of year t, from the National Income and Product Accounts.

   As described below, we also use data on the CPI inflation rate before 1970. We measure CPI inflation in fiscal year t with the inflation rate from the last month of $t-1$ to the last month of t (from June to June given the dating of fiscal years before 1970). We use CPI data from the BLS.

One-Year Expectations

For $t \geq 1970$, we measure one-year expected inflation $\mathbb{E}_t[\pi_{t+1}]$ with the median forecast of inflation over the next four quarters reported in the Survey of Professional Forecasters for the last quarter of fiscal year t.

   For $t < 1970$, we create a proxy for expected GDP deflator inflation from forecasts of CPI inflation in the Livingston survey of business economists. We use forecasts from the first of each calendar year's semi-annual surveys, which are published in June. Before 1970, June is the last month of the fiscal year. The raw data are forecasts of the CPI level in the following June. The FRB of Philadelphia, which maintains the SPF, computes an inflation rate forecast following the methodology of Carlson (1977). The method assumes that forecasters have observed the actual CPI for April, and therefore uses the CPI for April of year t and the forecast for June of $t+1$ to compute a forecast for annualized inflation over

14 months. We use this as a proxy for expected CPI inflation over fiscal year $t + 1$, from June of t to June of $t + 1$.

As discussed in the text, we compute a forecast of GDP deflator inflation for $t < 1970$ by assuming that the expectation error $\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]$ is the same for the GDP deflator as for the CPI (even though the level of CPI inflation is on average several tenths of a percentage point higher than that of GDP deflator inflation). We can see that this approximation is reasonable from Figure A.6, which plots the expectation errors for the two inflation rates from 1970 to 1998, when we can compute both using the SPF and Livingston data. The two series are usually close.

Ten-Year Expectations

As discussed in the text, for $t \geq 1968$, we measure ten-year expected inflation $\mathbb{E}_t[\pi^{10}]$ with long-term expected inflation from the Fed's database for its FRB/US Model[12]. These expectations are forecasts of inflation in the PCE deflator, but we use them as expectations of GDP deflator inflation because the actual inflation rates for these two deflators move closely together; see Figure A.7.

We do not have data on long-term expectations before 1968, so we construct a proxy using the series on one-year expectations and the relation between one-year and ten-year expectations. In developing this procedure, we first smooth the series for one-year expectations using the HP filter with $\lambda = 100$. Figure A.8 shows the smoothed series along with the actual series for one-year expectations and for ten-year expectations after 1968. We see that ten-year expectations generally follow the trend in one-year expectations, but lag behind somewhat as one-year expectations rise from 1968 to the early 1980s and then as they fall to the late 1990s. To capture this relationship, we regress the difference between ten-year and smoothed one-year expectations on the change in smoothed one-year expectations for the period 1968-1997, which yields the results shown in Table A.1. Notice there is a negative coefficient on the change in one-year expectations, capturing the tendency of long-term expectations to lag behind short-term expectations when the latter are trending up or down. Figure A.9 shows the fitted values of long-term expected inflation based on the equation in Table A.1 along with actual long-term expected inflation. We can see that the fitted values are close to actual long-term expectations over the estimation period. The Figure extends the fitted values back to 1952 and we use this fitted path as our proxy for ten-year expectations before 1968.

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[12]More specifically, we use the historical values of the PTR variable, which come from several sources. Since 1991Q4, the source is the Survey of Professional Forecasters (SPF), first for expected CPI inflation and then, when it becomes available in 2007, for expected PCE deflator inflation. PTR data from 1981Q1 to 1991Q3 is primarily from a survey conducted by Richard Hoey. The Hoey and SPF CPI observations are reduced by 40 basis points to account for the average difference between CPI and PCE inflation. Values of PTR before 1981 are constructed in a manner similar to the one described in Kozicki and Tinsley (2001).

The Term Structure of Inflation Expectations

Given our series on one-year and ten-year inflation expectations, we make assumptions about the shape of the term structure of expectations that allow us to estimate the entire term structure $\mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[\pi_{t+2}], ...$ Here, we describe our approach for all fiscal years except those from 1972 through 1976. The proximity of those years to the Transitional Quarter produces a complication discussed below.

For the years we consider here, we assume:

$$\mathbb{E}_t[\pi_{t+j}] = \mathbb{E}_t[\pi_{t+1}] + (j-1)k_t \text{ for } 2 \leq j \leq 5 \tag{A.9}$$

$$\mathbb{E}_t[\pi_{t+j}] = \mathbb{E}_t[\pi_{t+5}] \text{ for } j > 5 \tag{A.10}$$

The first equation says that inflation is expected to follow a linear path over the next five years, and the second says that inflation is then expected to remain constant. We view these assumptions as roughly consistent with term structures of expectations estimated by the Federal Reserve Banks of Philadelphia and Cleveland, which typically show that inflation is expected to change monotonically for roughly five years and then flatten out[13].

Given these assumptions, the term structure is determined by k_t, the rate at which inflation is expected to rise from $t+1$ to $t+5$. To determine k_t, we use the fact that long-term expected inflation $\mathbb{E}_t[\pi^{10}]$ is the average of one-year inflation rates expected over the next ten years:

$$\mathbb{E}_t[\pi^{10}] = \frac{1}{10} \sum_{j=1}^{10} \mathbb{E}_t[\pi_{t+j}] \tag{A.11}$$

Substituting equations (A.9) and (A.10) for $j = 2, 3, ...$ into the last equation yields an equation defining k_t in terms of $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[\pi^{10}]$, for which we have data. The solution is:

$$k_t = \frac{\mathbb{E}_t[\pi^{10}] - \mathbb{E}_t[\pi_{t+1}]}{3} \tag{A.12}$$

This solution and equations (A.9) and (A.10) define the term structure of expectations for $t < 1972$, $t =$TQ, and $t \geq 1977$. For $t < 1972$, expected inflation in the TQ is the non-annualized rate in that quarter implied by the constant annual rate expected for $t+5$ and later.

The Term Structure Near the Transitional Quarter

We have a special procedure for determining expectations set in fiscal years from 1972 through 1976. In those years, the periods $t + 1, .., t + 5$ include the Transitional Quar-

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[13]See the historical data on the term structure of expectations from the Federal Reserve Bank of Philadelphia, which uses the methodology developed in Aruoba (2020), and from the Federal Reserve Bank of Cleveland series based on Haubrich et al. (2012).

ter, and since that period is shorter than the others, it is no longer natural to assume that expected inflation changes linearly with the horizon measured in periods. Therefore, for 1972 through 1976, we use the quarterly data on expectations from the SPF to compute a term structure at the quarterly frequency. (This is not possible for our entire sample, because we have only the semi-annual Livingston survey of expectations before 1968.)

We index quarters by s. $\mathbb{E}_s[\pi_{s+j}]$ is the expectation in quarter s of annualized inflation in quarter $s + j$.

For each quarter, the SPF gives forecasts of the inflation rates in the next four quarters, $s+1, \dots s+4$. To construct a term structure for later quarters, we assume that inflation is expected to change linearly from $s+4$ through $s+20$ and then remain constant:

$$\mathbb{E}_s[\pi_{s+j}] = \mathbb{E}_s[\pi_{s+4}] + (j-4)k_s \text{ for } 5 \leq j \leq 20 \tag{A.13}$$
$$\mathbb{E}_s[\pi_{s+j}] = \mathbb{E}_s[\pi_{s+20}] \text{ for } j > 20 \tag{A.14}$$

We assume that long-term expected inflation equals the average of inflation expected over the next forty quarters:

$$\mathbb{E}_s[\pi^{40}] = \frac{1}{40}\sum_{j=1}^{40}\mathbb{E}_s[\pi_{s+j}] \tag{A.15}$$

where $\mathbb{E}_s[\pi^{40}]$ is the expectation at s of inflation over the next forty quarters (ten years), which we measure with the FRB/US quarterly series for long-term expectations. These equations lead to:

$$k_s = \frac{1}{456}\left[40\,\mathbb{E}_s[\pi^{40}] - \sum_{j=1}^{3}\mathbb{E}_s[\pi_{s+j}] - 37\mathbb{E}_s[\pi_{s+4}]\right] \tag{A.16}$$

which defines the quarterly term structure of expectations.

For fiscal years from $t = 1972$ through $t = 1976$, $\mathbb{E}_t[\pi_{t+j}]$ is the expectation in the last quarter of t of cumulated inflation over the four quarters of fiscal year $t + j$. To write this expectation in terms of our quarterly series for expectations, let $s = (t, q)$ denote quarter q of fiscal year t. With this notation,

$$\mathbb{E}_t[\pi_{t+j}] = \left[\prod_{q=1}^{4}\left(1 + \mathbb{E}_{(t,4)}[\pi_{(t+j,q)}]\right)\right]^{1/4} - 1 \tag{A.17}$$

We compute this expectation for $1972 \leq t \leq 1976$ and $j > 0$, accounting for which quarters belong to each fiscal year given the switch in timing in 1977.

Finally, we need to measure non-annualized expected inflation in the TQ for $1972 \leq t \leq 1976$ to compute the inflation surprises $\pi_{TQ} - \mathbb{E}_t[\pi_{TQ}]$ that determine the interest rate adjustments for TQ in our counterfactuals. We do so by converting the expected annualized

inflation rate from the quarterly term structure into a non-annualized rate:

$$\mathbb{E}_t[\pi_{TQ}] = \left[1 + \mathbb{E}_{(t,4)}[\pi_{TQ}]\right]^{1/4} - 1 \tag{A.18}$$

A.4 An Endogenous Reverse Maturity Structure

In our main analysis, we measure the reverse maturity structure of the debt, the weights w_t^τ, with data on actual debt and hold the weights fixed in all counterfactuals. Here we consider a variation in which we account for the fact that a change in the path of aggregate debt affects the amount of new debt issued each period and therefore influences the reverse maturity structure. We capture this effect under the assumption that the forward maturity structure of new debt in each period is the same in a counterfactual as in reality. For example, if 20 percent more debt is issued in year τ in a counterfactual than in history, we assume that the amount of debt issued in τ and maturing at $\tau + j$ is 20 percent higher for all j.

Debt Dynamics

Letting $\hat{D}_t$ equal the level of debt at time t in a counterfactual, the equation for debt dynamics in the counterfactual is:

$$\hat{D}_t = (1 + \hat{i}_t)\hat{D}_{t-1} - \hat{P}_t + \epsilon_t \tag{A.19}$$

where $\hat{i}_t$ and $\hat{P}_t$ are the counterfactual interest rates and primary surpluses and we again hold the residual ϵ_t constant at its historical levels. The interest rate is a weighted average of rates on debt issued in different periods $\tau < t$, with the weights given by the counterfactual reverse maturity structure $\hat{w}_{t-1}^\tau$:

$$\hat{i}_t = \sum_{\tau=t-M}^{t-1} \hat{w}_{t-1}^\tau \hat{i}_t^\tau = \sum_{\tau=t-M}^{t-1} \hat{w}_{t-1}^\tau \left(i_t^\tau + x_t^\tau\right) \tag{A.20}$$

where the second equality uses our decomposition of the counterfactual $\hat{i}_t^\tau$ into the actual rate i_t^τ and the adjustment x_t^τ. We continue to measure x_t^τ with equation (A.2).

Measuring i_t^τ

In our counterfactual analysis with a fixed maturity structure, we eliminate i_t^τ, the interest rates on debt with different reverse maturities, from our expression for the aggregate interest rate (see equation (5)). This simplification is not possible with an endogenous maturity structure, so we must measure the i_t^τ's. We do so with our data on interest rates on individual securities. For fiscal years up to 1960, we use the coupon rates on securities in the Hall et al (2018) database (or, when the coupon rate is missing, the interest rates by maturity from Friedman and Schwartz (1963)), and for years after 1960 we use the interest rates on securities in the CRSP database. As an initial measure of the interest rate i_t^τ, we use the average of the rates on all securities issued at τ and outstanding at t.

These measures are imperfect because the Hall et al. and CRSP data sets do not include every government security; the most important omission is that CRSP does not include non-marketable debt. As a result, with i_t^τ measured with these data, equation (4) relating the aggregate interest rate i_t to the i_t^τ does not hold exactly. To ensure consistency of our interest rate measures, we multiply our initial rates for a given year t by a factor ξ_t such that equation (4) holds. That is, if $\tilde{i}_t^\tau$ is the interest rate calculated from Hall et al. or CRSP, our final measure of i_t^τ is $\xi_t \tilde{i}_t^\tau$ with ξ_t defined by the condition $i_t = \xi_t \sum_{\tau=t-M}^{t-1} w_{t-1}^\tau \tilde{i}_t^\tau$.

In all counterfactuals, we keep s_{t-1}, the shares of T-bills in debt issued during $t-1$, and z_{t-1}^τ, the shares of TIPS in debt issued at τ, fixed at their levels in actual history.

Iterative Construction of the $\hat{w}_t^\tau$'s

The counterfactual weights $\hat{w}_{t-1}^\tau$ are determined by the interplay of the reverse maturity structure captured by the weights and the forward maturity structure of debt issued in each year. This forward maturity structure is defined by D_{t+j}^t, the amount of debt issued in year t that will still be outstanding at $t+j$, for $j \geq 0$. To derive the forward and reverse maturity structures in a counterfactual, we start with these maturity structures in 1946 and earlier, which we take from the data, and then perform an iterative procedure: Given the two maturity structures in years $t-1$ and earlier, we derive them for year t. Each iteration has the following steps:

1. Compute $\hat{D}_t$, the amount of aggregate debt in period t, from equations (A.19) and (A.20). (Note that (A.20) includes the weights $\hat{w}_{t-1}^\tau$ for $\tau \leq t-1$, which come from the previous iteration).

2. Compute $\hat{D}_t^t$, which is the amount of debt outstanding at t and issued during t, that is, the new debt at t. This quantity is given by $\hat{D}_t^t = (\hat{D}_t - \hat{D}_{t-1}) + \sum_{\tau=t-M}^{t-1} \left(\hat{D}_{t-1}^\tau - \hat{D}_t^\tau \right)$. In this expression, the first term is the amount of new debt that must be issued to accommodate the increase in total debt at t. The second term is the amount of new debt that must be issued to roll over debt that matures at t. It is the sum over issue dates τ of debt that was outstanding at $t-1$ but is no longer outstanding at t.

3. Compute $\hat{D}_{t+j}^t$ for all $j > 0$, which define the forward maturity structure of the debt issued at t. To perform this step, we assume that $\hat{D}_{t+j}^t = D_{t+j}^t \frac{\hat{D}_t^t}{D_t^t}$. That is, the amount of debt issued at t that is outstanding at $t+j$ is scaled up relative to actual history by the ratio of new debt at t in the counterfactual and in actual history.

4. The reverse maturity structure at t is defined by $\hat{D}_t$ and $\hat{D}_t^\tau$ for $\tau \leq t$. $\hat{D}_t$ and $\hat{D}_t^t$ are derived in steps 1 and 2. $\hat{D}_t^\tau$ for $\tau < t$ is available from the forward maturity structure at τ, which was derived in a previous iteration.

Results

The results of this exercise prove somewhat anticlimactic. While the endogenous reverse maturity structures differ from the fixed maturity structures in our main analysis, the differences are modest. Figure A.10 illustrates this result by comparing the two reverse maturity structures in three typical years.

Because the endogenous and fixed reverse maturity structures are similar, it does not matter much which we use in our counterfactual analysis. For our combined counterfactual, Figure A.11 shows that the paths of debt/GDP with the two reverse maturity structures are almost indistinguishable. With the fixed structure, debt/GDP is 73.8% in 1974 and 84.1% in 2022. With an endogenous structure, debt/GDP is 74.3% in 1974 and 87.4% in 2022.

A.5 Robustness to Alternative Assumptions About Equilibrium Interest Rates

A central part of our analysis is estimating undistorted real interest rates for various periods. Here we consider the robustness of our results to varying some of the assumptions we make in that exercise. In particular, we consider alternative assumptions about inflation expectations before 1943, equilibrium real rates during the peg period of 1943-1951, and the effects of quantitative easing since 2009.

Inflation Expectations Before 1943

In our main counterfactual simulations, we do not adjust the interest rates on bonds issued before the start of the peg. In principle, it would be more accurate to treat the pre-1943 period in the same way as the post-peg period and adjust rates based on inflation surprises relative to expectations when securities were issued. Our main approach is equivalent to assuming that expectations before 1943 of inflation after 1946 (the start of our simulations) equal actual inflation: there were no surprises relative to pre-1943 expectations. Here we examine the implications of more reasonable conjectures about these expectations (which were not measured directly in surveys). These exercises yield somewhat different results from our main case, but the differences are not large, reflecting the fact that only 20% of the debt outstanding in 1946 had been issued before 1943.

One reasonable conjecture is that, for $\tau < 1943$ and $t > 1946$, $\mathbb{E}_\tau[\pi_t] = 0$: no inflation was expected. This idea is suggested by the behavior of actual inflation in the decades before 1942. Barsky (1987) estimates the univariate process for inflation over the period 1860-1939 and finds it is close to a random walk without drift, implying zero expected inflation five or more years in the future. Before the peg there were episodes of inflation or deflation over several years, which produced substantial changes in the price level, but no period of persistently positive inflation.

We also consider an alternative assumption of $\mathbb{E}_\tau[\pi_t] = 3\%$ for $\tau < 1943$ and $t > 1946$. We view this as a generous upper bound on expected inflation: it means that markets expected a shift to a regime of persistent inflation that had not been seen in the past.

Table A.2 shows the implications of these assumptions. For our combined counterfactual with interest rate adjustments and primary balance, we compare debt/GDP in 1974 and 2022 in our main analysis to debt/GDP with the two alternative assumptions about pre-1943 expectations. The assumption of zero expected inflation raises D/Y in both 1974 and 2022 by about 2.5 percentage points, and 3% expected inflation raises D/Y by about 1 point. These results reflect the fact that the average inflation rate over 1947-1952, the period when ten-year bonds issued before 1943 were still outstanding, was about 4.5%. Any reasonable calibration of expected inflation before 1943 is below 4.5%, so adjusting for surprises relative to those expectations yields higher interest rates and debt levels.

Equilibrium Interest Rates in the Peg Period

As discussed in the text, we do not have direct evidence on the real interest rates that would have prevailed on securities issued during the pre-Accord period if the Fed had not pegged rates. As a baseline measure, we assume that the rate for any security of a given maturity would have equaled the average of the ex-ante real rates on securities with that maturity issued over 1952-1961, the decade after the peg ended. This assumption yields a term structure of undistorted real interest rates that ranges from 1.7% at a one-year horizon to 2.7% at thirty years.

As a robustness check, we calibrate r^* under the peg with average real rates over longer time periods: 1952-1980 and 1952-2022. We also consider the individual decades of the 1960s and 1970s. Our estimates of equilibrium real rates based on those periods are generally close to or a bit higher than those for 1952-1961: one-year rates range from 1.4 to 2.7 (for the 70s and 60s, respectively) and thirty-year rates range from 3.1 to 3.4 (for 1952-2022 and the 70s).

For each of these calibrations, Table A.3 shows D/Y in 1974 and 2022 in our combined counterfactual. Because the assumed interest rates are mostly higher than in our main case, the levels of D/Y are somewhat higher. The calibrations based on 1952-1980 and 1952-2022 raise D/Y by 2 or 3 percentage points.

As another robustness check, we simply increase or decrease the real rates assumed in our main case by 1 or 2 percentage points at all maturities. Table A.3 shows that these changes have significant effects on our results. Reducing the assumed rates by 2 points reduces the levels of D/Y in our combined counterfactual by more than 10 percentage points. However, the assumed real interest rates in that case are extremely low by historical standards: they range from -0.3% at one year to 0.7% at thirty years. There is no reason to think that r^* was unusually low during the peg period; if anything, we presume that wartime spending raised r^* above typical levels.

Quantitative Easing

Starting in 2009, the Fed addressed the constraints arising from the zero bound on the federal funds rate through quantitative easing, and there is a consensus that this policy reduced long-term interest rates somewhat. As discussed in footnote 6, it is questionable

that this effect was a distortion of interest rates in the sense of our analysis, but here we suppose that it is and add estimates of the effects of QE to the interest-rate adjustments in our counterfactuals.

We focus on the effects of QE over the period 2009-2015, when the federal funds rate was at its lower bound. Based on a survey of estimates, Ball et al. (2016) conclude that QE reduced ten-year interest rates by 50 basis points in calendar years 2009 and 2010, 75 points in 2011, 100 points in 2012, and 125 points from 2013 through 2015. We estimate the effect on the ten-year rate on bonds issued in fiscal year τ by $A^\tau = (3/4)A^{C,\tau} + (1/4)A^{C,\tau-1}$, where $A^{C,\tau}$ is the effect in calendar year τ. We assume these effects apply to all bonds with maturities of 10 years or more. For bonds with shorter maturities, we assume a linear effect on the term structure from the overnight rate to the ten-year rate. The effect for a bond issued at τ with maturity m is:

$$A^{\tau,m} = \begin{cases} A^\tau & \text{for } m \geq 10 \\ A^\tau \times \frac{m}{10} & \text{for } 1 \leq m < 10 \end{cases} \tag{A.21}$$

In our counterfactuals, the adjustment x_t^τ applies to the average interest rate on all bonds issued at τ and outstanding at t. The addition to x_t^τ to account for QE is a weighted average of the adjustments $A^{\tau,m}$:

$$\bar{A}_t^\tau = \sum_{m=t-\tau}^{M} f_t^{\tau,m} \times A^{\tau,m} \tag{A.22}$$

where $f_t^{\tau,m}$ is the fraction of debt issued at τ and outstanding at t with maturity m.

Adding $\bar{A}_t^\tau$ to our interest rate adjustment has no effect on our counterfactuals up to 2009, but it raises the debt/GDP ratio after that. In our combined counterfactual, debt/GDP in 2022 is 84.2 percent in our baseline case and 87.5 percent with the adjustment for QE.

Conclusion

All in all, we find that reasonable variations on our interest rate adjustments either have little effect on our results or modestly raise the levels of debt/GDP in our counterfactuals.

A.6   The Role of the Debt-Dynamics Residual

The exact equation for the evolution of the debt, (A.1), includes a residual ϵ that captures factors other than interest rates and primary surpluses. Figure A.12 shows the series for ϵ as a share of GDP. This residual is small in most years, but it is sizable in 1947 and in some years since 2008. The large residuals are explained by changes in the operating cash held by the Treasury: an increase in cash holdings requires an increase in debt, and running down cash reduces debt. The 1947 residual is negative because cash holdings fell as military operations were wound down. The residuals since 2008 reflect two factors: changes in cash as the Treasury coped with debt ceiling crises, and the flow of stimulus payments during the 2008-2009 recession and the COVID pandemic.

The residual averaged -0.19 percent of GDP over 1947-1974 and 0.25 percent over 1975-2022. Therefore, the residual contributed somewhat to both the decline in debt/GDP in the first period and the rise in the second. To assess the importance of this factor, Figure A.13 compares the actual debt/GDP path to a counterfactual in which ϵ is set to zero in all years, but interest rates and primary surpluses are kept at their historical levels. The Figure also compares the combined counterfactual in our main analysis–a case with primary balance, no interest-rate distortions, and the historical values of ϵ–to a variation on that case with ϵ set to zero. This last counterfactual reveals the exact path that debt/GDP would have followed if the only factor driving it were $r^* - g$, the difference between the undistorted real interest rate and the growth rate.

In the combined counterfactual with $\epsilon = 0$, debt/GDP falls only to 78 percent in 1974. Thus the debt reduction from $r^* < g$ is even smaller than the debt reduction in the combined counterfactual with historical ϵ's, in which debt/GDP is 74 percent in 1974. On the other hand, in the combined counterfactual with $\epsilon = 0$, debt/GDP reaches only 77 percent in 2022, somewhat lower than the 84 percent with historical ϵ's.

A.7   Comparison to Blanchard (2019)

As discussed in the text, we find that the real interest rate has exceeded the growth rate on average since 1979, either with or without adjustments to the real rate for surprise inflation. This result appears to differ from Blanchard (2019), who reports that real rates have consistently been lower than growth rates. As discussed in the text, the different results are explained by two differences in how interest rates are measured:

- We measure the interest rate as the government's interest payments divided by outstanding debt, which yields the interest rates set when the debt was issued. Blanchard uses current market yields on debt, specifically a weighted average of the one-year and ten-year Treasury rates. Since 1979, these yields have usually been lower than the interest rates paid by the government because interest rates have trended downward.

- We ignore the taxation of interest income. In some of his analysis, Blanchard examines after-tax interest rates that he calculates from estimates of the relevant tax rates.

Figure A.14 shows how these differences matter. The Figure presents scenarios for the evolution of the debt/GDP ratio since 1979, with the initial level normalized to 100 as in Blanchard's Figures 5-6. In all cases, we assume a zero primary surplus and use actual interest rates without any adjustment for surprise inflation–our "primary balance" scenario–for comparability with Blanchard. We also set the residual ϵ to zero. With these assumptions, the path of debt/GDP is driven by $r - g$, the difference between the actual interest rate and the growth rate. We show the path of debt/GDP with our measure of interest rates and with Blanchard's market-yield measure with and without his tax adjustment (taken from the replication package available here).

The Figure confirms the results in both our Figure 7 and Blanchard's Figures 5-6. With our interest rate measure, debt/GDP rises from 1979 to 2022 because r usually exceeds g

over this period. With Blanchard's measure of pre-tax interest rates, the ratio rises until 2002 and then falls, leaving it close to its 1979 level in the last few years. With Blanchard's after-tax interest rates, the ratio falls significantly from 1979 to 2022 because r is usually less than g.

For the analysis in this paper, the relevant interest rates are the rates paid by the government, not market yields. The rates paid by the government are the ones for which equation (1) for debt dynamics holds in the data.

The appropriate treatment of taxes is not obvious. Blanchard points out that taxes collected on the interest on government bonds reduce the debt. However, the issuance of government bonds crowds out capital, and the government loses the taxes it would have collected on the lost capital income. The relative sizes of the gain and loss in revenue is ambiguous. On the one hand, crowding out of capital by debt is likely to be less than one-for-one. On the other hand, the returns on capital are higher on average than the interest rate on debt (because of risk), so a dollar of capital produces more tax revenue than a dollar of debt. A natural baseline, we think, is to assume that debt has no net effect on tax revenue. In this case, the evolution of debt is determined by the pre-tax interest rate.

Appendix Tables and Figures

Table A.1 LONG-TERM AND SMOOTHED SHORT-TERM EXPECTATIONS

VARIABLES	$\mathbb{E}_t[\pi^{10}] - \tilde{\mathbb{E}}_t[\pi_{t+1}]$
$\Delta\tilde{\mathbb{E}}_t[\pi_{t+1}]$	-1.549
	[0.217]
Observations	30
R-squared	0.637

Standard errors in brackets

NOTE. $\tilde{\mathbb{E}}_t[\pi_{t+1}]$ indicates smoothed one-year expected inflation. The table shows the results of a regression of the difference between ten-year and smoothed one-year inflation expectations on the change in smoothed one-year inflation expectations for the period 1968-1997.

Table A.2 ROBUSTNESS CHECK - ALTERNATIVE ASSUMPTIONS ABOUT PRE-1943 INFLATION EXPECTATIONS

		Debt/GDP (%)		
Year	Actual	Combined Counterfactual		
		Baseline	Expected Inflation	
			3%	0%
1974	23.2	73.9	74.8	76.5
2022	97.0	84.2	85.0	86.8

NOTE. This table examines the implications for our counterfactuals of different treatments of debt issued before 1942. In our baseline we do not adjust the interest rates on this debt. Alternatively, we adjust these rates to eliminate the effects of surprise inflation, for the cases where expectations before 1943 of inflation after 1946 are either 3% or 0%. For each of these cases, the table shows the levels of debt/GDP in 1974 and 2022 in the combined counterfactual.

Table A.3 ROBUSTNESS CHECK - ALTERNATIVE ASSUMPTIONS ABOUT UNDISTORTED REAL INTEREST RATES UNDER THE PEG

Debt/GDP (%)

Year	Actual	Baseline	Combined Counterfactual							
			Robustness							
			1952-1980	1952-2022	1960s	1970s	(-2%)	(-1%)	(+1%)	(+2%)
1974	23.2	73.9	76.9	76.0	79.3	77.4	61.6	67.5	80.9	88.5
2022	97.0	84.2	87.1	86.3	89.6	87.6	71.9	77.8	91.1	98.7

NOTE. This table examines the implications of assuming higher or lower levels of undistorted real interest rates on securities issued during the peg period. Specifically, we measure undistorted rates with the average values of ex-ante real rates during different historical periods (1952-1980, 1952-2022, 1960s, and 1970s). We also add or subtract 1% or 2% to the entire term structure of undistorted rates in our baseline case, which is based on ex-ante real rates for 1952-1961. For each of these adjustments, the table shows the levels of debt/GDP in 1974 and 2022 in the combined counterfactual.

Figure A.1 Par Value and Market Value of Federal Debt Held by the Public as a Percent of GDP
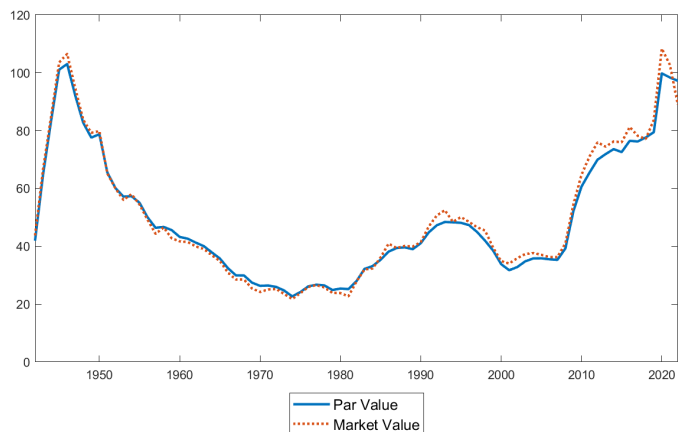
NOTE. The lines represent the ratio of the federal debt held by the public to GDP with debt measured at its par value and at its market value. Source: Hall, Authors' calculations.

Figure A.2 Alternative Measures of the Aggregate Interest Rate
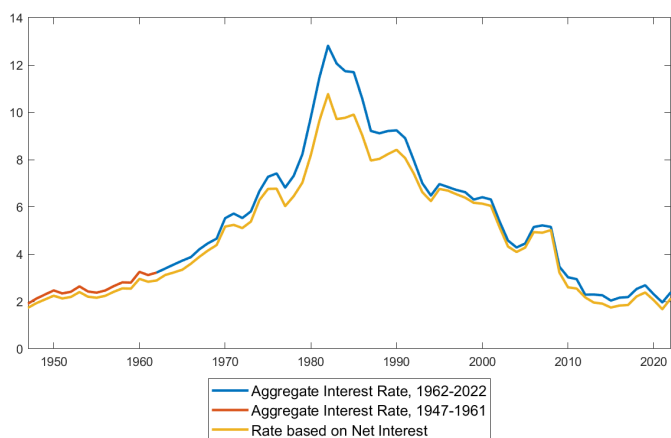
NOTE. The aggregate interest rate is our measure of the interest rate on the public debt. We compare it to an alternative measure computed from net interest payments as reported by OMB. For 1962-2022, our aggregate interest rate is computed from gross interest payments minus intra-governmental payments. For 1947-1961, our interest rate is computed as 1.1 times the rate based on net interest. Sources: OMB historical database, authors' calculations.

**Figure A.3** REVERSE MATURITY STRUCTURE OF MARKETABLE PUBLIC DEBT, 1942-1960
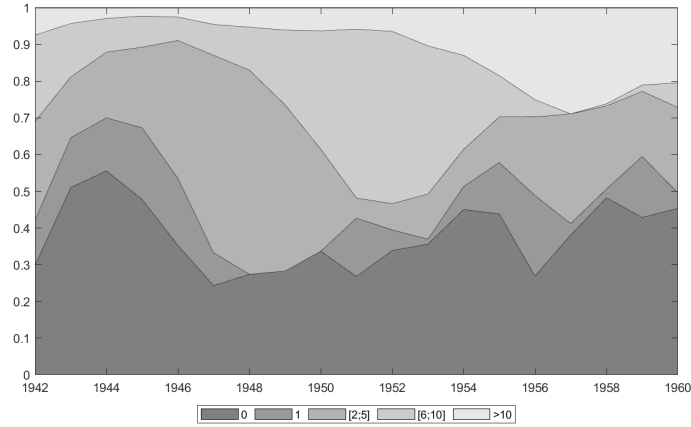
NOTE. This chart shows the reverse maturity structure of marketable debt held by the public between 1942 and 1960. The different shades represent the share of the debt at the end of the fiscal year which was issued in the same year, the previous year, 2 to 5 years earlier, 6 to 10 years earlier, and more than 10 years earlier. Lighter shades indicate longer reverse maturities. Source: Authors' calculations.

Figure A.4 REVERSE MATURITY STRUCTURE OF NON-MARKETABLE PUBLIC DEBT, 1942-1960

NOTE. This chart shows the reverse maturity structure of non-marketable debt held by the public between 1942 and 1960. The different shades represent the share of the debt at the end of the fiscal year which was issued in the same year, the previous year, 2 to 5 years earlier, 6 to 10 years earlier, and more than 10 years earlier. Lighter shades indicate longer reverse maturities. Source: Authors' calculations.

Figure A.5 NON-MARKETABLE DEBT AS A SHARE OF TOTAL DEBT (%)
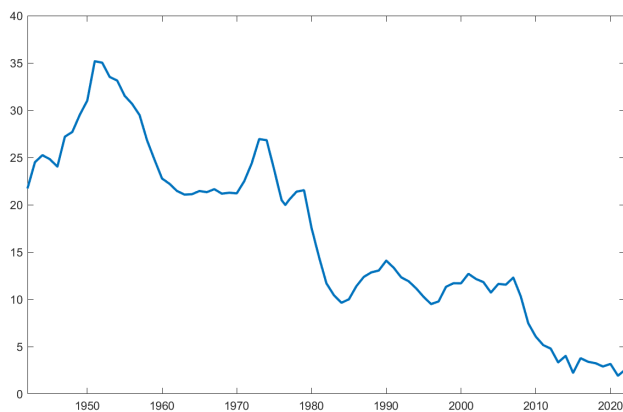
NOTE. The line represents the ratio of the par value of non-marketable Treasury securities held by the public to total debt held by the public. Source: MSPD, Hall.

Figure A.6 ONE-YEAR INFLATION EXPECTATION ERRORS $\mathbb{E}_{t-1}[\pi_t] - \pi_t$, 1970 - 1998
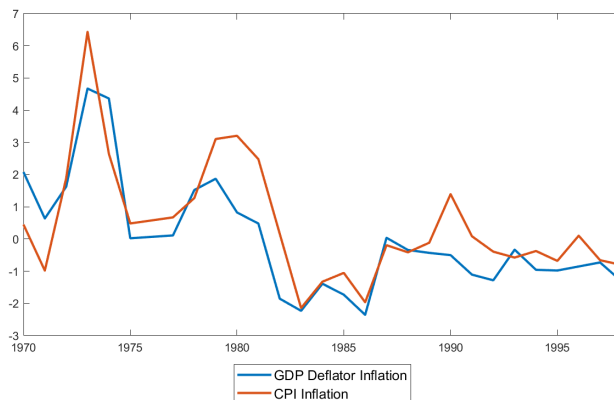
NOTE. The line for CPI inflation expectation errors is computed as the actual CPI inflation rate minus the expected CPI inflation rate (Livingston Survey), from FY 1970 to FY 1998. The line for GDP inflation expectation errors is computed as the GDP deflator inflation rate minus the expected GDP deflator inflation rate (Survey of Professional Forecasters). Sources: FRED, Livingston Survey, Survey of Professional Forecasters.

Figure A.7 GDP Deflator and PCE Deflator Inflation Rates
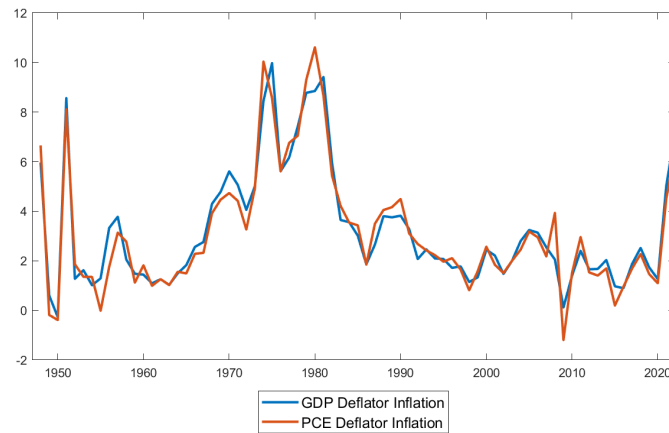
NOTE. The blue and orange lines represent, respectively, the GDP deflator inflation rate and the PCE deflator inflation rate. Sources: FRED, Bureau of Economic Analysis.

**Figure A.8** Short-term, Smoothed Short-term, and Long-term Inflation Expectations
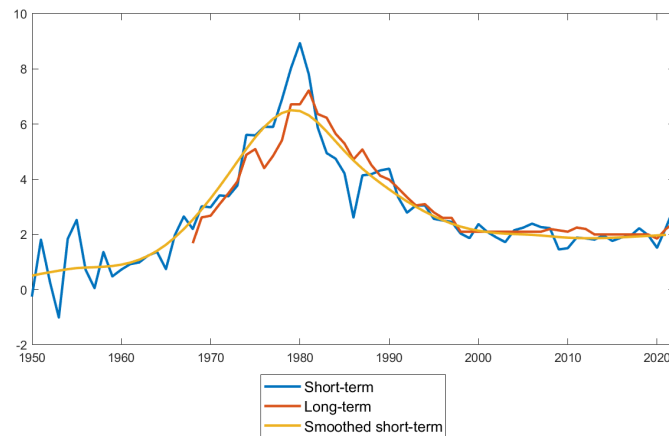
NOTE. Sources: Livingston Survey, Survey of Professional Forecasters, Federal Reserve FRB/US Model, authors' calculations.

Figure A.9 ACTUAL AND FITTED LONG-TERM INFLATION EXPECTATIONS
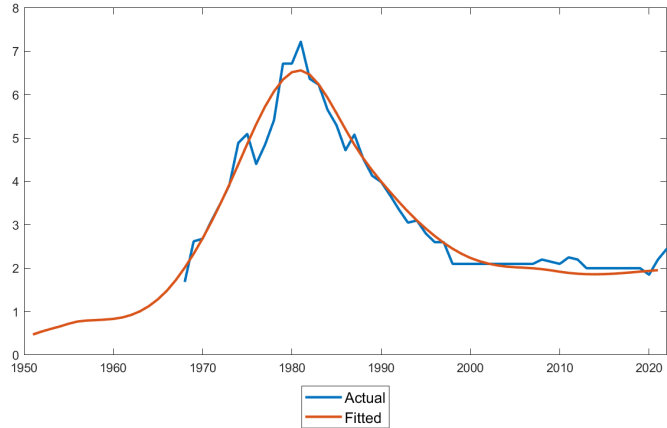
NOTE. Sources: Livingston Survey, Survey of Professional Forecasters, Federal Reserve FRB/US Model, authors' calculations.

Figure A.10 FIXED AND ENDOGENOUS REVERSE MATURITY STRUCTURES
(SELECTED YEARS)
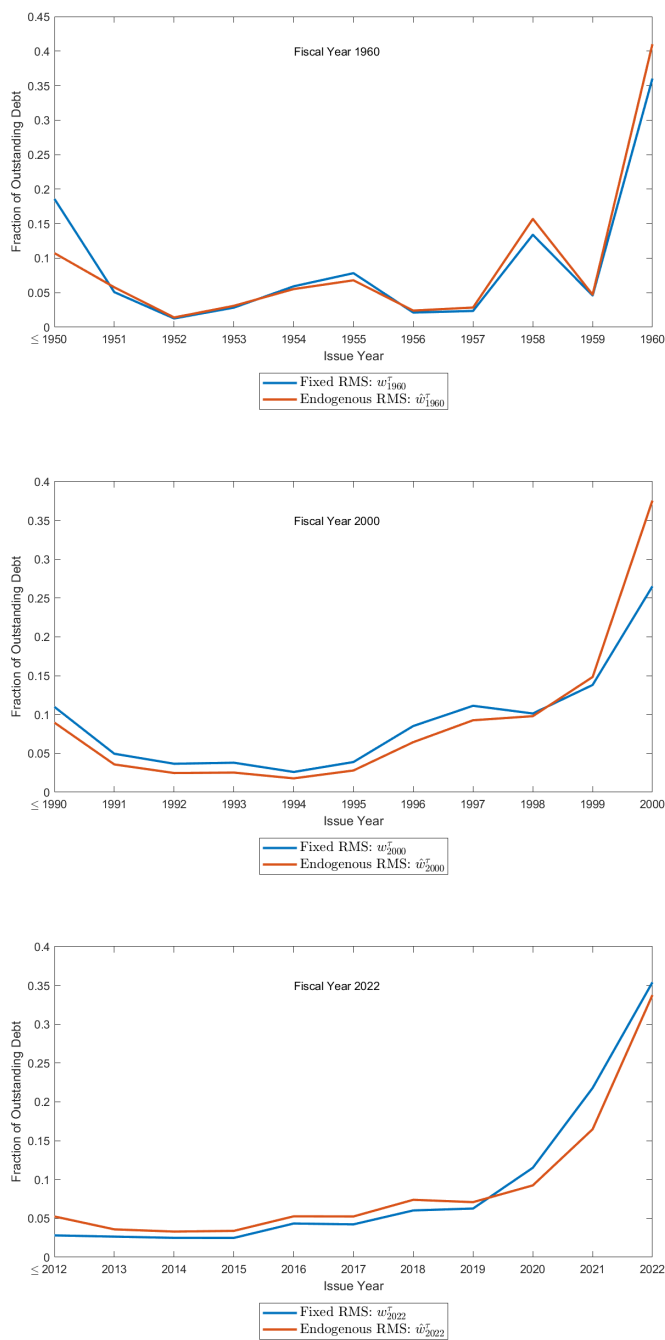
NOTE. These graphs compare the fixed and endogenous reverse maturity structures for fiscal years 1960, 2000, and 2022. The graph for year t shows the fractions of outstanding debt issued at each $\tau \leq t$ in the two cases. Source: Authors' calculations.

**Figure A.11** Debt/GDP Paths with Fixed and Endogenous Reverse Maturity Structures
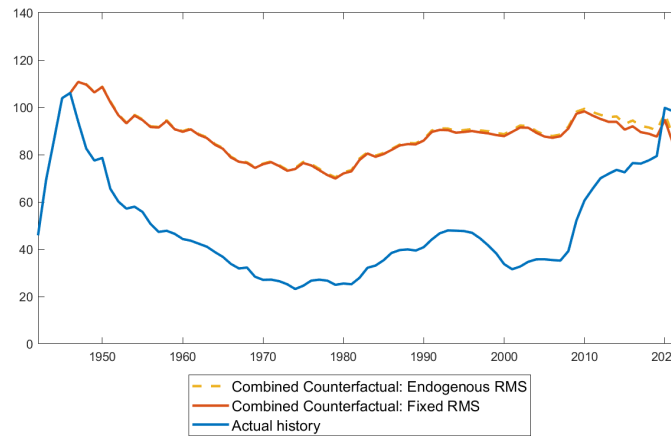
Note. The lines represent the path of the debt/GDP ratio in actual history and in our combined counterfactual with fixed and endogenous reverse maturity structures. Source: Authors' calculations.

Figure A.12 Residual in the Debt Dynamics Equation (% GDP)
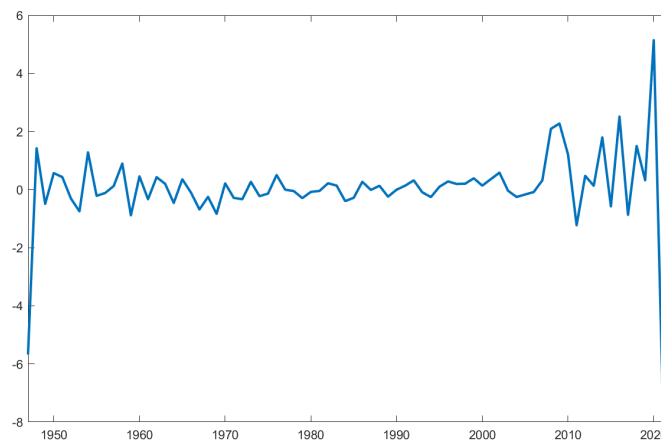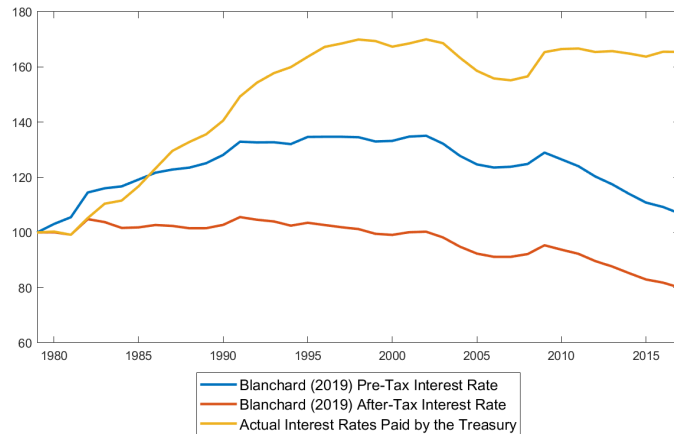

Note. The line represents the residual ϵ_t such that equation (A.1) holds exactly.

Figure A.13 DEBT/GDP PATHS - ACTUAL AND COMBINED COUNTERFACTUAL
SCENARIO
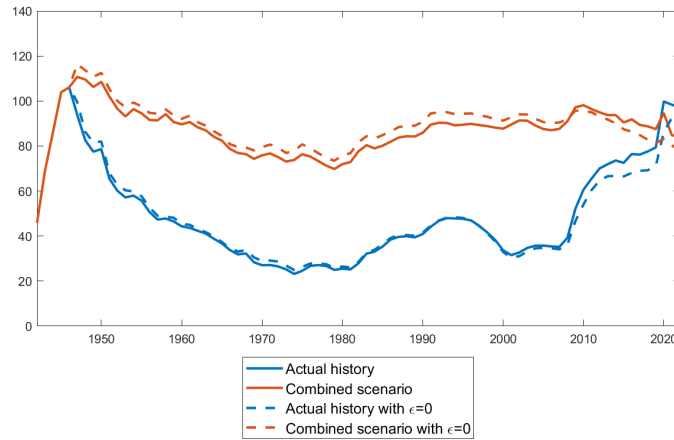
NOTE. The lines represent the path of the debt-GDP ratio in actual history and our combined counterfactual scenario with ϵ equal to either its actual value or zero. Source: Authors' calculations.

Figure A.14 DEBT/GDP WITH ZERO PRIMARY BALANCE AND ALTERNATIVE
MEASURES OF INTEREST RATES, 1979 - 2017

NOTE. The lines represent paths of debt/GDP with primary balance and the residual ϵ_t set to zero. Each line shows the path for a different measure of the interest rate. Debt/GDP is normalized to 100 in 1979. Source: Authors' calculations.