SVERIGES RIKSBANK
OCCASIONAL PAPER SERIES 12

Ramses II - Model Description

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February 2013

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Ramses II - Model Description*

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February 2013

Abstract

This paper describes Ramses II, the dynamic stochastic general equilibrium (DSGE) model currently in use at the Monetary Policy Department of Sveriges Riksbank. The model is used to produce macroeconomic forecasts, alternative scenarios, and for monetary policy analysis. The model was initially developed by Christiano, Trabandt and Walentin (2011). This paper describes the version of the model used for policy and differs in some respects from Christiano, Trabandt and Walentin. Compared with the earlier DSGE model at Sveriges Riksbank, the Ramses model developed by Adolfson et. al. (2008), Ramses II differs in three important respects: *i*) financial frictions are introduced in the accumulation of capital following Bernanke, Gertler and Gilchrist (1999), *ii*) the labor market block includes search and matching following Mortensen and Pissarides (1994), and *iii*) imports are allowed to enter export production as well as in the aggregate consumption and investment baskets.

Keywords: DSGE, financial frictions, labor market frictions, unemployment, small open economy, Bayesian estimation.
JEL codes: E0, E3, F0, F4, G0, G1, J6.

*The model description draws heavily on Christiano, Trabandt, and Walentin (2011).
The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank or those of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. We are grateful to Ulf Söderström for comments and suggestions.
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1. Introduction

This paper describes Ramses II, the dynamic stochastic general equilibrium (DSGE) model currently in use at the Monetary Policy Department of Sveriges Riksbank. The model is used to produce macroeconomic forecasts, to construct alternative scenarios, and for monetary policy analysis. The model was initially developed by Christiano, Trabandt, and Walentin (2011), but the current version of the model differs from CTW in some respects.

Compared with the earlier DSGE model at the Riksbank, the Ramses model developed by Adolfson, Laséen, Lindé and Villani (2008), Ramses II differs in three important respects. First, financial frictions are introduced in the accumulation of capital, following Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2003, 2008). Second, the labor market block includes search and matching frictions following Gertler, Sala, and Trigari (2008). Third, imported goods are used for exports as well as for consumption and investment.

The paper is organized as follows. In Section 2 we describe the theoretical structure of Ramses II. Section 3 describes the Bayesian estimation of the model, and discusses calibration and the choice of priors. This section also displays how we connect the data to the model through measurement equations. Section 4 contains the estimation results and discusses model fit, impulse responses, variance decompositions and some forecasts. Finally, Section 5 concludes. The bulk of the derivations are in various Appendices.

2. Ramses II: A Small Open Economy Model

The model builds on Christiano, Eichenbaum and Evans (2005) and Adolfson, Laséen, Lindé and Villani (2008) from which it inherits most of its open economy structure. The three final goods, consumption, investment and exports, are produced by combining the domestic homogenous good with specific imported inputs for each type of final good. Specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic import retailers. There are three types of import retailers. One uses the specialized import goods to create a homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. See Figure A in the Appendix for a graphical illustration. Exports involve a Dixit-Stiglitz continuum of exporters, each of which is a monopolist that produces a specialized export good. Each monopolist produces its export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign, competitive retailers which create a homogeneous good that is sold to foreign citizens.

Below we will describe the production of all these goods.

2.1. Intermediate input goods

2.1.1. Production of the Domestic Homogeneous Good

A homogeneous domestic good, Y_t, is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_d}} di \right]^{\lambda_d}, \ 1 \leq \lambda_d < \infty. \tag{2.1}$$

The domestic good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given.

The i^{th} intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi, \tag{2.2}$$

where $K_{i,t}$ denotes the capital services rented by the i^{th} intermediate good producer, $\log(z_t)$ is a technology shock whose first difference has a positive mean, $\log(\epsilon_t)$ is a stationary neutral technology shock and ϕ denotes a fixed production cost. In general, the economy has two sources of growth: a positive drift in $\log(z_t)$ and a positive drift in $\log(\Psi_t)$, where Ψ_t is the state of an investment-specific technology shock discussed below. The object, z_t^{+}, in (2.2) is defined as:[1]

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

In (2.2), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. Firms must borrow a fraction of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu_t^f R_t + 1 - \nu_t^f, \tag{2.3}$$

where W_t is the aggregate wage rate, R_t is the nominal interest rate, and ν_t^f corresponds to the fraction that must be financed in advance ($\nu_t^f = 1$ in this version).

By combining the two first-order conditions with respect to capital and labor in the firm's cost minimization problem we obtain the firm's marginal cost, which divided by the price of the homogeneous good is denoted by mc_t:

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \left(r_t^k \right)^{\alpha} \left(\bar{w}_t R_t^f \right)^{1-\alpha} \frac{1}{\epsilon_t}, \tag{2.4}$$

where r_t^k is the nominal rental rate of capital scaled by P_t, and $\bar{w}_t = W_t/(z_t^{+}/P_t)$. τ_t^d is a tax-like shock, which affects marginal cost, but does not appear in the production function. If there are

[1] All the details regarding the scaling of variables are collected in section B.1 in the Appendix. In general lower-case letters denote scaled variables throughout.

no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

Cost minimization (specifically the first order condition for labor) also yields another expression for marginal cost that must be satisfied:

$$
\begin{aligned}
mc_t &= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{MP_{l,t}} \\
&= \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t \left(1-\alpha\right) \left(\frac{k_{i,t}}{\mu_{z^+,t}} / H_{i,t}\right)^{\alpha}}
\end{aligned}
\tag{2.5}
$$

where $MP_{l,t}$ denotes the marginal product of labor.[2]

The i^{th} firm is a monopolist in the production of the i^{th} good and so it sets its price. Price setting is subject to Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case the price is set according to the following indexation scheme:

$$
\begin{aligned}
P_{i,t} &= \tilde{\pi}_{d,t} P_{i,t-1}, \\
\tilde{\pi}_{d,t} &\equiv \left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} \left(\breve{\pi}\right)^{\varkappa_d},
\end{aligned}
$$

where κ_d, $\varkappa_d$,are parameters and κ_d, $\varkappa_d, \kappa_d + \varkappa_d \in (0,1)$, π_{t-1} is the lagged inflation rate, $\bar{\pi}_t^c$ is the central bank's target inflation rate and $\breve{\pi}$ is a scalar. Note that in the current version of the model $\bar{\pi}_t^c = \breve{\pi} = 1.005$ (i.e., the inflation target is constant at 2%).[3]

With probability $1 - \xi_d$ the firm can optimize its price and maximize discounted profits,

$$
E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j}\}, \tag{2.6}
$$

subject to the indexation scheme above and the requirement that production equals demand

$$
Y_{i,t} = \left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} Y_t, \tag{2.7}
$$

where υ_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits, in terms of currency. The equilibrium conditions associated with price setting problem and their derivation are reported in section B.3.1 in the Appendix.

The domestic intermediate output good is allocated among alternative uses as follows:

$$
Y_t = G_t + C_t^d + I_t^d + X_t^d + D_t \tag{2.8}
$$

[2] In Ramses I the combination of equation (2.4) and (2.5) defines the rental rate of capital.

[3] $\breve{\pi}$ is a scalar which allows us to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = 1$, $\varkappa_d = 1$) or index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$). Note that we get price dispersion in steady state if $\varkappa_d > 0$ and if $\breve{\pi}$ is different from the steady state value of π. See Yun (1996) for a discussion of steady state price dispersion.

Here, C_t^d denotes intermediate domestic consumption goods used together with foreign consumption goods to produce the final household consumption good. Also, I_t^d is the amount of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. X_t^d is domestic resources allocated to exports, Finally, D_t is the costs of the real frictions in the model (investment adjustment costs, capital utilization costs and vacancy posting costs). The determination of consumption, investment and export demand is discussed below.

2.1.2. Production of Imported Intermediate Goods

We now turn to a discussion of imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input (they "brand name" it) and supply that input monopolistically to domestic retailers. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment, and (iii) one produces goods used to produce an intermediate good for the production of exports. All importers are subject to Calvo price setting frictions.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is:

$$C_t^m = \left[\int_0^1 \left(C_{i,t}^m \right)^{\frac{1}{\lambda^{m,c}}} di \right]^{\lambda^{m,c}},$$

where $C_{i,t}^m$ is the output of the i^{th} specialized producer and C_t^m is the intermediate good used in the production of consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i^{th} intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. In the usual way, the demand curve for specialized inputs is given by the domestic retailer's first order condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,c}}{\lambda^{m,c}-1}}.$$

We now turn to the producer of $C_{i,t}^m$, who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good firm must pay the inputs in advance at the beginning of the period with foreign currency, and finance this abroad. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*}, \tag{2.9}$$

where

$$R_t^{\nu,*} = \nu_t^* R_t^* + 1 - \nu_t^*, \tag{2.10}$$

R_t^* is the foreign nominal interest rate, and S_t the exchange rate (domestic currency per unit foreign currency). There is no risk to this firm, because all shocks are realized at the beginning of the period, and so there is no uncertainty within the duration of the cash in advance loan about the realization of prices and exchanges rates. Also, $\tau_t^{m,c}$ is a tax-like shock, which affects marginal cost but does not appear in the production function. If there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a markup shock.

Now consider (ii). The production function for the domestic retailer of imported investment goods, I_t^m, is:

$$I_t^m = \left[\int_0^1 \left(I_{i,t}^m \right)^{\frac{1}{\lambda_t^{m,i}}} di \right]^{\lambda_t^{m,i}}.$$

The retailer of imported investment goods is competitive and takes output prices, $P_t^{m,i}$, and input prices, $P_{i,t}^{m,i}$, as given.

The producer of the i^{th} intermediate imported investment input buys the homogeneous foreign good and converts it one-for-one into the differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is

$$\tau_t^{m,i} S_t P_t^* R_t^{\nu,*}.$$

Note that this implies the importing investment firm's cost is P_t^* (before borrowing costs and exchange rate conversion), which is the same cost for the specialized inputs used to produce C_t^m. This may seem inconsistent with the property that domestically produced consumption and investment goods have different relative prices. Below, we suppose that the efficiency of imported investment goods grows over time, in a way that makes our assumptions about the relative costs of consumption and investment, whether imported or domestically produced.

Now consider (iii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m, for the production of export goods is:

$$X_t^m = \left[\int_0^1 \left(X_{i,t}^m \right)^{\frac{1}{\lambda_t^{m,x}}} di \right]^{\lambda_t^{m,x}}.$$

The imported good retailer is competitive, and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*}.$$

Each of the above three types of intermediate good firms is subject to Calvo price-setting frictions. With probability $1-\xi_{m,j}$, the j^{th} type of firm can reoptimize its price and with probability $\xi_{m,j}$ it sets price according to:

$$\begin{aligned} P_{i,t}^{m,j} &= \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j}, \\ \tilde{\pi}_t^{m,j} &\equiv \left(\pi_{t-1}^{m,j} \right)^{\kappa_{m,j}} \left(\bar{\pi}_t^c \right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \breve{\pi}^{\varkappa_{m,j}}. \end{aligned} \tag{2.11}$$

for $j = c, i, x$, and $\kappa_{m,j}, \varkappa_{m,j}, \kappa_{m,j} + \varkappa_{m,j} \in (0,1)$. Note also that in the current version of the model $\bar{\pi}_t^c = \breve{\pi} = 1.005$.

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers and are reported in section B.3.5 in the Appendix. The real marginal cost is

$$\begin{aligned}
mc_t^{m,j} &= \tau_t^{m,j} \frac{S_t P_t^*}{P_t^{m,j}} R_t^{\nu,*} \\
&= \tau_t^{m,j} \frac{S_t P_t^* P_t^c P_t}{P_t^c P_t^{m,j} P_t} R_t^{\nu,*} \\
&= \tau_t^{m,j} \frac{q_t p_t^c}{p_t^{m,j}} R_t^{\nu,*}
\end{aligned} \tag{2.12}$$

for $j = c, i, x$.

2.2. Production of Final Consumption Goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm with the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d \right)^{\frac{(\eta_c - 1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m \right)^{\frac{(\eta_c - 1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}, \tag{2.13}$$

using two inputs. The first, C_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, C_t^m, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$, as well as the output price of the final consumption good, P_t^c, as given. Profit maximization leads to the following demand for the intermediate inputs (in scaled form):

$$\begin{aligned}
c_t^d &= (1-\omega_c) \left(p_t^c \right)^{\eta_c} c_t, \\
c_t^m &= \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta c} c_t.
\end{aligned} \tag{2.14}$$

where $p_t^c = P_t^c / P_t$ and $p_t^{m,c} = P_t^{m,c} / P_t$. The price of C_t is related to the price of inputs by:

$$p_t^c = \left[(1-\omega_c) + \omega_c \left(p_t^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \tag{2.15}$$

The rate of inflation of the consumption good is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c} \right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c} \right)^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \tag{2.16}$$

2.3. Production of Final Investment Goods

Investment goods are produced by a representative competitive firm using the following technology:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}},$$

where we define investment to be the sum of investment goods, I_t, used in the accumulation of physical capital, plus investment goods used in capital maintenance, $a(u_t)\bar{K}_t$; where $\bar{K}_t$ is the physical capital stock and section B.2 in the Appendix defines the functional form of $a(u_t)$. Capital maintenance are expenses that arise from varying the utilization of capital, discussed in section 2.5 below. The utilization rate of capital, u_t, is defined from

$$K_t = u_t \bar{K}_t.$$

To accommodate the observation that the price of investment goods relative to the price of consumption goods is declining over time, we assume that Ψ_t is a unit root process with positive drift. The details of the law of motion of this process is discussed below. (In the current version of Ramses II this is not stochastic). As in the consumption good sector the representative investment goods producers takes all relevant prices as given. Profit maximization leads to the following demand for the intermediate inputs in scaled form:

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i) \tag{2.17}$$

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}} \right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) \tag{2.18}$$

where $p_t^i = \Psi_t P_t^i / P_t$ and $p_t^{m,i} = P_t^{m,i}/P_t$.

The price of I_t is related to the price of the inputs by:

$$p_t^i = \left[(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}. \tag{2.19}$$

The rate of inflation of the investment good is:

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{m,i}\right)^{1-\eta_i}} \right]^{\frac{1}{1-\eta_i}}. \tag{2.20}$$

2.4. Production of Final Export Goods

Total foreign demand for domestic exports is:

$$X_t = \left(\frac{P_t^x}{P_t^*} \right)^{-\eta_f} Y_t^*.$$

In scaled form, this is

$$x_t = (p_t^x)^{-\eta_f}\, y_t^*. \tag{2.21}$$

Here, Y_t^* is foreign GDP and P_t^* is the foreign currency price of foreign homogeneous goods. P_t^x is an index of export prices, whose determination is discussed below. The goods, X_t, are produced by a representative, competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}}\, di\right]^{\lambda_x}. \tag{2.22}$$

where $X_{i,t},\ i \in (0,1)$, are exports of specialized goods. The retailer that produces X_t takes its output price, P_t^x, and its input prices, $P_{i,t}^x$, as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_x}{\lambda_x - 1}} X_t. \tag{2.23}$$

Combining (2.22) and (2.23), we obtain:

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_x}}\, di\right]^{1-\lambda_x}.$$

The i^{th} export monopolist produces its differentiated export good using the following CES production technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x - 1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the i^{th} exporter's use of the imported and domestically produced goods, respectively. We derive the marginal cost from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$\min \quad \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d\right] + \lambda \left\{ X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x - 1}} \right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic good. It is assumed that the exporters must finance a fraction of their production costs in advance implying that R_t^x enters the input cost. Using the first order conditions of this problem we derive the real marginal cost, mc_t^x :

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}, \tag{2.24}$$

where lower case letters denote scaled variables and

$$R_t^x = \nu_t^x R_t + 1 - \nu_t^x, \tag{2.25}$$

where $\nu_t^x = 1$ in the current version, and where we have used

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x. \tag{2.26}$$

From the solution to the same problem we also get the demand for domestic inputs for export production:

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t} \right)^{\eta_x} X_{i,t} \left(1 - \omega_x\right) \tag{2.27}$$

The aggregate export demand for the *domestic* homogeneous input good is

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t-1}}} \left(p_t^x\right)^{-\eta_f} Y_t^*, \tag{2.28}$$

where $\mathring{p}_t^x$ is a measure of the price dispersion, which is not active in this version of the model and hence equal to one (see also section B.3.3 in the Appendix).

The aggregate export demand for the *imported* input good is:

$$X_t^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_x}{\lambda_x - 1}} \left(p_t^x\right)^{-\eta_f} Y_t^* \tag{2.29}$$

The i^{th} export firm takes (2.23) as its demand curve, and sets the price subject to Calvo frictions. With probability ξ_x the i^{th} export good firm cannot reoptimize its price, in which case it update its price as:

$$\begin{aligned} P_{i,t}^x &= \tilde{\pi}_t^x P_{i,t-1}^x, \\ \tilde{\pi}_t^x &= \left(\pi_{t-1}^x\right)^{\kappa_x} \left(\pi^x\right)^{1-\kappa_x-\varkappa_x} \left(\breve{\pi}\right)^{\varkappa_x}, \end{aligned} \tag{2.30}$$

where $\kappa_x, \varkappa_x, \kappa_x + \varkappa_x \in (0,1)$ .Note also that in the current version of the model $\pi^x = \breve{\pi} = 1.005$.

The equilibrium conditions associated with price setting by exporters that do get to reoptimize their prices are analogous to the ones derived for domestic intermediate good producers and are reported in section B.3.2 in the Appendix.

2.5. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln(C_t - bC_{t-1}) - \zeta_t^h A_L \left(\sum_{i=0}^{N-1} \frac{\left(\varsigma_{i,t}\right)^{1+\sigma_L}}{1+\sigma_L} l_t^i \right) \right], \tag{2.31}$$

where ζ_t^c is a shock to consumption preferences, ζ_t^h is labor supply shock, $\varsigma_{i,t}$ is hours worked per employee and l_t^i is the number of workers in cohort $i \in \{0, ...N-1\}$ (see Section 2.7). The household owns the stock of net foreign assets and determines its rate of accumulation.

2.5.1. Household Consumption Decision

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c \left(1 + \tau_t^c\right) = 0. \tag{2.32}$$

where

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+$$

is the marginal value of one unit of the homogenous domestic good at time t.

2.5.2. Financial Assets and Interest Rate Parity

The household does the economy's saving. Period t saving occurs by the acquisition of net foreign assets, A_{t+1}^*, and a domestic asset. The domestic asset is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to $t+1$, R_t. The first order condition associated with this asset is:

$$-\psi_{z^+,t} + \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}} \left[\frac{R_t - \tau_t^b \left(R_t - \pi_{t+1}\right)}{\pi_{t+1}}\right] = 0, \tag{2.33}$$

where τ_t^b is the tax rate on the real interest rate on bond income (for additional discussion of τ^b, see section 2.9.) A consequence of our treatment of the taxation on domestic bonds is that the steady state real after tax return on bonds is invariant to π.

In the model the tax treatment of domestic agents' earnings on foreign bonds is the same as the tax treatment of agents' earnings on foreign bonds. The scaled date t first order condition associated with A_{t+1}^* that pays R_t^* in terms of foreign currency is:

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1} \left[S_{t+1} R_t^* \Phi_t - \tau^b \left(S_{t+1} R_t^* \Phi_t - \frac{S_t}{P_t} P_{t+1}\right)\right]. \tag{2.34}$$

Recall that S_t is the domestic currency price of a unit of foreign currency. On the left side of this expression, we have the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the Lagrange multiplier on the household's budget constraint, υ_t. The term in square brackets is the after tax payoff of the foreign asset, in domestic currency units. The first term is the period $t+1$ pre-tax interest payoff on A_{t+1}^*, which is $S_{t+1} R_t^* \Phi_t$. Here, R_t^* is the foreign nominal rate of interest, which is risk free in foreign currency units. The term, Φ_t represents a risk adjustment, so that a unit of the foreign asset acquired in t pays off $R_t^* \Phi_t$ units of foreign currency in $t+1$. The determination of Φ_t is discussed below. The remaining term pertains to the impact of taxation on the return on foreign assets. If we ignore the term after the minus sign within the set of parentheses, we see that taxation is applied to the whole nominal payoff on the bond, including principle. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t+1$ coincides with the real value of the currency used to purchase the asset in period t. In particular, recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So, the period t real cost of the asset is S_t/P_t. The domestic currency value in period $t+1$ of this real quantity is $P_{t+1}S_t/P_t$.

We scale the first order condition, eq. (2.34), by multiplying both sides by $P_t z_t^+/S_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}}[s_{t+1}R_t^*\Phi_t - \tau_t^b\left(s_{t+1}R_t^*\Phi_t - \pi_{t+1}\right)], \tag{2.35}$$

where

$$s_t = \frac{S_t}{S_{t-1}}.$$

The risk adjustment term has the following form:

$$\Phi_t = \Phi\left(a_t, E_t s_{t+1}s_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(a_t - \bar{a}\right) - \tilde{\phi}_s\left(E_t s_{t+1}s_t - s^2\right) + \tilde{\phi}_t\right), \tag{2.36}$$

where, recall,

$$a_t = \frac{S_t A_{t+1}}{P_t z_t^+},$$

and $\tilde{\phi}_t$ is a mean zero shock whose law of motion is discussed below. In addition, $\tilde{\phi}_a$, $\tilde{\phi}_s$, $\bar{a}$ are positive parameters. In the steady state discussion in the Appendix, we derive the equilibrium outcomes that a_t coincides with $\bar{a}$ and $\Phi_t = 1$ in non-stochastic steady state.

The dependence of Φ_t on a_t ensures, in the usual way, that there is a unique steady state value of a_t that is independent of the initial net foreign assets and capital of the economy. The dependence of Φ_t on the anticipated growth rate of the exchange rate is designed to allow the model to reproduce two types of observations. The first concerns observations related uncovered interest parity. The second concerns the hump-shaped response of output to a monetary policy shock.

A log linear approximation of the model (in which ϕ_t corresponds to the log deviation of Φ_t about its steady state value of unity) implies the following representation of the uncovered interest parity condition:

$$\begin{aligned} R_t - R_t^* &= E_t \log S_{t+1} - \log S_t + \phi_t, \\ &= E_t \log S_{t+1} - \log S_t - \tilde{\phi}_s \Delta E_t \log S_{t+1} - \tilde{\phi}_s \Delta \log S_t - \tilde{\phi}_a\left(a_t - \bar{a}\right) + \tilde{\phi}_t, \\ &= \left(1 - \tilde{\phi}_s\right)\Delta E_t \log S_{t+1} - \tilde{\phi}_s \Delta \log S_t - \tilde{\phi}_a\left(a_t - \bar{a}\right) + \tilde{\phi}_t, \end{aligned}$$

where Δ is the difference operator and ϕ_t denotes the risk premium on domestic assets.[4] Consider first the case in which $\phi_t \equiv 0$ (and $\tilde{\phi}_s = 0$). In this case, a fall in R_t relative to R_t^* produces an anticipated appreciation of the currency. This drop in $E_t \log S_{t+1} - \log S_t$ is accomplished in part by an instantaneous depreciation in $\log S_t$. The idea behind this is that asset holders respond to

[4] Note that the risk premium has an endogenous part, namely $-\tilde{\phi}_s \Delta E_t \log S_{t+1} - \tilde{\phi}_s \Delta \log S_t - \tilde{\phi}_a\left(a_t - \bar{a}\right)$ as well as an exogenous part, namely $\tilde{\phi}_t$ which we refer to as the risk premium shock below.

the unfavorable domestic rate of return by attempting to sell domestic assets and acquire foreign exchange for the purpose of acquiring foreign assets. This selling pressure pushes $\log S_t$ up, until the anticipated appreciation precisely compensates traders in international financial assets holding domestic assets.

There is evidence that the preceding scenario does not hold in the data. Vector autoregression evidence on the response of financial variables to an expansionary domestic monetary policy shock suggests that $E_t \log S_{t+1} - \log S_t$ actually rises for a period of time (see, e.g., Eichenbaum and Evans (1995)). One interpretation of these results is that when the domestic interest rate is reduced, say by a monetary policy shock, then risk in the domestic economy falls and that alone makes traders happier to hold domestic financial assets in spite of their lower nominal return and the losses they expect to make in the foreign exchange market. Our functional form for ϕ_t is designed to capture this idea when $\phi_t \neq 0$ (and $\tilde{\phi}_s \neq 0$).

2.6. Capital Accumulation and Financial Frictions

We assume that only the accumulation and management of capital involves frictions, but that working capital loans are frictionless. Our strategy of introducing frictions in the accumulation and management of capital follows the variant of the Bernanke, Gertler and Gilchrist (1999) (henceforth BGG) model implemented in Christiano, Motto and Rostagno (2003). The discussion here borrows heavily from the derivation in Christiano, Motto and Rostagno (2008) (henceforth CMR).

The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce 'entrepreneurs'. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, 'banks', can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. An alternative arrangement that is more efficient is one in which banks extend entrepreneurs a 'standard debt contract', which specifies a loan amount and a given interest payment. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are 'bankrupt'. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed. For a graphical illustration of the financing problem in the capital market, see Figure B.

The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneur's net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneur's assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

The ultimate source of funds for lending to entrepreneurs is the household. The standard debt contracts extended by banks to entrepreneurs are financed by issuing liabilities to households. Although individual entrepreneurs are risky, banks themselves are not. We suppose that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. Extensions of the model that introduce risk into banking have been developed, but it is not clear that the added complexity is justified.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced.[5]

As we shall see, entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumptions have been made in the model, which guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by an entrepreneur is proportional to his level of net worth. These are enough to guarantee the aggregation result.

[5] With this model, it is typically the practice to compare the net worth of entrepreneurs with a stock market quantity (index), and we follow this route. Whether this is really appropriate is uncertain. A case can be made that the 'bank loans' of entrepreneurs in the model correspond well with actual bank loans *plus* actual equity. It is well known that dividend payments on equity are very smooth. Firms work hard to accomplish this. For example, during the US Great Depression some firms were willing to sell their own physical capital in order to avoid cutting dividends. That this is so is perhaps not surprising. The asymmetric information problems with actual equity are surely as severe as they are for the banks in our model. Under these circumstances one might expect equity holders to demand a payment that is not contingent on the realization of uncertainty within the firm (payments could be contingent upon publicly observed variables). Under this vision, the net worth in the model would correspond not to a measure of the aggregate stock market, but to the ownership stake of the managers and others who exert most direct control over the firm. The 'bank loans' in this model would, under this view of things, correspond to the actual loans of firms (i.e., bank loans and other loans such as commercial paper) plus the outstanding equity. While this is perhaps too extreme, these observations highlight that there is substantial uncertainty over exactly what variable should be compared with net worth in the model. It is important to emphasize, however, that whatever the right interpretation is of net worth, the model potentially captures balance sheet problems very nicely.

2.6.1. Capital Accumulation and Investment Decision

The stock of physical capital is owned by the entrepreneur, who determines the rate at which the capital stock is accumulated and its utilization rate. The law of motion of the physical stock of capital is subject to investment adjustment costs as introduced by Christiano, Eichenbaum and Evans (2005):

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t,$$

where Υ_t is a stationary investment-specific technology shock that affects the efficiency of transforming investments into capital. In scaled terms the law of motion of capital can be written[6]

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{2.38}$$

The first order condition with respect to I_t (derived from the Lagrangian representation of the investment purchase and the law of motion for capital) is in scaled terms:

$$\begin{aligned} -\psi_{z^+,t}p_t^i + \psi_{z^+,t}p_{k',t}\Upsilon_t\left[1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] & \\ +\beta\psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{\Psi,t+1}\mu_{z^+,t+1} &= 0. \end{aligned} \tag{2.39}$$

2.6.2. The Individual Entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1}. The entrepreneur's net worth, N_{t+1}, constitutes his state at this time, and nothing else about his history is relevant. We imagine that there are many entrepreneurs for each level of net worth and that for each level of net worth, there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with a particular level of net worth, N_{t+1}. The entrepreneur combines this net worth with a bank loan, B_{t+1}, to purchase new, installed physical capital, $\bar{K}_{t+1}$, from capital producers. The loan the entrepreneur requires for this is:

$$B_{t+1} = P_tP_{k',t}\bar{K}_{t+1} - N_{t+1}. \tag{2.40}$$

The entrepreneur is required to pay a gross interest rate, Z_{t+1}, on the bank loan at the end of period $t+1$, if it is feasible to do so. After purchasing capital the entrepreneur experiences an idiosyncratic

[6] See subsection B.2 in the Appendix for the functional form of the investment adjustment costs, $\tilde{S}$.
Note that the first order condition for capital in the baseline model (i.e. the model without financial frictions and the labour market block) implies:

$$\psi_{z^+,t} = \beta E_t\psi_{z^+,t+1}\frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{2.37}$$

productivity shock which converts the purchased capital, $\bar{K}_{t+1}$, into $\bar{K}_{t+1}\omega$. Here, ω is a unit mean, lognormally and independently distributed random variable across entrepreneurs. The variance of $\log \omega$ is σ_t^2. The t subscript indicates that σ_t is itself the realization of a random variable. This allows us to consider the effects of an increase in the riskiness of individual entrepreneurs. We denote the cumulative distribution function of ω by $F(\omega;\sigma)$. and its partial derivatives as e.g. $F_\omega(\omega;\sigma)$, $F_\sigma(\omega;\sigma)$

After observing the period $t+1$ shocks, the entrepreneur sets the utilization rate, u_{t+1}, of capital and rents capital out in competitive markets at nominal rental rate, $P_{t+1}r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in (B.4). The first order condition associated with capital utilization is, in scaled terms:

$$\bar{r}_t^k = p_t^i a'(u_t), \tag{2.41}$$

$\bar{r}_t^k = \Psi_t r_t^k$ is the scaled real rental rate of capital.[7] The entrepreneur then sells the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic shock ω earns a return (after taxes), of $R_{t+1}^k\omega$, where R_{t+1}^k is the rate of return on a period t investment in a unit of physical capital:

$$R_{t+1}^k = \frac{(1-\tau_t^k)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau_t^k\delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{2.42}$$

where

$$\frac{p_t^i}{\Psi_t}P_t = P_t^i,$$

is the date t price of the homogeneous investment good. Here, $P_{k',t}$ denotes the price of a unit of newly installed physical capital, which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R_{t+1}^k represents the period $t+1$ payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k.[8]

[7] The tax rate on capital income does not enter here because maintenance costs are assumed to be deductible from taxes.

[8] It is convenient to express R_t^k in scaled terms:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{(1-\tau_t^k)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau_t^k\delta\frac{\mu_{\Psi,t+1}}{\pi_{t+1}}p_{k',t}}{p_{k',t}}. \tag{2.43}$$

After entrepreneurs sell their capital, they settle their bank loans. At this point, the resources available to an entrepreneur who has purchased $\bar{K}_{t+1}$ units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R^k_{t+1} \omega \bar{K}_{t+1}$. There is a cutoff value of ω, $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R^k_{t+1} P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{2.44}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R^k_{t+1} \omega P_t P_{k',t} \bar{K}_{t+1},$$

which is less than $Z_{t+1} B_{t+1}$, to the bank. In this case, the bank monitors the entrepreneur, at cost

$$\mu R^k_{t+1} \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

Banks obtain the funds loaned in period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest, R_t. The subscript on R_t indicates that the payoff to households in $t+1$ is not contingent on the period $t+1$ uncertainty. This feature of the relationship between households and banks is simply assumed. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as (2.33).

We suppose that there is competition and free entry among banks, and that banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs.[9] It follows that the bank's cash flow in each state of period $t+1$ is zero, for each loan amount.[10] For loans in the amount, B_{t+1}, the bank receives gross interest, $Z_{t+1} B_{t+1}$, from the $1 - F(\bar{\omega}_{t+1}; \sigma_t)$ entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is:

$$[1 - F(\bar{\omega}_{t+1}; \sigma_t)] Z_{t+1} B_{t+1} + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega; \sigma_t) R^k_{t+1} P_t P_{k',t} \bar{K}_{t+1} = R_t B_{t+1},$$

or, after making use of (2.44) and rearranging,

$$[\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t = \varrho_t - 1 \tag{2.45}$$

where $p_{k',t} = \Psi_t P_{k',t}$.

[9] If banks also had access to state contingent securities, then free entry and competition would imply that banks earn zero profits in an ex ante expected sense from the point of view of period t.

[10] Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state by state zero profit condition quoted in the text.

where

$$G(\bar{\omega}_{t+1};\sigma_t) = \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega;\sigma_t).$$

$$\Gamma(\bar{\omega}_{t+1};\sigma_t) = \bar{\omega}_{t+1}\left[1 - F(\bar{\omega}_{t+1};\sigma_t)\right] + G(\bar{\omega}_{t+1};\sigma_t),$$

$$\varrho_t = \frac{P_t P_{k',t}\bar{K}_{t+1}}{N_{t+1}}.$$

The expression, $\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)$ is the share of revenues earned by entrepreneurs that borrow B_{t+1}, which goes to banks. Note that $\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) = 1 - F(\bar{\omega}_{t+1};\sigma_t) > 0$ and $G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) = \bar{\omega}_{t+1}F_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) > 0$. It is thus not surprising that the share of entrepreneurial revenues accruing to banks is non-monotone with respect to $\bar{\omega}_{t+1}$. BGG argue that the expression on the left of (2.45) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1} = \omega^*$, say. The expression is increasing for $\bar{\omega}_{t+1} < \omega^*$ and decreasing for $\bar{\omega}_{t+1} > \omega^*$. Thus, for any given value of the leverage ratio, ϱ_t, and R^k_{t+1}/R_t, generically there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy (2.45). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of the inverted 'U' shape. This is because, according to (2.44), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. As discussed below, the equilibrium contract is one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period $t+1$ shock that drives R^k_{t+1} up. The fraction of entrepreneurs that experience bankruptcy is $F(\bar{\omega}_{t+1};\sigma_t)$, so it follows that a shock which drives up R^k_{t+1} has a negative contemporaneous impact on the bankruptcy rate. According to (B.28), shocks that drive R^k_{t+1} up include anything which raises the value of physical capital and/or the rental rate of capital.

As just noted, we suppose that the equilibrium debt contract maximizes entrepreneurial welfare, subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt, B_{t+1} and a state $t+1-$contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\begin{aligned}
&\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty}\left[R^k_{t+1}\omega P_t P_{k',t}\bar{K}_{t+1} - Z_{t+1}B_{t+1}\right]dF(\omega;\sigma_t)}{R_t N_{t+1}} \\
&= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty}\left[\omega - \bar{\omega}_{t+1}\right]dF(\omega;\sigma_t)R^k_{t+1}P_t P_{k',t}\bar{K}_{t+1}}{R_t N_{t+1}} \\
&= E_t\left\{\left[1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)\right]\frac{R^k_{t+1}}{R_t}\right\}\varrho_t,
\end{aligned}$$

after making use of (2.40), (2.44) and

$$1 = \int_0^{\infty}\omega dF(\omega;\sigma_t) = \int_{\bar{\omega}_{t+1}}^{\infty}\omega dF(\omega;\sigma_t) + G(\bar{\omega}_{t+1};\sigma_t).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to $R_t N_{t+1}$), subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t, \{\bar{\omega}_{t+1}\}} E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 \right) \right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The first order conditions for this problem are:

$$\begin{aligned} E_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} - 1 \right) \right\} &= 0 \\ -\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) \frac{R^k_{t+1}}{R_t} + \lambda_{t+1} [\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} &= 0 \\ [\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 &= 0, \end{aligned}$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that we assume $\lambda_{t+1} > 0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the first order conditions reduce to:

$$E_t \left\{ \begin{array}{c} [1 - \Gamma(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1}; \sigma_t)} \\ \left([\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} - 1 \right) \end{array} \right\} = 0, \tag{2.46}$$

$$[\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 = 0, \tag{2.47}$$

for $t = 0, 1, 2, ...\infty$ and for $t = -1, 0, 1, 2, ...$ respectively.

Since N_{t+1} does not appear in the last two equations, we conclude that ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs, regardless of their net worth. The results for ϱ_t implies that

$$\frac{B_{t+1}}{N_{t+1}} = \varrho_t - 1,$$

i.e. that an entrepreneur's loan amount is proportional to his net worth. Rewriting (2.40) and (2.44) we see that the rate of interest paid by the entrepreneur is

$$Z_{t+1} = \frac{\bar{\omega}_{t+1} R^k_{t+1}}{1 - \frac{N_{t+1}}{P_t P_{k',t} \bar{K}_{t+1}}} = \frac{\bar{\omega}_{t+1} R^k_{t+1}}{1 - \frac{1}{\varrho_t}}, \tag{2.48}$$

which is the same for all entrepreneurs, regardless of their net worth.

2.6.3. Aggregation Across Entrepreneurs and the External Financing Premium

The law of motion for the net worth of an individual entrepreneur is

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t, \tag{2.49}$$

Each entrepreneur faces an identical and independent probability $1 - \gamma_t$ of being selected to exit the economy. With the complementary probability, γ_t, each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is simply $\gamma_t \bar{V}_t$. A fraction, $1 - \gamma_t$, of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer, W_t^e. This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or,

$$\begin{aligned} \bar{N}_{t+1} &= \gamma_t \{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \} \\ &\quad + W_t^e, \end{aligned} \tag{2.50}$$

where upper bar over a variable denotes its aggregate average value. For a derivation of the aggregation across entrepreneurs see Appendix B.4.1.

We now turn to the external financing premium for entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate, R_t, which he loses by applying it to capital rather than just depositing it in the bank. The average payment by all entrepreneurs to the bank is the entire object in square brackets in equation (2.50). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the risk premium in the model. Another is the excess of the interest rate paid by entrepreneurs who are not bankrupt, over R_t :

$$Z_{t+1} - R_t = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t} k_{t+1}}} - R_t,$$

according to (2.48).

2.7. Wage Setting and Employment Frictions

The labor market is modeled through the search and matching framework of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005 and 2012) - following the GST strategy implemented in Christiano, Ilut, Motto, and Rostagno (2007). This framework allows for variation in both the extensive (employment) and intensive (hours per worker) margin, which is an important empirical observation. Most of the variation in hours worked in Sweden appears to be generated by the extensive margin.[11]

[11] A simple data analysis on Swedish data 1995q1-2009q2, following the method of Hansen (1985), using the decomposition

In the model, labor services are supplied to the homogeneous labor market by 'employment agencies' (see Figure B for a graphical illustration). This leaves the equilibrium conditions associated with the production of the homogeneous good unaffected. Key labor market activities - vacancy postings, layoffs, labor bargaining, setting the intensity of labor effort - are all carried out inside the employment agencies.[12]

Each household is composed of many workers, each of which is in the labor force. A worker begins the period either unemployed or employed with a particular employment agency. Unemployed workers do undirected search. They find a job with a particular agency with a probability that is proportional to the efforts made by the agency to attract workers. Workers are separated from employment agencies either exogenously, or because they are actively cut. Workers pass back and forth between unemployment and employment with an agency. There are no agency to agency transitions.

The events during the period in an employment agency are displayed in Figure C. Each employment agency begins a period with a stock of workers. That stock is immediately reduced by exogenous separations and it is increased by new arrivals that reflect the agency's recruiting efforts in the previous period. Then, the economy's aggregate shocks are realized.

At this point, each agency's wage is set. The agencies are allocated permanently into N equal-sized cohorts and each period $1/N$ agencies establish a new wage by Nash bargaining. When a new wage is set, it evolves over the subsequent $N-1$ periods according to:

$$\begin{aligned} W_{j,t+1} &= \tilde{\pi}_{w,t+1} W_{j,t} && (2.51) \\ \tilde{\pi}_{w,t+1} &= \left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}, && (2.52) \end{aligned}$$

where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes. The wage negotiated in a given period covers all workers employed at an agency for each of the subsequent $N-1$ periods, even those that will not arrive until later. The bargaining arrangement is unionized, so that a union representing the 'average worker' bargains with the employment agency.

Next, if we allow for endogenous layoffs, each worker draws an idiosyncratic productivity shock. A cutoff level of productivity is determined, and workers with lower productivity are laid off. From a

$$var\left(H_t\right) = var\left(\varsigma_t\right) + var\left(L_t\right) + 2covar\left(\varsigma_t, L_t\right),$$

where H_t denotes total hours worked, ς_t hours per worker and L_t number of people employed. H_t and L_t are in per capita terms (of the adult population) and all series are HP-filtered with $\lambda = 1600$, indicates that roughly 4/5th of the variation in total hours worked comes from variation in employment and 1/5th from variation in hours per worker. The covariance term is close to 0, which is in line with previous Swedish evidence and institutional factors that discourage over-time work.

[12] An alternative, perhaps more natural, formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem.

technical point of view this modelling is symmetric to the modeling of entrepreneurial idiosyncratic risk and bankruptcy. We consider two mechanisms by which the cutoff is determined. One is based on the total surplus of a given worker and the other is based purely on the employment agency's interest.[13] After this endogenous layoff decision, the employment agency posts vacancies and the intensity of work effort is chosen efficiently, i.e. so that the value of labor services to the employment agency is equated to the cost of providing it by the household. At this point the employment agency supplies labor to the labor market. We now describe these various labor market activities in greater detail. We begin with the decisions at the end of the period and work backwards to the bargaining problem. This is a convenient way to develop the model because the bargaining problem internalizes everything that comes after. The actual equilibrium conditions are displayed in the Appendix.

2.7.1. Labor Hours

Labor intensity is chosen to equate the value of labor services to the employment agency with the cost of providing it by the household. To explain the latter, we again display the utility function of the household:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{ \zeta_{t+l}^c \ln(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \left[\sum_{i=0}^{N-1} \frac{(\varsigma_{i,t+l})^{1+\sigma_L}}{1+\sigma_L} \left[1 - \mathcal{F}\left(\bar{a}_{t+l}^i\right)\right] l_{t+l}^i \right] \}, \tag{2.53}$$

Here, $i \in \{0, ..., N-1\}$ indexes the cohort to which the employment agency belongs. The index, $i = 0$ corresponds to the cohort whose employment agency renegotiates the wage in the current period, $i = 1$ corresponds to the cohort that renegotiated in the previous period, and so on. The object, l_t^i denotes the number of workers in cohort i, after exogenous separations and new arrivals from unemployment have occurred.

$$\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i \tag{2.54}$$

denotes the number of workers with an employment agency in the i^{th} cohort who survive the endogenous layoffs.[14] It should be noted that the current version of Ramses II does not allow for endogenous layoffs, so $\mathcal{F}_t^i = 0$ for all j and t, in the subsequent equations.

Let $\varsigma_{i,t}$ denote the number of hours supplied by a worker in the i^{th} cohort. The absence of the index, a, on $\varsigma_{i,t}$ reflects our assumption that each worker who survives endogenous layoffs in

[13] In the current version of Ramses II we do not consider endogenous layoffs, where each worker draws an idiosyncratic productivity shock, a cutoff level of productivity is determined, and workers with lower productivity are laid off. There are two mechanisms by which the cutoff can be determined. One is based on the total surplus of a given worker and the other is based purely on the employment agency's interest.

[14] Let a_t^i denote an idiosyncratic productivity shock drawn by a worker in cohort i. Then, $\bar{a}_t^i$, denotes the endogenously-determined cutoff such that all workers with $a_t^i < \bar{a}_t^i$ are laid off from the firm. Also, let

$$\mathcal{F}\left(\bar{a}_t^i\right) = P\left[a_t^i < \bar{a}_t^i\right]$$

denote the cumulative distribution function of the idiosyncratic productivity shock. (In practice, we assume that $\mathcal{F}$ is lognormal with $Ea = 1$ and standard deviation of $log\,(a)$ equal to σ_a.)

cohort i works the same number of hours, regardless of the realization of their idiosyncratic level of productivity. The disutility experienced by a worker that works $\varsigma_{i,t}$ hours is:

$$\zeta_t^h A_L \frac{(\varsigma_{i,t})^{1+\sigma_L}}{1+\sigma_L}.$$

The utility function in (2.53) sums the disutility experienced by the workers in each cohort.

Although the individual worker's labor market experience - whether employed or unemployed - is determined by idiosyncratic shocks, each household has sufficiently many workers that the total fraction of workers employed,

$$L_t = \sum_{i=0}^{N-1} \left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i,$$

as well as the fractions allocated among the different cohorts, $\left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i$, $i = 0, ..., N-1$, are the same for each household. We suppose that all the household's workers are supplied inelastically to the labor market (i.e., labor force participation is constant).

The household's currency receipts arising from the labor market are:

$$(1-\tau_t^y)(1-L_t) P_t b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i \left[1 - \mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} \tag{2.55}$$

where W_t^i is the nominal wage rate earned by workers in cohort $i = 0, ..., N-1$. The presence of the term involving b^u indicates the assumption that unemployed workers, $1 - L_t$, receive a pre-tax payment of $b^u z_t^+$ final consumption goods. These unemployment benefits are financed by lump sum taxes. As in our baseline model, there is a labor income tax τ_t^y and a payroll tax τ_t^w that affect the after-tax wage.

Let W_t denote the price received by employment agencies for supplying one unit of labor service. It represents the marginal gain to the employment agency that occurs when an individual worker increases time spent working by one unit. Because the employment agency is competitive in the supply of labor services, it takes W_t as given. We treat W_t as an unobserved variable in the data. In practice, it is the shadow value of an extra worker supplied by the human resources department to a firm.

Following GST, we assume that labor hours are chosen to equate the worker's marginal cost of working with the agency's marginal benefit:

$$W_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\upsilon_t \frac{1-\tau_t^y}{1+\tau_t^w}} \tag{2.56}$$

for $i = 0, ..., N-1$. Here, $\mathcal{G}_t^i$ denotes expected productivity of workers who survive endogenous separation:

$$\mathcal{G}_t^i = \frac{\mathcal{E}_t^i}{1 - \mathcal{F}_t^i}, \tag{2.57}$$

where

$$\mathcal{E}_t^i \equiv \mathcal{E}\left(\bar{a}_t^i;\sigma_{a,t}\right) \equiv \int_{\bar{a}_t^i}^{\infty} a d\mathcal{F}\left(a;\sigma_{a,t}\right) \tag{2.58}$$

$$\mathcal{F}_t^i = \mathcal{F}\left(\bar{a}_t^i;\sigma_{a,t}\right) = \int_0^{\bar{a}_t^i} d\mathcal{F}\left(a;\sigma_{a,t}\right). \tag{2.59}$$

To understand the expression on the right of (2.56), note that the marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is $\zeta_t^h A_L \varsigma_{i,t}^{\sigma_L}$. This is converted to currency units by dividing by the multiplier, υ_t, on the household's nominal budget constraint, and by the tax wedge $\left(1-\tau_t^y\right)/\left(1+\tau_t^w\right)$. The left side of (2.56) represents the increase in revenues to the employment agency from increasing hours worked by one unit (recall, all workers who survive endogenous layoffs work the same number of hours.) Division by $1-\mathcal{F}_t^i$ is required in (2.57) so that the expectation is relative to the distribution of a conditional on $a \geq \bar{a}_t^j$.

Labor intensity is the same in all cohorts since Ramses II does not allow for endogenous layoffs.

2.7.2. Vacancies and the Employment Agency Problem

The employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies, v_t^i. The vacancy posting costs associated with v_t^i are:

$$\frac{\kappa z_t^+}{\varphi}\left(\frac{Q_t^{\iota} v_t^i}{\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i}\right)^{\varphi}\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i,$$

units of the domestic homogeneous good. The parameter φ determines the curvature of the cost function and in practice we set $\varphi = 2$. Also, $\kappa z_t^+/\varphi$ is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate and, as noted above, $\left[1-\mathcal{F}\left(\bar{a}_t^i\right)\right] l_t^i$ denotes the number of employees in the i^{th} cohort after endogenous separations have occurred. Also, Q_t is the probability that a posted vacancy is filled, a quantity that is exogenous to an individual employment agency. The functional form of our cost function reduces to the function used in GT and GST when $\iota = 1$. With this parameterization, costs are a function of the number of people hired, not the number of vacancies per se. We interpret this as reflecting that the GT and GST specifications emphasize internal costs (such as training and other) of adjusting the work force, and not search costs. In models used in the search literature (see, e.g., Shimer (2005a)), vacancy posting costs are independent of Q_t, i.e., they set $\iota = 0$. To understand the implications for our type of empirical analysis, consider a shock that triggers an economic expansion and also produces a fall in the probability of filling a vacancy, Q_t. We expect the expansion to be smaller in a version of the model that emphasizes search costs (i.e., $\iota = 0$) than in a version that emphasizes internal costs (i.e., $\iota = 1$).

To further describe the vacancy decisions of the employment agencies, we require their objective function. We begin by considering $F\left(l_t^0, \omega_t\right)$, the value function of the representative employment agency in the cohort, $i=0$, that negotiates its wage in the current period. The arguments of F are the agency's workforce after beginning-of-period exogenous separations and new arrivals, l_t^0, and an arbitrary value for the nominal wage rate, ω_t. That is, we consider the value of the firm's problem after the wage rate has been set.

We suppose that the firm chooses a particular monotone transform of vacancy postings, which we denote by $\tilde{v}_t^i$:

$$\tilde{v}_t^i \equiv \frac{Q_t^\iota v_t^i}{\left(1-\mathcal{F}_t^j\right) l_t^i},$$

where $1-\mathcal{F}_t^j$ denotes the fraction of the beginning-of-period t workforce in cohort j which survives endogenous separations. The agency's hiring rate, χ_t^i, is related to $\tilde{v}_t^i$ by:

$$\chi_t^i = Q_t^{1-\iota} \tilde{v}_t^i. \tag{2.60}$$

To construct $F\left(l_t^0, \omega_t\right)$, we must derive the law of motion of the firm's work force, during the period of the wage contract. If l_t^i is the period t work force just after exogenous separations and new arrivals, then (2.54) is the size of the workforce after endogenous separations. The time $t+1$ workforce of the representative agency in the i^{th} cohort at time t is denoted l_{t+1}^{i+1}. That workforce reflects the endogenous separations in period t as well as the exogenous separations and new arrivals at the start of period $t+1$. Let ρ denote the probability that an individual worker attached to an employment agency at the start of a period survives the exogenous separation. Then, given the hiring rate, χ_t^i, we have

$$l_{t+1}^{j+1} = \left(\chi_t^j + \rho\right)\left(1-\mathcal{F}_t^j\right) l_t^j, \tag{2.61}$$

for $j=0,1,...,N-1$, with the understanding here and throughout that $j=N$ is to be interpreted as $j=0$. Expression (2.61) is deterministic, reflecting the assumption that the representative employment agency in cohort j employs a large number of workers.

The value function of the firm is:

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\left(\tilde{v}_{t+j}^j, \bar{a}_{t+j}^j\right)} [\int_{\bar{a}_{t+j}^j}^{\infty} \left(W_{t+j} a - \Gamma_{t,j}\omega_t\right) \varsigma_{j,t+j} d\mathcal{F}(a) \\ &\quad - P_{t+j} \frac{\kappa z_{t+j}^{+}}{\varphi} \left(\tilde{v}_{t+j}^j\right)^{\varphi} \left(1-\mathcal{F}_{t+j}^j\right)] l_{t+j}^j \\ &\quad + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned} \tag{2.62}$$

where l_t^j evolves according to (2.61), $\varsigma_{j,t}$ satisfies (2.56) and

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}, & j>0 \\ 1 & j=0 \end{cases}. \tag{2.63}$$

Here, $\tilde{\pi}_{w,t}$ is defined in (2.52). The term, $\Gamma_{t,j}\omega_t$, represents the wage rate in period $t+j$, given the wage rate was ω_t at time t and there have been no wage negotiations in periods $t+1$, $t+2$, up to and including period $t+j$. In (2.62), $\tilde{W}_{t+N}$ denotes the Nash bargaining wage that is negotiated in period $t+N$, which is when the next round of bargaining occurs. At time t, the agency takes the state $t+N-$contingent function, $\tilde{W}_{t+N}$, as given. The vacancy decision of employment agencies solve the maximization problem in (2.62).

It is easily verified using (2.62) that $F\left(l_t^0, \omega_t\right)$ is linear in l_t^0:

$$F\left(l_t^0, \omega_t\right) = J\left(\omega_t\right) l_t^0, \tag{2.64}$$

where $J\left(\omega_t\right)$ is not a function of l_t^0. The function, $J\left(\omega_t\right)$, is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is ω_t. Although later in the period workers become heterogeneous when they draw an idiosyncratic shock to productivity, the fact that that draw is i.i.d. over time means that workers are all identical at the time that (2.64) is evaluated.

2.7.3. Worker Value Functions

Let V_t^i denote the period t value of being a worker in an agency in cohort i, after that worker has survived that period's endogenous separation:

$$\begin{aligned} V_t^i &= \Gamma_{t-i,i}\tilde{W}_{t-i}\varsigma_{i,t}\frac{1-\tau_t^y}{1+\tau_t^w} - A_L\frac{\zeta_t^h \varsigma_{i,t}^{1+\sigma_L}}{\left(1+\sigma_L\right)\upsilon_t} \\ &\quad +\beta E_t\frac{\upsilon_{t+1}}{\upsilon_t}\left[\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right)V_{t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right)U_{t+1}\right], \end{aligned} \tag{2.65}$$

for $i = 0, 1, ..., N-1$. In (2.65), $\tilde{W}_{t-i}$ denotes the wage negotiated i periods in the past, and $\Gamma_{t-i,i}\tilde{W}_{t-i}$ represents the wage received in period t by workers in cohort i. The two terms after the equality in (2.65) represent a worker's period t flow utility, converted into units of currency.[15] The terms in square brackets in (2.65) correspond to utility in the two possible period $t+1$ states of the world. With probability $\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right)$ the worker survives the exogenous and endogenous separations in period $t+1$, in which case its value function in $t+1$ is V_{t+1}^{i+1}. With the complementary probability, $1-\rho+\rho\mathcal{F}_{t+1}^{i+1}$, the worker separates into unemployment in period $t+1$, and enjoys utility, U_{t+1}.

The currency value of being unemployed in period t is:

$$U_t = P_t z_t^+ b^u\left(1-\tau_t^y\right) + \beta E_t\frac{\upsilon_{t+1}}{\upsilon_t}[f_t V_{t+1}^x + \left(1-f_t\right)U_{t+1}], \tag{2.66}$$

where f_t is the probability that an unemployed worker will land a job in period $t+1$. Also, V_{t+1}^x is the period $t+1$ value function of a worker who knows that he has matched with an employment agency at the start of $t+1$, but does not know which one. In particular,

$$V_{t+1}^x = \sum_{i=0}^{N-1}\frac{\chi_t^i\left(1-\mathcal{F}_t^i\right)l_t^i}{m_t}\tilde{V}_{t+1}^{i+1}. \tag{2.67}$$

[15] Note the division of the disutility of work in (2.65) by υ_t, the multiplier on the budget constraint of the household optimization problem.

Here, total new matches at the start of period $t+1$, m_t, is given by:

$$m_t = \sum_{j=0}^{N-1} \chi_t^j \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{2.68}$$

In (2.67),

$$\frac{\chi_t^i \left(1 - \mathcal{F}_t^i\right) l_t^i}{m_t}$$

is the probability of finding a job in $t+1$ in an agency belonging to cohort i in period t. Note that this is a proper probability distribution because it is positive for each i and it sums to unity by (2.68).

In (2.67), $\tilde{V}_{t+1}^{i+1}$ is the analog of V_{t+1}^{i+1}, except that the former is defined before the worker knows if he survives the endogenous productivity cut, while the latter is defined after survival. The superscript $i+1$ appears on $\tilde{V}_{t+1}^{i+1}$ because the probabilities in (2.67) refer to activities in a particular agency cohort in period t, while in period $t+1$ the index of that cohort is incremented by unity.

We complete the definition of U_t in (2.66) by giving the formal definition of $\tilde{V}_t^j$:

$$\tilde{V}_t^j = \mathcal{F}_t^j U_t + \left(1 - \mathcal{F}_t^j\right) V_t^j. \tag{2.69}$$

That is, at the start of the period, the worker has probability $\mathcal{F}_t^j$ of returning to unemployment, and the complementary probability of surviving in the firm to work and receive a wage in period t.

2.7.4. Bargaining Problem

We assume that bargaining occurs between a union representing the 'average worker' and the employment agency, and that it ignores the impact of the wage bargain on decisions like vacancies and separations, taken by the firm. The Nash bargaining problem that determines the wage rate is a combination of the worker surplus and firm surplus

$$\max_{\omega_t} \left(\tilde{V}_t^0 - U_t\right)^{\eta} J\left(\omega_t\right)^{(1-\eta)},$$

where η represents the bargaining power of the workers, $\tilde{V}_t^0 - U_t$ is the worker surplus (where U_t is the outside option of unemployment), and $J\left(\omega_t\right)$ is the firm surplus, which reflects that the outside option of the firm in the bargaining problem is zero. We denote the wage that solves this problem by $\tilde{W}_t$. The first order condition of this problem can be found in the appendix. The first derivative of the surplus with respect to the wage rate, $J_{w,t}$, is.

$$\begin{aligned} J_{w,t} &= -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\ &+\beta\frac{\upsilon_{t+1}}{\upsilon_t}[-\Gamma_{t,1}\varsigma_{1,t+1}\rho\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right)] \\ &+\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[-\Gamma_{t,2}\varsigma_{2,t+2}\right]\rho^2\left(1-\mathcal{F}_{t+2}^2\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\ &+...+ \\ &+\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\left[-\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\right]\rho^{N-1}\left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right), \end{aligned}$$

where it should be noted that there are no endogenous layoffs so that $\mathcal{F}_t^j = 0$ for all j and t.A rise in the wage reduces J_t only in future states of the world in which the worker survives both exogenous $(1-\rho)$ and endogenous separation $(1-\mathcal{F}_t^j)$. If we abstract from taxes it is easy to verify that $J_{w,t} = -\tilde{V}_{w,t}$. That is, a contemplated increase in the wage simply reallocates resources between the firm and the worker.

2.8. Monetary Policy

We model monetary policy according to an instrument rule of the following form:

$$\begin{aligned} \ln\left(\frac{R_t}{R}\right) &= \rho_R\ln\left(\frac{R_{t-1}}{R}\right)+(1-\rho_R)[\ln\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right)+r_\pi\ln\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right) \\ &\quad +r_y\ln\left(\frac{h_{t-1}}{h}\right)]+r_{\Delta\pi}\Delta\ln\left(\frac{\pi_t^c}{\pi^c}\right)+r_{\Delta y}\Delta\ln\left(\frac{h_t}{h}\right)+\varepsilon_{R,t}, \end{aligned} \tag{2.70}$$

where the policy parameters are estimated to capture the historical behavior of the Riksbank between 1995 and 2008. Notice that we use hours worked instead of output as a measure of the utilization of resources. The two reasons for this is that, *i*) filtered hours worked is an observed variable (where the filter is an HP-trend or a KAMEL-trend[16]) which enable judgments of this measure of resource utilization to directly influence monetary policy (which is only implicitly the case with the (unobserved) model output gap), and *ii*) this specification had a slight empirical advantage.

2.9. Fiscal Authorities

Government consumption expenditures are modeled as

$$G_t = g_t z_t^+,$$

where g_t is an exogenous stochastic process, orthogonal to the other shocks in the model. We suppose that

$$\ln g_t = \left(1-\rho_g\right)\ln g + \rho_g \ln g_{t-1} + \varepsilon_t^g,$$

[16]KAMEL is a model developed by the National Institute of Economic Research for demographic description of labor market variables.

where $g = \eta_g Y$. We set $\eta_g = 0.3$, the sample average of government consumption as a fraction of GDP.

The tax rates in our model are:

$$\tau_t^k,\ \tau_t^b,\ \tau_t^y,\ \tau_t^c,\ \tau_t^w.$$

We set the tax rate on capital income, $\tau^k = 0.25$; the payroll tax rate, $\tau^w = 0.35$; the value-added tax on consumption, $\tau^c = 0.25$; and the personal income tax rate that applies to labor, $\tau^y = 0.3$. We set the tax rates on bonds to zero, $\tau^b = 0$, to be able to match the pre-tax real rate on bonds of 2.25% in the data. Setting $\tau^b = 0$ is required to get the interest rate on bonds to be this low, given the high GDP growth rate, log utility of consumption and β not too close to 1. All the tax rates are held constant in the model, implying that there are no stochastic tax shocks.

2.10. Foreign Variables

Our representation of the stochastic processes driving the foreign variables takes into account that foreign output, Y_t^*, is affected by disturbances to z_t^+, just as domestic variables are. In particular, our model of Y_t^* is:

$$\begin{aligned} \ln Y_t^* &= \ln y_t^* + \ln z_t^+ \\ &= \ln y_t^* + \ln z_t + \frac{\alpha}{1-\alpha} \ln \psi_t, \end{aligned}$$

where $\log(y_t^*)$ is assumed to be a stationary process. We assume:

$$\begin{pmatrix} \ln\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \ln\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \ln\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \ln\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \ln\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \ln\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \end{pmatrix} + \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\psi,t} \end{pmatrix}, \tag{2.71}$$

where the ε_t's are mean zero, unit variance, i.i.d. processes uncorrelated with each other. In matrix form,

$$X_t^* = AX_{t-1}^* + C\varepsilon_t,$$

in obvious notation. Note that the matrix C has 10 elements, so that the order condition for identification is satisfied, since CC' represents 15 independent equations.

We now briefly discuss the intuition underlying the zero restrictions in A and C. First, we assume that the shock, $\varepsilon_{y^*,t}$, affects the first three variables in X_t^*, while $\varepsilon_{\pi^*,t}$ only affects the second two and

$\varepsilon_{R^*,t}$ only affects the third. The assumption about $\varepsilon_{R^*,t}$ corresponds to one strategy for identifying a monetary policy shock, in which it is assumed that inflation and output are predetermined relative to the monetary policy shock. Under this interpretation of $\varepsilon_{R^*,t}$, our treatment of the foreign monetary policy shock and the domestic one are inconsistent because in our model domestic prices are not predetermined in the period of a monetary policy shock. Second, note from the zeros in the last two columns of the first row in A and C, that the technology shocks do not affect y_t^*. This reflects our assumption that the impact of technology shocks on Y_t^* is completely taken into account by z_t^+, while all other shocks to Y_t^* are orthogonal to z_t^+ and they affect Y_t^* via y_t^*. Third, the A and C matrices capture the notion that innovations to technology affect foreign inflation and the interest rate via their impact on z_t^+. Fourth, our assumptions on A and C imply that $\ln\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right)$ and $\ln\left(\frac{\mu_{z,t}}{\mu_z}\right)$ are univariate first order autoregressive processes driven by $\varepsilon_{\mu_\psi,t}$ and $\varepsilon_{\mu_z,t}$, respectively. This is a standard assumption made on technology shocks in DSGE models.

2.11. Resource Constraints

2.11.1. Resource Constraint for Domestic Homogeneous Output

Resources expressed from the production side defines domestic homogeneous good, Y_t, in terms of aggregate factors of production. The scaled version of the production function (2.2) yields real, scaled GDP:

$$y_t = \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}k_t\right)^\alpha (H_t)^{1-\alpha} - \phi\right]. \tag{2.72}$$

where it should be noted that in the current version of Ramses II there is no price dispersion ($\mathring{p}_t = 1$).

It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (2.8) and (2.28),

$$z_t^+ y_t = G_t + C_t^d + I_t^d + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} Y_t^*,$$

or, after scaling by z_t^+ and using (2.14) and (2.17):

$$\begin{aligned} y_t \quad = \quad & g_t + (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i}\left(i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \\ & + \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} y_t^*. \end{aligned} \tag{2.73}$$

where it should be noted that in the current version of Ramses II there is no price dispersion ($\mathring{p}_t = 1$).

When we match GDP to the data we use subtract capital utilization costs, recruitment costs and monitoring/bankruptcy costs from y_t. See section 3.6 for details.

$$
\begin{aligned}
gdp_t &= y_t - \left(p_t^i\right)^{\eta_i}\left(a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \\
&\quad -\frac{\kappa}{2}\sum_{j=0}^{N-1}\left(\tilde{v}_t^j\right)^2\left[1-\mathcal{F}_t^j\right]l_t^j - \left(p_t^i\right)^{\eta_i} - \mu\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1})R_t^k P_{t-1}P_{k',t-1}\bar{K}_t
\end{aligned}
$$

2.11.2. Trade Balance

We begin by developing the link between net exports and the current account. Expenses on imports and new purchases of net foreign assets, A_{t+1}, must equal income from exports and interest from previously purchased net foreign assets:

$$
S_tA_{t+1} + \textit{expenses} \text{ on imports}_t = \textit{receipts} \text{ from exports}_t + R_{t-1}^*\Phi_{t-1}S_tA_t^*,
$$

where Φ_t is the risk premium defined in (2.36). Expenses on imports correspond to the purchases of the specialized importers in the consumption, investment and export sectors, so that the current account can be written as

$$
\begin{aligned}
& S_tA_{t+1}^* + S_tP_t^*R_t^{\nu,*}\left(C_t^m\left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + I_t^m\left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + X_t^m\left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}}\right) \\
= \; & S_tP_t^xX_t + R_{t-1}^*\Phi_{t-1}S_tA_t^*,
\end{aligned}
$$

where $\mathring{p}_t^{m,c} = \mathring{p}_t^{m,i} = \mathring{p}_t^{m,x} = 1$. With price distortions among the imported intermediate goods, the expenses of the homogeneous import goods would be higher for any given value of C_t^m. Writing the current account in scaled form and dividing by $P_tz_t^+$, we obtain using (2.26)

$$
\begin{aligned}
a_t + q_tp_t^cR_t^{\nu,*}\left(c_t^m\left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + i_t^m\left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x_t^m\left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}}\right) \\
= q_tp_t^cp_t^xx_t + R_{t-1}^*\Phi_{t-1}s_t\frac{a_{t-1}}{\pi_t\mu_{z^+,t}},
\end{aligned}
\tag{2.74}
$$

where $a_t = S_tA_{t+1}^*/(P_tz_t^+)$.

2.12. Exogenous Shock Processes

The structural shock processes in the model are given by the univariate representation

$$
\hat{\varsigma}_t = \rho_\varsigma\hat{\varsigma}_{t-1} + \varepsilon_{\varsigma t}, \qquad \varepsilon_{\varsigma t} \overset{iid}{\sim} N\left(0,\sigma_\varsigma^2\right) \tag{2.75}
$$

where $\varsigma_t = \{\ \mu_{zt},\ \epsilon_t, \tau_t^j,\ \zeta_t^c,\ \zeta_t^h,\ \Upsilon_t,\ \tilde{\phi}_t,\ \varepsilon_{Rt}, \gamma_t, \varepsilon^g, \varepsilon_{y^*t}, \varepsilon_{\pi^*t}, \varepsilon_{R^*t}\},\ j = \{d, x, mc, mi; mx\}\ ,\ \mu_{zt} = z_t/z_{t-1}$,and a hat denotes the deviation of a log-linearized variable from a steady-state level ($\hat{v}_t \equiv dv_t/v$ for any variable v_t, where v is the steady-state level). τ_t^j,$\varepsilon_{Rt}, \varepsilon_t^g, \varepsilon_{y^*t}, \varepsilon_{\pi^*t}, \varepsilon_{R^*t}$ are all assumed to be white noise (that is, $\rho_{\tau^j} = 0$, $\rho_{\varepsilon_R} = 0$, etc.).

3. Estimation

We estimate the model using Bayesian techniques. The equilibrium conditions of the model are summarized in Appendix B.7.

3.1. Data

We estimate the model using quarterly Swedish data for the period $1995Q1-2008Q2$. We do not at this stage want to include the extraordinary financial crisis after the collapse of Lehman Brothers why we have cut the sample short. Compared to ALLV's 15 macro variables, three additional variables are included among the observed variables: unemployment, the spread between the risk-free rate (i.e., the interest rate on government bonds with a maturity of 6 months to match the duration of the corporate debt) and the loan rate entrepreneurs face (i.e, the interest rate on all outstanding loans to non-financial corporations). The vector of 18 observed variables are therefore

$$\tilde{Y}_t = \begin{array}{lcccccc} [& R_t^{data} & \pi_t^{c,data} & \pi_t^{data} & \pi_t^{i,data} & \pi_t^{*,data} & R_t^{*,data} \\ & \hat{H}_t^{data} & \Delta\ln Y_t^{data} & \Delta\ln C_t^{data} & \Delta\ln I_t^{data} & \Delta\ln X_t^{data} & \Delta\ln M_t^{data} \\ & \Delta\ln(W_t/P_t)^{data} & \Delta\ln q_t^{data} & \Delta\ln Unemprate_t^{data} & \Delta\ln spread_t^{data} & \Delta\ln G^{data} & \Delta\ln Y_t^{*,data} \end{array}]', \tag{3.1}$$

where the first seven variables are matched in levels; the repo rate, CPI inflation, GDP deflator, investment deflator, foreign inflation, foreign interest rate, and the hours gap (hours deviation from an hp-trend). The inflation and interest rates are measured as annualized quarterly rates. The rest of the variables are matched in growth rates measured as quarter-to-quarter log-differences; GDP, consumption, investment, exports, imports, real wage, real exchange rate, unemployment rate, interest rate spread, government consumption, and foreign output. All real quantities (except hours and foreign output) are in per capita terms.

All variables are seasonally adjusted but no other pre-filtering of the data is done (such as demeaning) except for exports, imports and government consumption. Since exports, imports and government consumption grow at substantially different rates compared to output we adjust the mean growth rates of these three series so that they are growing at the same pace as output (i.e., we take out the excess trends in exports and imports and add an extra trend to government consumption). We also extract an obvious outlier in 1997 from the government consumption series.

The data are taken from Statistics Sweden and Sveriges Riksbank (i.e., repo rate, interest rate spread, foreign variables). The foreign variables on output, the interest rate and inflation are weighted together across Sweden's 20 largest trading partners in 1991 using weights from the IMF.

The thick black line in Figure D in the Appendix plots the data used in the estimation.

3.2. Calibration

We choose to calibrate the parameters related to the steady-state values of the observable quantities, for example the "great ratios" (i.e., C/Y, I/Y and G/Y). Table 1 shows the calibrated parameters.

The discount factor β and the tax rate on bonds τ_b are calibrated to yield a real interest of rate equal to 2.14 percent annually. We calibrate the capital share α to 0.35 which yields a capital-output ratio slightly below 2 on an annual basis. The capital share is set higher than most of the literature to compensate for the effect of a positive external finance premium.

Sample averages are used when available, e.g. for the various import shares ω_i, ω_c, ω_x (obtained from input-output tables), the remaining tax rates, the government consumption share of GDP, η_g, growth rates of technology (using investment prices to disentangle neutral from investment-specific technology) and several other parameters. To calibrate the steady value of the inflation target we simply use the inflation target stated by Sveriges Riksbank.

We let the markup of export good producers λ_x be low so as to avoid double marking up of these goods. All other price markups are set to 1.2, following a wide literature. We require full working capital financing in all appropriate sectors. The indexation parameters $\varkappa^j$, $j = d, x, mc, mi, mx, w$ are set so that there is no indexation to the inflation target, but instead to $\breve{\pi}$ which is set equal to the steady state inflation. This implies that we do not allow for partial indexation in this estimation, which would result in steady state price and wage dispersion.

The curvature parameter determining the cost of varying the capacity utilization, σ_a, is calibrated to 0.2 to allow for a varying degree of utilization of the capital stock. Bayesian posterior odds indicate that data are strongly against having a fixed capacity utilization ($\sigma_a = 10^6$) when we compared two calibrated values. We did not include this parameter in the estimation because in ALLV (2007) σ_a turned out to generate convergence problems in the Metropolis chain.

For the financial block of the model we set $F(\bar{\omega})$ equal to the sample average bankruptcy rate according to microdata from the leading Swedish credit registry, called "UC AB". W_e/y has no other noticeable effect than jointly with γ determining the $n/(p_{k'}k)$ and is set to yield at the prior mean.

For the labor block, the steady state unemployment rate is to 7% which is 1.13% below the sample average ($1995Q1 - 2008Q2$) but more or less equal to the average over a longer horizon ($1986Q1 - 2008Q2$). The length of a wage contract N is set to an annual negotiation frequency, $\varphi = 2$ to yield quadratic recruitment costs, and ρ is set so that it takes an unemployed person on average 3 quarters to find a job (i.e. $f = 1/3$), in line with the evidence presented in Forslund and Johansson (2007) for completed unemployment spells. Holmlund (2006) present evidence of unemployment duration for all unemployment spells being slightly higher, around 4 quarters. The matching function parameter σ is set to 0.5 so that number of unemployed and vacancies have equal factor shares in the production of matches. σ_m is calibrated to match the probability $Q = 0.9$ of filling a vacancy within a quarter, although this is merely a normalization. We assume hiring costs, and not search costs by setting $\iota = 1$ and thereby follow GST. We are reinforced in this calibration by the limited importance of search costs that has been documented using Swedish microdata by Carlsson, Eriksson and Gottfries (2006).

Parameter	Value	Description
α	0.35	Capital share in production
β	0.9999	Discount factor
ω_i	0.43	Import share in investment goods
ω_c	0.25	Import share in consumption goods
ω_x	0.35	Import share in export goods
η_g	0.3	Government consumption share of GDP
τ_k	0.25	Capital tax rate
τ_w	0.35	Payroll tax rate
τ_c	0.25	Consumption tax rate
τ_y	0.30	Labor income tax rate
τ_b	0	Bond tax rate
μ_z	1.005	Steady state growth rate of neutral technology
μ_ψ	1.0004	Steady state growth rate of investment technology
$\bar{\pi}, \breve{\pi}$	1.005	Steady state gross inflation target
λ_x	1.05	Export price markup
λ_j	1.2	Price markups, $j = d, mc, mi, mx$
$\nu_t^*, \nu_t^x, \nu_t^f$	1	Working capital shares
σ_a	0.2	Capacity utilization (curvature)
$\widetilde{\phi}_a$	0.01	Risk premium dependence on net foreign assets
ϑ_w, κ_w	0	Wage indexation to real growth trend and lagged inflation
$\varkappa^j$	$1 - \kappa^j$	Indexation to inflation target for $j = d, x, mc, mi, mx, w$
$F(\bar{\omega})$	0.0063	Steady state bankruptcy rate
W_e/y	0.001	Transfers to entrepreneurs
L	$1 - 0.07$	Steady state fraction of employment
N	4	Number of agency cohorts/length of wage contracts
φ	2	Curvature of recruitment costs
ρ	0.976	Exogenous survival rate of a match
σ	0.5	Unemployment share in matching technology
σ_m	0.5559	Level parameter in matching function
ι	1	Employment adj. costs dependence on tightness

Table 1. Calibrated parameters.

Throughout the estimation, four observable ratios are chosen to be exactly matched in our steady-state solution and accordingly four corresponding 'steady-state' parameters are recalibrated for each (estimated) parameter draw. We set the depreciation rate δ to match the ratio of investment over output, $p_i i/y$, the entrepreneurial survival rate γ to match the net worth to assets ratio[17],

[17] We used micro data to calculate the average equity/total assets during the sample period both for all Swedish firms and for only the stock market listed firms. In the first case book values where used, and in the second case market value of equity was used. Both ratios where close to 0.5.

$n/(p_{k'}k)$,the steady state real exchange rate $\tilde{\varphi}$ to match the export share $P^x X/(PY)$ in the data, and finally we set the disutility of labor scaling parameter A_L to fix the fraction of their time that individuals spend working. The values of these four calibrated parameters (evaluated at the posterior mode) are presented in Table 2.

	Parameter description	Calibrated value	Moment	Moment value
δ	Depreciation rate of capital	0.012	$p_i i/y$	0.17
γ	Entrepreneurial survival rate	0.969	$n/(p_{k'}k)$	0.5
$\tilde{\varphi}$	Real exchange rate	0.287	$P^x X/(PY)$	0.44
A_L	Scaling of disutility of work	46.912	$L\varsigma$	0.27

Table 2. Matched moments and corresponding parameters (evaluated at the posterior mode).

3.3. Choice of priors

In total we estimate 64 parameters, of which 16 are VAR parameters for the foreign economy, 8 are AR1-coefficients and 17 are standard deviations of the shocks. The priors are displayed in Tables A1 and A2.

Compared to the old model (Ramses) the prior distribution is similar for many of the parameters. For example, the Calvo price stickiness parameters are estimated with a beta distribution with mean 0.75 and standard deviation 0.075, corresponding to an adjustment of prices once a year based on the micro evidence in Apel, Friberg and Hallsten (2005). As in Ramses, but in contrast to CTW, we let the indexation parameters to past inflation in the price setting, κ^j $j\epsilon(d, mc, mi, mx)$, be the same in all sectors and estimate it with a relatively diffuse beta prior centered at 0.5.

There are also some notable exceptions compared to Ramses as well as to CTW:

The inverse of the Frisch elasticity of labor supply, σ_L, which, in contrast to Ramses, is now estimated. We use a gamma distribution with prior mode 2 and standard deviation 0.5. The prior mode follows Smets and Wouters (2003) and falls between the calibrated value of 1 in Ramses as well as in Christiano, Eichenbaum and Evans (2005) and CTW:s prior mode of 7.5. Micro evidence tend to find lower Frisch elasticities (i.e., $1/\sigma_L$) than normally used in DSGE models. Typically micro estimates of the Frisch elasticity lie in the range of 0.05-0.3, see e.g. MaCurdy (1986) who reports a Frisch elasticity of 0.15 for U.S. men.[18] However, Flodén and Domeij (2006) show that estimates of the labor supply elasticity is biased downward if borrowing constraints are ignored. They report an elasticty of 0.36 for U.S. married men when they take this into account, implying a value of 2.7 for σ_L.

There are two new parameters related to the labor model compared to Ramses that are being estimated. For the fraction of GDP spent on vacancy costs, *recshare*, we use a prior with a mode

[18] See Mulligan (1998) for an alternative view on the small micro estimates and Rogerson and Wallenius (2009) on the relation between micro and macro estimates.

of 0.1% corresponding to $\kappa = 2.3$.[19] This is slightly below the value of 0.14% used by Galí (2010). We set the mode for the replacement rate for unemployed workers, $bshare$, to 0.75 which is slightly above the average statutory replacement ratio after tax for this time period which is 0.71. The reason to put the prior above the statutory rate is that the latter ignores the utility value of leisure and any private unemployment insurance, which is reasonably common.

Regarding the financial model there are two new parameters being estimated. The prior mode for μ is set to 0.33 to yield a 1.6% annual external finance premium, as this is the sample average. We choose a diffuse prior so as to let data determine the elasticity of the finance premium in terms of basis points, as this is what affects the dynamics of the economy.[20] For the shock to entrepreneurs idiosyncratic productivity (i.e., the survival rate of the entrepreneur) we use an uninformative inverse gamma distribution with prior mode 0.5. The prior mode of the corresponding persistence parameter is 0.85.

In Ramses the instrument rule responded to output, whereas in Ramses II it responds to hours worked. We set the prior mode of the response coefficients to the resource utilization to almost the same values, however. We use a normal distribution with prior mode 0.125 for r_y, and a gamma distribution with prior mode 0.05 for $r_{\Delta y}$.

3.4. Shocks

In total, there are 23 exogenous stochastic variables in the model. 12 of these evolve according to AR(1) processes:

$$\epsilon, \Upsilon, \bar{\pi}^c, \zeta^c, \zeta^h, \tilde{\phi}, \sigma, \gamma, g, \eta, \sigma_m, \sigma_a$$

Further, we have 6 shock processes that are i.i.d.:

$$\tau^d, \tau^x, \tau^{mi}, \tau^{mc}, \tau^{mx}, \varepsilon_R.$$

Finally, the last 5 shock processes are assumed to follow a VAR(1):

$$y^*, \pi^*, R^*, \mu_z, \mu_\Psi.$$

In the estimation we only allow for 17 shocks. Accordingly we do not allow six shocks present in the theoretical model: the inflation target shock $\bar{\pi}^c$, the shock to bargaining power η, the shock to matching technology σ_m, the shock to the standard deviation of idiosyncratic productivity of workers σ_a, the unit root shock to investment-specific technology μ_Ψ and the idiosyncratic entrepreneur risk shock σ. Indeed for our sample, $1995 - 2008$, the *de jure* inflation target has

[19] Formally the steady state recruitment share is defined as

$$recruitshare = \frac{\frac{\kappa}{2} N \tilde{v}^2 l}{y}$$

[20] In this way we are not constrained by the assumption for the functional form of the idiosyncratic risk.

been in place the entire period and has been constant. η also seems superfluous as we already have the standard labor supply shock - the labor preference shock ζ^h. We excluded μ_Ψ as it did not contribute substantially to explaining any variable in preliminary estimations. For σ the reason for exclusion was the high correlation with the other financial shock, γ.

3.5. Measurement errors

Since Swedish macro data is measured with substantial noise, we allow for measurement errors in all variables except for the nominal interest rates in Sweden and abroad. The variance of the measurement errors is calibrated so that it corresponds to 10% of the variance in each data series.

3.6. Measurement equations

Below we report how the model is linked to the observable data through the 18 measurement equations. The data is measured in percentages so the model variables are accordingly multiplied by 100. Furthermore the data series for inflation and interest rates are annualized, so these model variables are multiplied by 400.

$$\begin{aligned}
R_t^{data} &= 400(R_t - 1) - \vartheta_1 400(R - 1) \\
R_t^{*,data} &= 400(R_t^* - 1) - \vartheta_1 400(R^* - 1) \\
\\
\pi_t^{data} &= 400 \ln \pi_t - \vartheta_1 400 \ln \pi + \varepsilon_{\pi,t}^{me} \\
\pi_t^{c,data} &= 400 \ln \pi_t^c - \vartheta_1 400 \ln \pi^c + \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{i,data} &= 400 \ln \pi_t^i - \vartheta_1 400 \ln \pi^i + \varepsilon_{\pi^i,t}^{me} \\
\pi_t^{*,data} &= 400 \ln \pi_t^* - \vartheta_1 400 \ln \pi^* + \varepsilon_{\pi^*,t}^{me},
\end{aligned}$$

$$
\begin{aligned}
\Delta \ln Y_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln(y_t - p_t^i a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - \\
&\quad -\frac{\kappa}{2}\sum_{j=0}^{N-1}\left(\tilde{v}_t^j\right)^2\left[1-\mathcal{F}_t^j\right] l_t^j - \mu\int_0^{\bar{\omega}_t}\omega dF(\omega;\sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1}\bar{K}_t) \\
&\quad \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{y,t}^{me} \\
\Delta \ln Y_t^{*,data} &= 100(\ln \mu_{z^+,t} + \Delta \ln y_t^*) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{y^*,t}^{me} \\
\Delta \ln C_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln c_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{c,t}^{me} \\
\Delta \ln X_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln x_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{x,t}^{me} \\
\Delta \ln q_t^{data} &= 100\Delta \ln q_t + \varepsilon_{q,t}^{me} \\
\Delta \ln H_t^{data} &= 100\Delta \ln H_t^{meas} + \varepsilon_{H,t}^{me} \\
\Delta \ln M_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln \text{ Imports}_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
&= 100\left[\ln \mu_{z^+,t} + \Delta \ln \begin{pmatrix} c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} \\ +i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \\ +x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \end{pmatrix}\right] - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
\Delta \ln I_t^{data} &= 100\left[\ln \mu_{z^+,t} + \ln \mu_{\psi,t} + \Delta \ln i_t\right] - \vartheta_2 100(\ln \mu_{z^+} + \ln \mu_{\psi}) + \varepsilon_{I,t}^{me} \\
\Delta \ln G_t^{data} &= 100(\ln \mu_{z^+,t} + \Delta \ln g_t) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{g,t}^{me}
\end{aligned}
$$

$$
\Delta \ln(W_t/P_t)^{data} = 100\Delta \ln \frac{\tilde{W}_t}{z_t^+ P_t} = 100(\ln \mu_{z^+,t} + \Delta \ln w_t^{avg}) - \vartheta_2 100(\ln \mu_{z^+}) + \varepsilon_{W/P,t}^{me}
$$

$$
\begin{aligned}
\Delta \ln Spread_t^{data} &= 100\Delta \ln(Z_{t+1} - R_t) = 100\Delta \ln\left(\frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t\right) + \varepsilon_{Spread,t}^{me} \\
\Delta \ln Unemp_t^{data} &= 100\Delta \ln(1 - L_t) + \varepsilon_{Unemp,t}^{me}.
\end{aligned}
$$

where $\varepsilon_{i,t}^{me}$ denote the measurement error for the respective variable. In addition, we introduce the parameters $\vartheta_1 \in \{0,1\}$ and $\vartheta_2 \in \{0,1\}$ which allows us to handle demeaned and non-demeaned data. However, in this version of the model we are only working with non-demeaned data; $\vartheta_1 = 0$, $\vartheta_2 = 0$.

Note that neither measured GDP nor measured investment include investment goods used for capital maintenance. The reason is that the documentation for calculation of the Swedish National Accounts (SOU (2002)) indicate that these are not included in the investment definition (and the national accounts are primarily based on the expenditure side). To calculate measured GDP we also exclude monitoring costs and recruitment costs.

Finally, define the measurement equation for real wages we have used the employment-weighted average Nash bargaining wage in the model:

$$w_t^{avg} = \frac{1}{L} \sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j} \bar{w}_{t-j}$$

4. Results

4.1. Posterior parameter values

In Tables A1 and A2 the posterior mode estimates of the parameters are shown. All Calvo price rigidity parameters have a posterior mode of roughly around 0.8, see Table A1. Compared to the old model (Ramses), the price stickiness in the domestic sector is a lot larger; 0.84 compared to 0.71. This means that the domestic Phillips curve will be much flatter in Ramses II. In contrast, the prices in the import and export sectors are substantially more flexible in Ramses II than in the old model. Now import and export prices are re-set optimally at least once a year compared to every second or even every fourth year as for the imported investment goods in the old model. There are several reasons for this finding. In Ramses II a substantial part of imports enter directly into exports, so a lot of the variation in the real series can be accounted for without creating a tension in matching also the consumption series, for instance. This also applies when the exchange rate fluctuates. Variation in imports due to exchange rate movements does not lead to the same extent of expenditure switching into the domestic good. The effect of exchange rate fluctuations are thus smaller in Ramses II, and prices need not be as rigid to avoid large movements in inflation. Altogether this spills over to the price setting and thus the degree of price stickiness.

The investment adjustment cost, S'', is estimated to be a lot lower compared to the literature as well as compared to Ramses. The parameter, S'', is estimated to be 2.09 in Ramses II, which is about four times smaller than in the old model. However, the financial frictions applied to the entrepreneur induce a gradual response of investment, which means that the investment adjustment costs take on a more superflous role in the new model.

Also the friction pertaining to consumption, the habit persistence b, turn out to be a lot lower than expected. The posterior mode for b equals 0.53 in Ramses II. Again it should be noted that a possible explanation for this is that part of the imports are used for exports. With lower expenditure switching effects both the substitution elasticity between domestic and imported goods, η_c, and the habit persistence can be lower without generating large fluctuations in the consumption series. η_c is now being estimated to 1.41 instead of being calibrated to 5 in the old model.

The posterior mode of the persistence in the instrument rule, ρ_R is estimated to 0.83, which makes the policy rule a little bit less persistent in the new model. However, since the instrument rule is specified in terms of hours worked instead of output as in the old model, and there is no exchange rate response, also the response coefficients on the real variables changes somewhat. The posterior mode estimate of the coefficient on the hours gap is $r_h = 0.05$.

$\tilde{\phi}_s$ determines how much the forward premium puzzle is allowed to affect the risk premium in the interest rate parity condition, and thereby the hump shape of the exchange rate to a monetary policy shock. The posterior mode estimate is 0.30 in Ramses II which is slightly lower than in the old model (0.48). This implies deviations from UIP, but as we shall see below, the hump shape of the real exchange rate is not as pronounced as in the old model.

We now turn to the new parameters in Ramses II. The degree of financial fricitions in the model is determined by μ, which captures the bank's monitoring costs and thereby the size of the spread between the risk free rate and the interest rate paid by the entrepreneurs. The posterior mode estimate of μ is 0.47, which implies a spread of 1.3% during the sample period.

Regarding the new parameters in the labor market block, the replacement rate for unemployed workers, *bshare*, is estimated to be 0.97, which is substantially higher than the replacement rate in the public Swedish unemployment insurance. The model needs *bshare* to be high in order for the household to be relatively indifferent between working and staying in unemployment in order to be able to explain variations in unemployment. The recruitment costs as a fraction of GDP, *recshare*, is estimated to be 0.09 percent.

It should be noted that the Frisch elasticity of labor supply, $1/\sigma_L$, is hard to identify and highly dependent on which prior we choose. With a prior mean of 2.0, we obtain an estimate of $\sigma_L = 2.53$. This implies that a larger part of fluctuations in hours worked are attributed to the intensive margin instead of unemployment compared to the case with a larger prior on σ_L, which would have generated more disutlity for the household in changing the working intensity. As mentioned in Section 3.3 this is a controversial parameter since the micro and macro evidence are so dispersed. We have taken a conservative view here and relied more on prior macro evidence and the results of Flodén and Domeij (2006) when they take incomplete markets into account.

We note from the posterior standard deviations in Table A1 that data seem informative about most of the estimated parameters and that the posterior distribution is more concentrated than the prior distribution.

4.2. Model fit

Figure D shows the data (thick line) used in the estimation and the one-sided Kalman-filtered one-step-ahead predictions from the model (thin line) computed at the posterior mode. We see that the model captures the low-frequency fluctuations in the data relatively well for most of the observed variables but misses out on many of the high-frequency movements, especially in the four inflation series as well as in exports and imports. In addition, the real wage grows too slowly in the model compared with the data, throughout the sample. One explanation to this is that the real wage is computed using the GDP deflator which is an extremely volatile series. Much of the variance in the data should thus not be attributed to the structural model. For the three 'new' variables (compared to Ramses I), the model can explain the growth rate in unemployment reasonably well but makes a bad job in explaining government consumption and the spread. However, fiscal policy is only rudimentary modeled so this is perhaps not so surprising. It is more cumbersome that the fit of the spread is not satisfactory.

Table A3 presents the first and second moments in the data and in the model (calculated at the posterior mode), as well as the importance of the measurement errors. We see that there is an excess trend in exports and imports that can not be matched by the model, where all real variables grow at the same pace. Government consumption on the other hand grows too slowly in the data compared with the model why we have adjusted this series too. As can be seen from the table there is also a clear downward trend in unemployment that can not be explained within the model. We have, however, not pre-filtered this series. The same applies to the investment series, which grows much faster in the data compared to the model.

Regarding the volatility in the model and the data we see that the second moments seems to be captured satisfactory.

The last column shows how much of the variance in the data that can be accounted for by the structural shocks in the model. We see that the measurement error in the wage equation is obviously too large since this accounts for almost 30% of the variation of the real wage growth in the data. As already mentioned above, this is a problematic series. For the other variables, the structural shocks account for about 95% or more of the variance in the data, with the exception of import and export growth which have a slightly lower ratio of structural explanation (84% and 91%, respectively). All in all, we therefore believe the size of measurement errors are appropriate.

An additional, perhaps more indirect, way to evaluate the way the addition of financial frictions to the baseline model fits the data is to compare data which was not used in the estimation of the model, such as bankruptcy data, with the smoothed, two-sided Kalman filtered, estimates of the bankruptcy rate in the model. Figure H shows the smoothed, two-sided Kalman filtered, estimates of the bankruptcy rate computed at the posterior mode and bankrupcy data taken from UC AB. We see that the model captures the low-frequency fluctuations in the bankruptcy data relatively well.

4.3. Smoothed shock processes

Figure E shows the smoothed, two-sided Kalman filtered, estimates of the shock processes (deviations from steady state). The unit-root technology shock, μ_z, appears to have a clear trend. The reason for this is the way the shock is identified through the foreign VAR and the measurement equations for the domestic and foreign real variables. Because the technology shock has a direct impact on the foreign interest rates in the VAR (see eq. (2.71) where $a_{34} \neq 0$ and $c_{34} \neq 0$), permanent technology shocks jointly explain both foreign interest rates as well as the real variables domestically and abroad. Since the foreign interest rate contains a downward trend in our sample, data forces the posterior mode estimate of the persistence in the technology process up to

$\rho_{\mu_z} = 0.93$, to be able to explain this movement in the interest rate and the smoothed estimate of μ_z turns out trending. This is also the reason for why the unit-root technology shock comes out so important in the variance decomposition for the three foreign variables.

In contrast, if the direct effect of the permanent technology on the foreign interest rate is turned off, $a_{34} = 0$ and $c_{34} = 0$, μ_z is only identified through the measurement equations (which is more like the setting in Ramses). This would imply a lower estimate of the persistence, ρ_{μ_z}, since technology is no longer forced to directly explain the interest rate, and the smoothed estimate of μ_z would no longer be trending.Impulse response functions

4.4. Impulse response functions

We plot impulse response functions at the posterior mode for all 17 shocks. The first figure for each shock shows the observed variables in levels (i.e., percentage deviations from steady-state for all shocks except the unit-root technology shock for which we plot the true level in percent). As an example, unemployment raises 0.1% to a positive monetary policy shock which means that unemployment increases from 7% to 7.1%. The second figure for each shock shows the impulse response functions for some key variables of interest rate related to the labor market and the capital market, such as for instance intensity, wages, value functions for the worker and the employer, and financial variables such as the spread, net worth, bankruptcy rate as well as the real rates domestically and abroad.

The impulse response functions to a **monetary policy shock** is relatively similar in Ramses II and Ramses, with a reasonable transmission mechanism. A temporary hike in the nominal interest rate with 25 basis points, lowers CPIF inflation with about 0.1%. The response in Ramses is similar but a little bit more hump-shaped. The same pertains to the real exchange rate which have a hump that is a lot more pronounced. The reason for this is the smaller estimate of ϕ_s, which determines the degree to which the UIP-condition in the model is modified. As stated above this parameter is 0.48 in Ramses while 0.3 in Ramses II. This implies less impact on the risk premium from exchange rate changes and hence the response is more like a spike. CTW uses another specification of the risk premium and appears not to obtain much of hump-shape. The effects of the positive monetary policy shock is amplified by the financial frictions. Entrepreneurial net worth is reduced both because of the falling price of capital and because of the surprise disinflation that increases the real value of the nominal debt. Accordingly the interest rate risk spread increase by about 5 basis points (annualized). This has impact foremost on the response of investment. We see that the investments decreases by almost 1% in Ramses II and by more than 1.5% in CTW compared to the modest response of 0.25% in Ramses. One should however remember that the monetary policy shocks explain relatively little of the variation in investement (see Table A4). The output response do not change much between RamsesII and Ramses which is probably due to the fact thet resources in Ramses II are used up because monitoring increases following the shock. It is also worth noting that we obtain a more sticky or slow response of hours in Ramses II than in Ramses. Changes in hours, for this particular shock, are due to variations in unemployment (rather than intensity). Since unemployment is predetermined (recall again that we only have exogenous layoffs), hours will also be rigid. This stems well with the fact the labor market lags production in the data.

The impulse responses to a s**tationary technology shock** are principally similar (qualitatively) in Ramses and Ramses II although the discrepancies are somewhat larger than for the monetary policy shock. It is mainly the responses to inflation that are somewhat smaller in Ramses II. The effects of differences in productivity ares maller because of the rigidities in the labor market (hours). However, for this shock both hours per employee (intensity) and employment changes. An increase in productivity leads to drop in hours worked. Since there is a positive correlation in the data between employment and output we have rigged the model so that unemployment decreases to this shock. However, this implies that hours per employee drops substantially to make hours work decrease. We see the opposite response in CTW; unemployment *increases* after a positive technology shock. If one interprets the stationary technology shock as positive business cycle chock, this would square well with the increase in employment in Ramses II. Notice also that the instrument rule in Ramses II responds to the gap and growth rate in hours worked, whereas Ramses rule responds to output. This means that monetary policy do not try to counteract technolgy shocks in Ramses II (they are accomodated by a decrease in interest rates), whereas monetary policy tries to balance the increase in the output gap in Ramses (where potential output by definition is not affected by stationary technology shocks so that the output gap increases.)

Comparing the impulse response functions to a **risk premium shock**, we see that the exchange rate channel is weaker in Ramses II than in Ramses. This is due to several reasons. First, we allow for a part of imports to enter exports directly. This implies less of a tension in the model when matching both consumption (small volatility) and aggregate imports (large volatilty). Part of the volatility can be "directed into" exports which yields a smaller estimate of the suibstitution elasticity in the consumption basket (η_c) and thereby smaller expenditure switching effects than in Ramses. Second, the risk premium shock is estimated to be smaller and less persistent. Third, and most important is, however, that the instrument rule differs in Ramses II and Ramses. We do not allow for a direct response to the real exchange rate in Ramses II. This implicitly yields lower inflation and lower volatility in output due to more emphasis on these variables.

The **entrepreneurial wealth shock** drives up CPIF inflation, consumption, investment and output. The responses to output is very persistent. The key difference versus the investment-specific shock is that the wealth shock implies an increase in net worth (the stock market).

4.5. Variance Decomposition

Table A4a and A4b presents the variance decomposition at 1, 4, 8 and 40 quarters ahead computed at the posterior mode. The first thing to note is that in the short run, 1 quarter ahead, monetary policy shocks explain the largest part (55%) of the variations in the interest rate, domestic and imported markup shocks explain almost 80% of the variation in CPIF inflation, and stationary technology shocks are the main determinant (30%) behind variations in output growth. However, markup shocks, especially imports-for-exports markup shocks are also important (25%) for the short-run variation in output growth. At longer horizons, 8 and 40 quarters, the picture is more dispersed and technology shocks, both unit-root and stationary shocks have a larger impact on all variables (nominal as well as real).

Unemployment is predetermined (since we only allow for exogenous layoffs), so the measurement error explains 100% of the variation one-step ahead. At longer horizons, the labor preference shock explains slightly below 20% of changes in the unemployment series, but technology shocks as well as markup shocks are also main factors explaining the development in the labor market. For hours worked also consumption preference shocks play an important role. Note that markup shocks can be important also at longer horizons even if they are i.i.d.

The shock related specifically to the financial block, that is the entrepreneurial wealth shock, explains 35-45% of the variation in investment growth and about 45% of the variation in the spread difference at the different horizons, but only about 3% of the variation in output growth. Notice, however, that the entrepreneurial wealth shock has a larger impact on the *level* of output, where it explains about 10% of the variance (not shown). As a side note; also during the financial crises, following the collapse of Lehman Brothers, shocks in the financial sector play only a minor role in explaining the large fall in output. The model instead explains this with a combination of foreign disturbances, export markup shocks, risk premium shocks and to a small extent, the entrepreneurial wealth shock.

The investment specific technology shock is still important in explaining fluctuations in investment growth as well as changes in the spread. It accounts for about 30-40% of the variation in investment and 15% of the variation in the spread. CTW reports that the investment technology shock is "crowded out" by the entrepreneurial shock, which holds true for the *level* of investment but not the growth rate.

It should be noted that the unit-root technology shock stands out as very important for the foreign variables, in particular the foreign interest rate. The permanent technology shock accounts for 55% of the variation in the foreign interest rate at 8 quarters horizon and the foreign output shock explains almost all of the rest (42%). The foreign interest rate is, hence, predominantly driven by movements in foreign output rather than the foreign "policy shock".

4.6. Forecasts

In Figure F, the recursive model forecasts with data up to and including $2003Q4 - 2008Q2$ are plotted against actual data. It should be noted that the model is not reestimated and that the projections are *in-of-sample* forecasts 1-12 quarters ahead based on the same posterior mode vector (see Tables A1 and A2). The model forecasts capture output growth, CPIF inflation, and in particular investment growth relatively well. The model also seems to capture the main movements in unemployment well. On the other hand, the model tends to overestimate the nominal interest rate. This is, however, not specific to Ramses II but was also the case for the old model (Ramses).

If one compares the (in-of-sample) forecast accuracy of Ramses II and Ramses, the root mean square error for the interest rate is in fact smaller for Ramses II (see Figure I). In terms of hours and CPIF inflation, Ramses II also does better for all horizons. The models have more or less the same accuracy for GDP growth, whereas Ramses II forecasts for the real exchange rate are slightly worse, at least for the very short horizons (1 to 4 quarters ahead).

4.7. Level data on unemployment

The model is estimated matching changes in unemployment (first differenced data). However, since the sample mean in the data is higher than our steady-state calibration (8.13% against 7%), this implies that the model Kalman-filters out an estimate of the unemployment level that is roughly 1% lower than the acutal unemployment level in the data. This also has consequences for many other unobserved levels, such as capital, investment, consumption, output etc., in the model. In the recent policy rounds the model has, in contrast to the strategy during estimation, been fed with the unemployment level as an observable variable Since the (observed) hours gap is the same in both cases, equation (B.94) implies that hours per employee (i.e., the intensity) must move around to compensate for the change in (un)employment. A different estimate of intensity, in turn, affects the consumption euler equation and we get, for example, another estimate of the consumption level in the model. To illustrate this, Figure G shows the two-sided (smoothed) Kalman filtered estimates of some of the state variables in the model when we use the unemployment growth or the unemployment level as an observable variable.[21] Notice that many of the level series are not centered around zero for the level code, which should be taken into account when analyzing the current state of the economy.

5. Conclusion

This paper describes Ramses II, the dynamic stochastic general equilibrium (DSGE) model currently in use at the Monetary Policy Department of Sveriges Riksbank. The model is used to produce macroeconomic forecasts, to construct alternative scenarios, and for monetary policy analysis. The model was initially developed by Christiano, Trabandt, and Walentin (2011), but the current version of the model differs from CTW in some respects.

Compared with the earlier DSGE model at the Riksbank, the Ramses model developed by Adolfson, Laséen, Lindé and Villani (2008), Ramses II differs in three important respects. First,

[21] The data used in this exercise is from Monetary Policy Report 2010:2.

financial frictions are introduced in the accumulation of capital, following Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2003, 2008). Second, the labor market block includes search and matching frictions following Gertler, Sala, and Trigari (2008). Third, imported goods are used for exports as well as for consumption and investment.

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A. Tables and Figures

		Prior		Posterior	
	Distr.	Mean	S.d.	Mode	S.d.
ξ_d	β	0.750	0.075	0.843	0.025
ξ_x	β	0.750	0.075	0.783	0.034
ξ_{mc}	β	0.750	0.075	0.828	0.026
ξ_{mi}	β	0.750	0.075	0.742	0.036
ξ_{mx}	β	0.750	0.150	0.735	0.050
κ	β	0.500	0.150	0.122	0.048
κ_w	β	0.500	0.150	0.343	0.137
σ_L	Γ	2.000	0.500	2.531	0.463
b	β	0.650	0.150	0.539	0.097
S''	Γ	8.000	2.000	2.090	0.453
ρ_R	β	0.850	0.100	0.833	0.020
r_π	N	1.700	0.150	1.733	0.093
$r_{\Delta\pi}$	N	0.3	0.1	0.098	0.033
r_y	N	0.125	0.05	0.051	0.027
$r_{\Delta y}$	N	0.05	0.025	0.103	0.024
η_x	Γ	1.500	0.250	1.216	0.179
η_c	Γ	1.500	0.250	1.413	0.141
η_i	Γ	1.500	0.250	1.466	0.167
η_f	Γ	1.500	0.250	1.543	0.183
$\tilde{\phi}_s$	Γ	0.500	0.150	0.300	0.064
μ	β	0.330	0.100	0.465	0.095
$recshare, \%$	Γ	0.100	0.075	0.094	0.034
$bshare$	β	0.750	0.075	0.967	0.010
ρ_{μ_z}	β	0.500	0.150	0.926	0.036
ρ_ε	β	0.850	0.075	0.942	0.018
ρ_Υ	β	0.850	0.075	0.444	0.080
ρ_{ζ^c}	β	0.850	0.075	0.824	0.066
ρ_{ζ^h}	β	0.850	0.075	0.900	0.033
$\rho_{\tilde{\phi}}$	β	0.850	0.075	0.721	0.064
ρ_g	β	0.850	0.075	0.947	0.032
ρ_γ	β	0.850	0.075	0.830	0.056
$a11$	N	0.500	0.500	1.041	0.045
$a22$	N	0.000	0.500	-0.089	0.169
$a33$	N	0.500	0.500	0.460	0.094
$a12$	N	0.000	0.500	-0.115	0.261
$a13$	N	0.000	0.500	-0.611	0.202
$a21$	N	0.000	0.500	0.127	0.052
$a23$	N	0.000	0.500	-0.143	0.250
$a24$	N	0.000	0.500	-0.085	0.302
$a31$	N	0.000	0.500	0.100	0.022
$a32$	N	0.000	0.500	0.050	0.033
$a34$	N	0.000	0.500	0.526	0.140
$c21$	N	0.000	0.500	-0.100	0.123
$c31$	N	0.000	0.500	0.036	0.024
$c32$	N	0.000	0.500	-0.016	0.033
$c24$	N	0.000	0.500	-0.424	0.432
$c34$	N	0.000	0.500	0.390	0.126

Table A1. Estimation results. Parameters.

	Prior			Posterior	
	Distr.	Mean	S.d.	Mode	S.d.
$100\sigma_{\mu_z}$	Inv-r	0.15	Inf	0.078	0.019
$100\sigma_{\varepsilon}$	Inv-r	0.50	Inf	0.418	0.043
$10\sigma_{\Upsilon}$	Inv-r	0.50	Inf	0.274	0.060
$10\sigma_{\zeta^c}$	Inv-r	0.15	Inf	0.140	0.029
$10\sigma_{\zeta^h}$	Inv-r	0.15	Inf	0.138	0.036
$100\sigma_{\tilde{\phi}}$	Inv-r	0.15	Inf	0.394	0.067
$100\sigma_{\varepsilon_R}$	Inv-r	0.15	Inf	0.075	0.010
$100\sigma_g$	Inv-r	0.50	Inf	0.545	0.044
τ^d	Inv-r	0.15	Inf	0.148	0.048
τ^x	Inv-r	0.15	Inf	0.169	0.053
τ^{mc}	Inv-r	0.15	Inf	0.221	0.064
τ^{mi}	Inv-r	0.15	Inf	0.101	0.032
τ^{mx}	Inv-r	0.15	Inf	0.609	0.279
$100\sigma_{\gamma}$	Inv-r	0.50	Inf	0.212	0.031
$100\sigma_{y^*}$	Inv-r	0.50	Inf	0.268	0.024
$100\sigma_{\pi^*}$	Inv-r	0.50	Inf	0.181	0.020
$1000\sigma_{R^*}$	Inv-r	0.50	Inf	0.192	0.042

Table A2. Estimation results. Standard deviation of shocks.

	Mean		Standard dev.		Structural explanation
	Data	Model	Data	Model	$1-\frac{\text{var(measure error)}}{\text{var(data)}}$
CPIF inflation	1.65	2.00	1.36	1.61	0.94
Domestic inflation	1.55	2.00	1.80	1.76	0.91
Invest. inflation	1.42	1.85	2.16	2.30	0.98
Nom. intrest rate	3.82	4.14	1.71	1.07	1
GDP growth	0.59	0.52	0.53	0.58	0.95
Real wage growth	0.68	0.52	0.72	0.55	0.70
Consumption growth	0.49	0.52	0.67	0.76	0.96
Investment growth	1.08	0.56	2.19	2.64	0.95
Gov. cons growth*	0.17	0.52	0.64	0.62	0.98
Import growth*	1.37	0.52	1.82	1.48	0.84
Export growth*	1.46	0.52	1.74	1.58	0.91
Total hours	0.19	0	1.56	1.53	0.96
Real exchange rate growth	-0.03	0	2.23	1.93	0.96
Spread growth	-0.70	0	11.03	12.25	0.98
Unemployment growth	-1.11	0	3.50	3.35	0.91
Foreign GDP growth	0.57	0.52	0.29	0.34	0.98
Foreign inflation	1.89	2.0	0.85	0.99	0.99
Foreign interest rate	3.89	4.14	1.03	0.85	1

*The trend above(under) the growth rate in output (0.59) is taken out before estimation.

Table A3: First and second moments in the data and in the model (in percent), and importance of measurement errors

Shock	R	ΔY	π_c	ΔC	ΔI	ΔX	ΔM	ΔG	H	ΔW	Δq	ΔU	Δspread	π d	π i	ΔY*	π*	R*
	1 quarter																	
Unit-root tech.	0,00	0,02	0,02	0,08	0,00	0,01	0,02	0,02	0,00	0,01	0,03	0,00	0,01	0,01	0,03	0,07	0,03	0,66
Stationary tech.	0,10	0,27	0,01	0,05	0,01	0,00	0,02	0,00	0,14	0,01	0,03	0,00	0,01	0,03	0,00	0,00	0,00	0,00
Investment tech.	0,01	0,00	0,00	0,00	0,39	0,00	0,07	0,00	0,01	0,00	0,00	0,00	0,15	0,00	0,00	0,00	0,00	0,00
Consumption pref	0,03	0,06	0,00	0,66	0,00	0,00	0,06	0,00	0,05	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Labor pref	0,04	0,06	0,01	0,00	0,01	0,00	0,03	0,00	0,09	0,02	0,00	0,00	0,02	0,02	0,00	0,00	0,00	0,00
Monetary policy	0,55	0,02	0,01	0,02	0,07	0,00	0,02	0,00	0,00	0,00	0,03	0,00	0,18	0,00	0,01	0,00	0,00	0,00
Gov. consumption	0,01	0,03	0,00	0,00	0,00	0,00	0,00	0,86	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Domestic markup	0,01	0,05	0,38	0,02	0,00	0,00	0,00	0,00	0,03	0,69	0,04	0,00	0,00	0,81	0,07	0,00	0,00	0,00
Export markup	0,04	0,10	0,00	0,00	0,00	0,69	0,19	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Import cons. markup	0,09	0,00	0,45	0,03	0,01	0,00	0,04	0,00	0,00	0,00	0,08	0,00	0,04	0,00	0,00	0,00	0,00	0,00
Import invest. markup	0,00	0,01	0,00	0,00	0,01	0,00	0,02	0,00	0,01	0,00	0,00	0,00	0,03	0,00	0,65	0,00	0,00	0,00
Imp-for-exp markup	0,08	0,24	0,00	0,00	0,00	0,06	0,29	0,00	0,13	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Risk premium	0,02	0,02	0,04	0,02	0,03	0,03	0,01	0,00	0,01	0,00	0,61	0,00	0,01	0,00	0,11	0,00	0,00	0,00
Entrepreneurial wealth	0,00	0,02	0,00	0,00	0,35	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,43	0,00	0,00	0,00	0,00	0,00
Foreign	0,01	0,01	0,00	0,00	0,00	0,05	0,01	0,00	0,01	0,00	0,01	0,00	0,00	0,00	0,01	0,81	0,85	0,34
Measurement errors	*0,00*	*0,11*	*0,09*	*0,10*	*0,12*	*0,15*	*0,19*	*0,12*	*0,46*	*0,25*	*0,16*	*1,00*	*0,09*	*0,12*	*0,12*	*0,12*	*0,12*	*0,00*
	4 quarters																	
Unit-root tech.	0,09	0,06	0,04	0,09	0,01	0,02	0,02	0,05	0,02	0,11	0,03	0,13	0,01	0,02	0,05	0,12	0,04	0,66
Stationary tech.	0,07	0,23	0,01	0,06	0,01	0,01	0,02	0,00	0,12	0,02	0,03	0,02	0,01	0,06	0,00	0,00	0,00	0,00
Investment tech.	0,01	0,01	0,00	0,00	0,30	0,00	0,07	0,00	0,02	0,00	0,00	0,02	0,15	0,00	0,00	0,00	0,00	0,00
Consumption pref	0,05	0,06	0,00	0,64	0,01	0,00	0,06	0,00	0,13	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00
Labor pref	0,01	0,05	0,02	0,00	0,01	0,00	0,03	0,00	0,12	0,07	0,00	0,12	0,02	0,03	0,00	0,00	0,00	0,00
Monetary policy	0,20	0,02	0,02	0,02	0,07	0,00	0,02	0,00	0,02	0,01	0,03	0,08	0,19	0,01	0,02	0,00	0,00	0,00
Gov. consumption	0,01	0,02	0,00	0,00	0,00	0,00	0,00	0,83	0,03	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00
Domestic markup	0,09	0,05	0,34	0,02	0,00	0,00	0,00	0,00	0,10	0,57	0,04	0,25	0,00	0,74	0,06	0,00	0,00	0,00
Export markup	0,02	0,09	0,00	0,00	0,00	0,66	0,19	0,00	0,07	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00	0,00
Import cons. markup	0,21	0,01	0,42	0,03	0,01	0,00	0,04	0,00	0,00	0,02	0,08	0,01	0,04	0,00	0,00	0,00	0,00	0,00
Import invest. markup	0,00	0,01	0,00	0,00	0,01	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,59	0,00	0,00	0,00
Imp-for-exp markup	0,03	0,24	0,00	0,00	0,00	0,08	0,28	0,00	0,10	0,01	0,01	0,09	0,00	0,00	0,00	0,00	0,00	0,00
Risk premium	0,16	0,02	0,06	0,02	0,04	0,03	0,02	0,00	0,05	0,00	0,62	0,03	0,01	0,01	0,15	0,00	0,00	0,00
Entrepreneurial wealth	0,01	0,03	0,00	0,00	0,45	0,00	0,04	0,00	0,01	0,00	0,00	0,01	0,42	0,00	0,00	0,00	0,00	0,00
Foreign	0,03	0,01	0,01	0,01	0,01	0,05	0,01	0,00	0,01	0,00	0,01	0,01	0,00	0,00	0,01	0,77	0,85	0,34
Measurement errors	*0,00*	*0,09*	*0,08*	*0,09*	*0,08*	*0,13*	*0,17*	*0,11*	*0,21*	*0,19*	*0,14*	*0,13*	*0,08*	*0,11*	*0,10*	*0,12*	*0,11*	*0,00*

Table A4a: Variance decomposition (at 1 and 4 quarters horizon)

Table A4b: Variance decomposition (at 8 and 40 quarters horizon)

Shock	R	ΔY	π_c	ΔC	ΔI	ΔX	ΔM	ΔG	H	ΔW	Δq	ΔU	Δspread	π d	π i	ΔY*	π*	R*
									8 quarters									
Unit-root tech.	0,14	0,06	0,04	0,09	0,01	0,02	0,02	0,08	0,03	0,12	0,04	0,12	0,01	0,02	0,05	0,13	0,09	0,55
Stationary tech.	0,08	0,23	0,02	0,06	0,01	0,01	0,02	0,00	0,09	0,03	0,03	0,05	0,01	0,07	0,00	0,00	0,00	0,00
Investment tech.	0,01	0,01	0,00	0,00	0,32	0,00	0,07	0,00	0,01	0,00	0,00	0,02	0,15	0,00	0,00	0,00	0,00	0,00
Consumption pref	0,06	0,06	0,00	0,65	0,00	0,00	0,07	0,00	0,15	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00
Labor perf	0,02	0,05	0,02	0,00	0,01	0,00	0,03	0,00	0,16	0,08	0,00	0,13	0,02	0,03	0,01	0,00	0,00	0,00
Monetary policy	0,15	0,02	0,02	0,02	0,07	0,00	0,02	0,00	0,04	0,01	0,03	0,08	0,19	0,01	0,02	0,00	0,00	0,00
Gov. consumption	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,81	0,03	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00
Domestic markup	0,07	0,05	0,33	0,02	0,00	0,00	0,00	0,00	0,12	0,55	0,04	0,23	0,00	0,73	0,06	0,00	0,00	0,00
Export markup	0,02	0,09	0,00	0,00	0,00	0,66	0,18	0,00	0,06	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00	0,00
Import cons. markup	0,15	0,01	0,41	0,03	0,01	0,00	0,05	0,00	0,00	0,02	0,08	0,01	0,04	0,00	0,01	0,00	0,00	0,00
Import invest. markup	0,00	0,01	0,00	0,00	0,01	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,58	0,00	0,00	0,00
Imp-for-exp markup	0,03	0,24	0,00	0,00	0,00	0,09	0,28	0,00	0,07	0,01	0,01	0,09	0,00	0,00	0,00	0,00	0,00	0,00
Risk premium	0,19	0,03	0,06	0,03	0,05	0,04	0,02	0,00	0,05	0,00	0,63	0,03	0,01	0,01	0,16	0,00	0,00	0,00
Entrepreneurial wealth	0,05	0,03	0,01	0,00	0,43	0,00	0,04	0,00	0,02	0,00	0,00	0,01	0,43	0,01	0,01	0,00	0,00	0,00
Foreign	0,05	0,01	0,01	0,01	0,01	0,05	0,01	0,00	0,01	0,00	0,01	0,01	0,00	0,00	0,01	0,76	0,81	0,45
Measurement errors	*0,00*	*0,08*	*0,07*	*0,08*	*0,08*	*0,12*	*0,16*	*0,11*	*0,16*	*0,18*	*0,13*	*0,12*	*0,08*	*0,11*	*0,09*	*0,11*	*0,10*	*0,00*
									40 quarters									
Unit-root tech.	0,12	0,08	0,04	0,09	0,02	0,02	0,03	0,11	0,04	0,14	0,04	0,13	0,01	0,02	0,05	0,24	0,30	0,58
Stationary tech.	0,13	0,22	0,03	0,06	0,00	0,01	0,02	0,00	0,12	0,03	0,03	0,06	0,01	0,07	0,02	0,00	0,00	0,00
Investment tech.	0,01	0,01	0,00	0,00	0,29	0,00	0,07	0,00	0,01	0,00	0,00	0,02	0,15	0,00	0,00	0,00	0,00	0,00
Consumption pref	0,06	0,06	0,00	0,65	0,00	0,00	0,07	0,00	0,13	0,00	0,00	0,06	0,00	0,00	0,00	0,00	0,00	0,00
Labor perf	0,03	0,05	0,02	0,01	0,01	0,00	0,03	0,00	0,22	0,08	0,00	0,13	0,02	0,03	0,01	0,00	0,00	0,00
Monetary policy	0,11	0,02	0,02	0,02	0,06	0,00	0,02	0,00	0,03	0,01	0,03	0,08	0,19	0,01	0,02	0,00	0,00	0,00
Gov. consumption	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,79	0,04	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00	0,00
Domestic markup	0,05	0,05	0,32	0,02	0,00	0,00	0,00	0,00	0,11	0,53	0,04	0,23	0,00	0,72	0,06	0,00	0,00	0,00
Export markup	0,01	0,08	0,00	0,00	0,00	0,64	0,18	0,00	0,04	0,00	0,00	0,04	0,00	0,00	0,00	0,00	0,00	0,00
Import cons. markup	0,12	0,01	0,40	0,03	0,01	0,00	0,04	0,00	0,00	0,02	0,08	0,01	0,04	0,00	0,01	0,00	0,00	0,00
Import invest. markup	0,00	0,01	0,00	0,00	0,01	0,00	0,02	0,00	0,00	0,00	0,00	0,00	0,03	0,00	0,56	0,00	0,00	0,00
Imp-for-exp markup	0,03	0,23	0,00	0,00	0,00	0,09	0,27	0,00	0,06	0,01	0,01	0,08	0,00	0,00	0,00	0,00	0,00	0,00
Risk premium	0,15	0,03	0,06	0,03	0,05	0,04	0,03	0,00	0,04	0,00	0,63	0,03	0,01	0,01	0,16	0,00	0,00	0,00
Entrepreneurial wealth	0,12	0,03	0,02	0,01	0,46	0,00	0,05	0,00	0,03	0,00	0,00	0,01	0,43	0,01	0,01	0,00	0,00	0,00
Foreign	0,05	0,01	0,01	0,01	0,01	0,05	0,01	0,00	0,01	0,00	0,01	0,01	0,00	0,01	0,02	0,66	0,62	0,42
Measurement errors	*0,00*	*0,08*	*0,07*	*0,08*	*0,07*	*0,12*	*0,15*	*0,11*	*0,12*	*0,17*	*0,13*	*0,11*	*0,08*	*0,10*	*0,09*	*0,10*	*0,07*	*0,00*

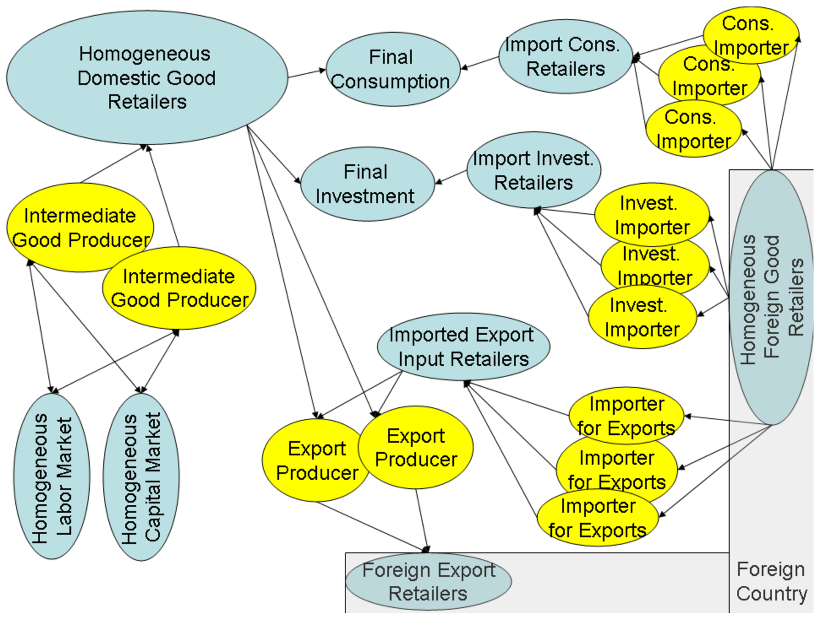


Figure A. Graphical illustration of the goods production part of the model.

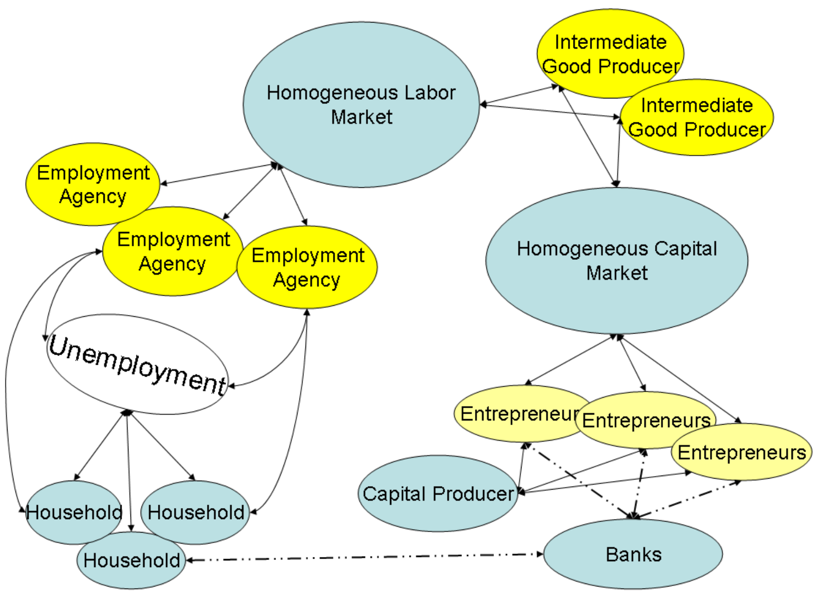


Figure B. Graphical illustration of the labor and capital markets of the model.

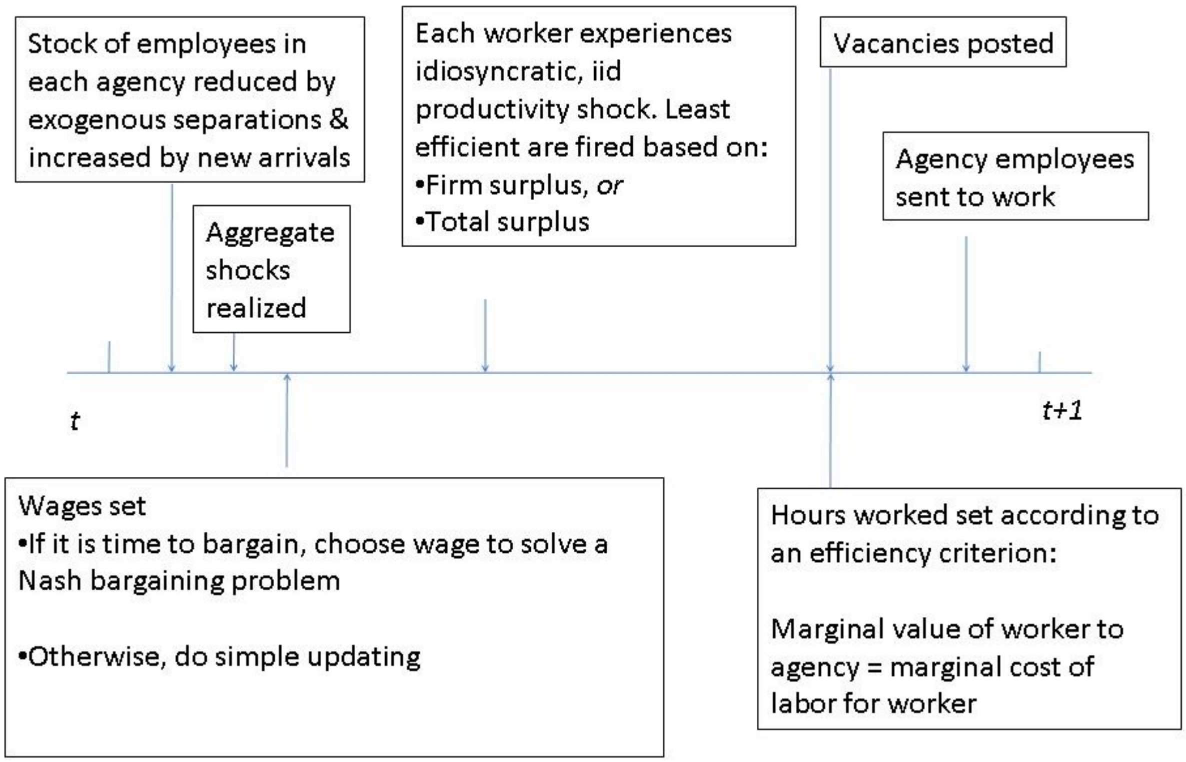


Figure C. Timeline for labor market in employment friction model.

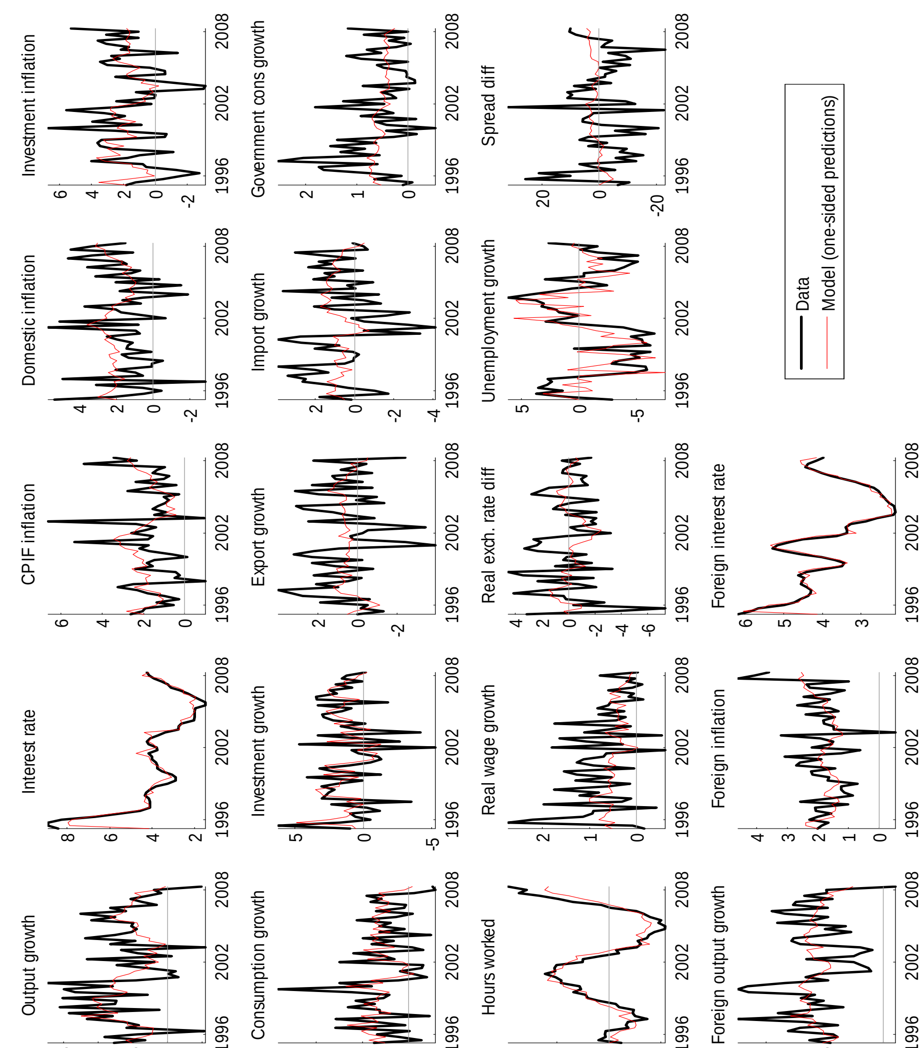


Figure D. Data (thick black) and one-sided Kalman-filtered predicitons (thin red).

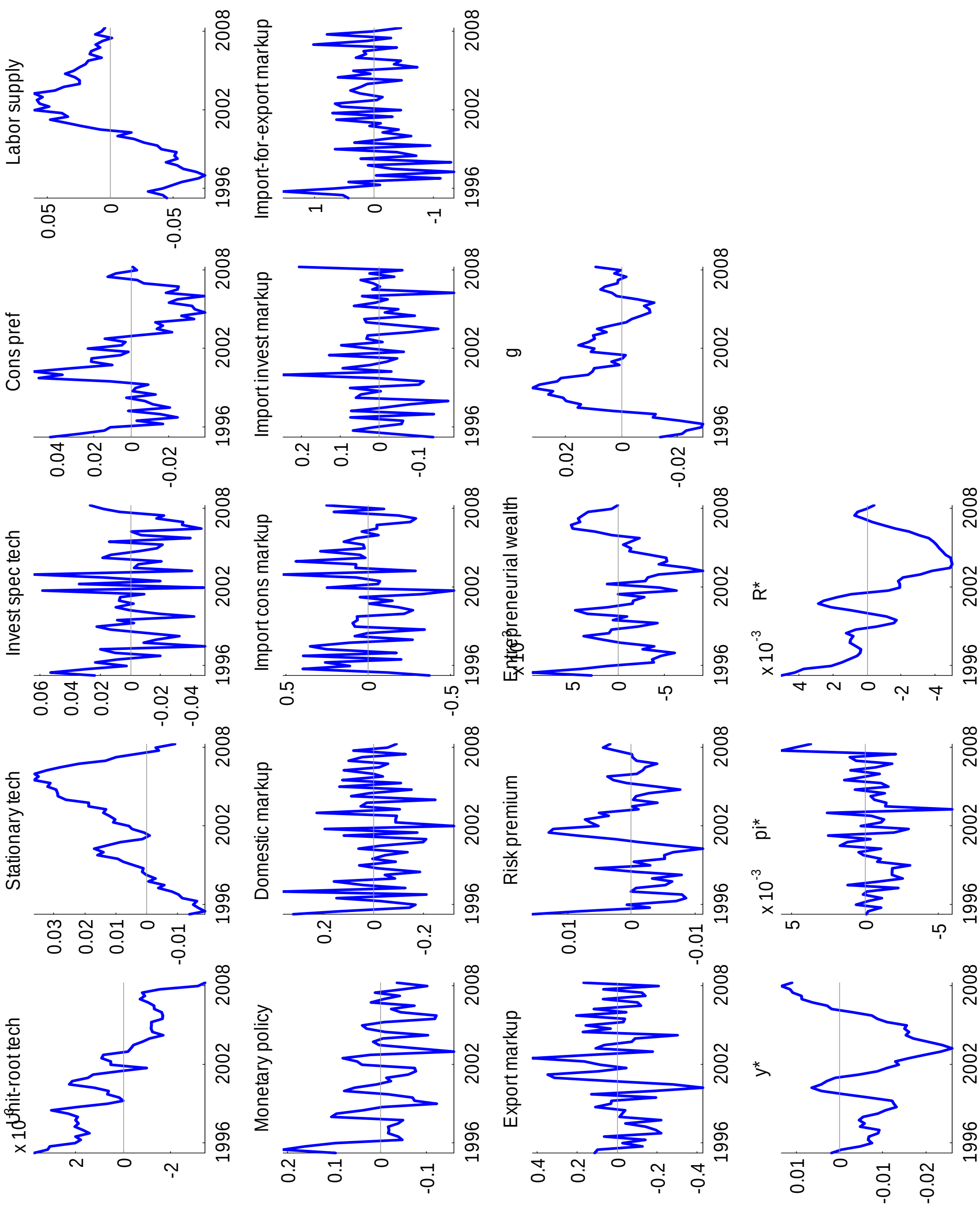


Figure E. Smoothed (two-sided Kalman filtered) estimates of the shocks (deviations from steady-state)

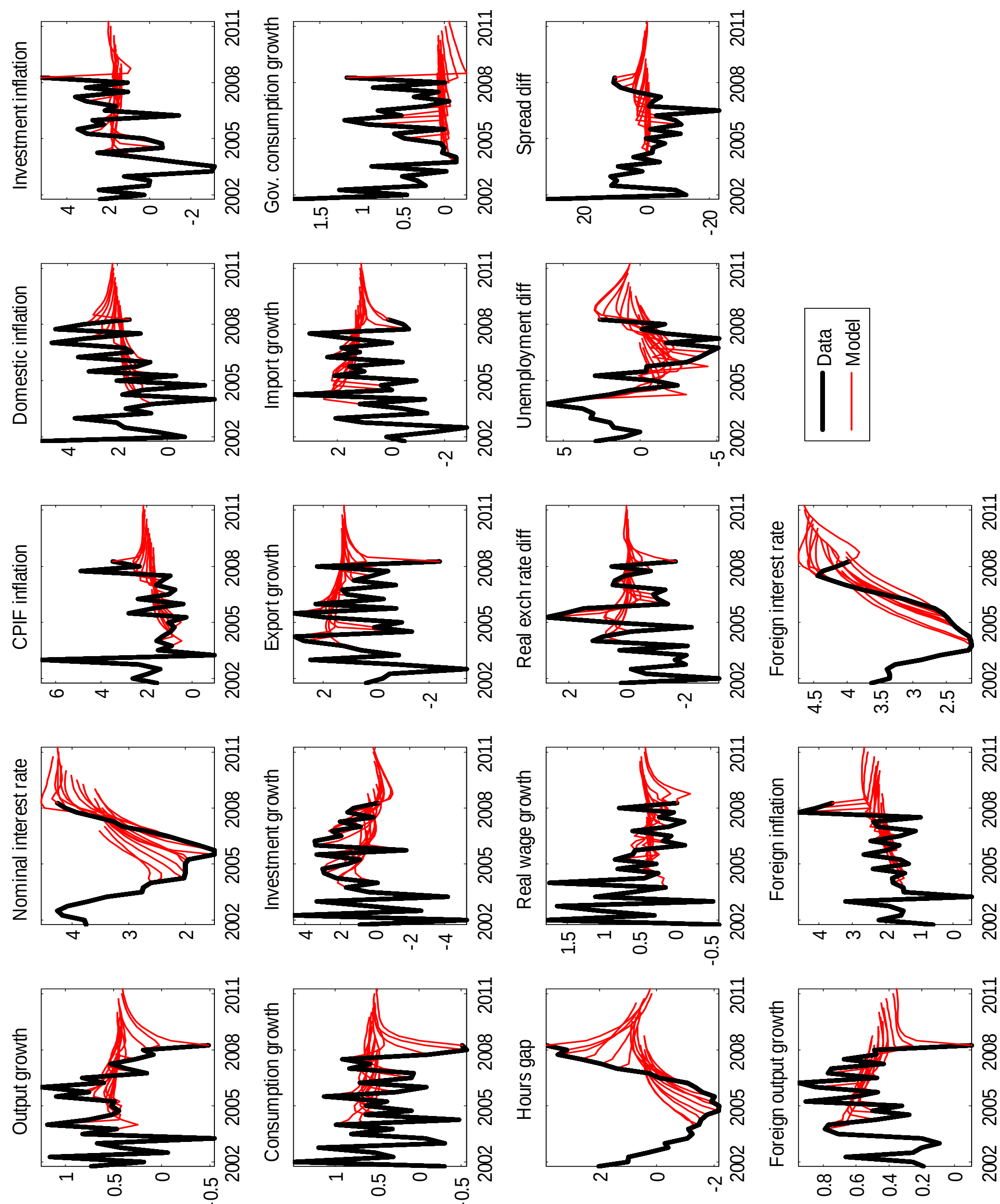


Figure F. Actual data (thick line) and (in-of-sample) model forecasts (thin line) with data up to 2003Q4-2008Q2

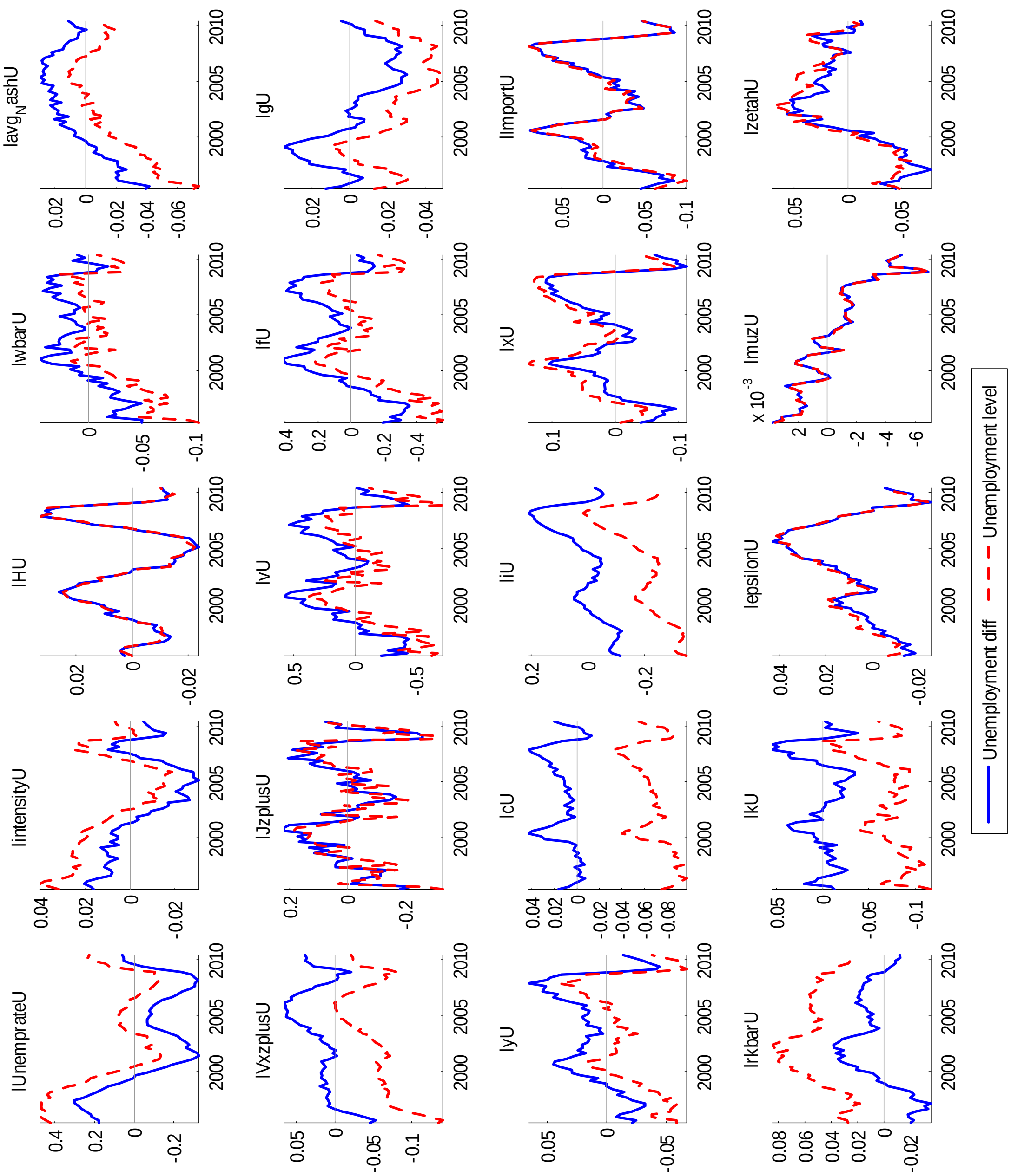


Figure G. Two-sided (smoothed) Kalman filtered estimates of some key state variables when either unemployment growth or unemployment is used as observable.

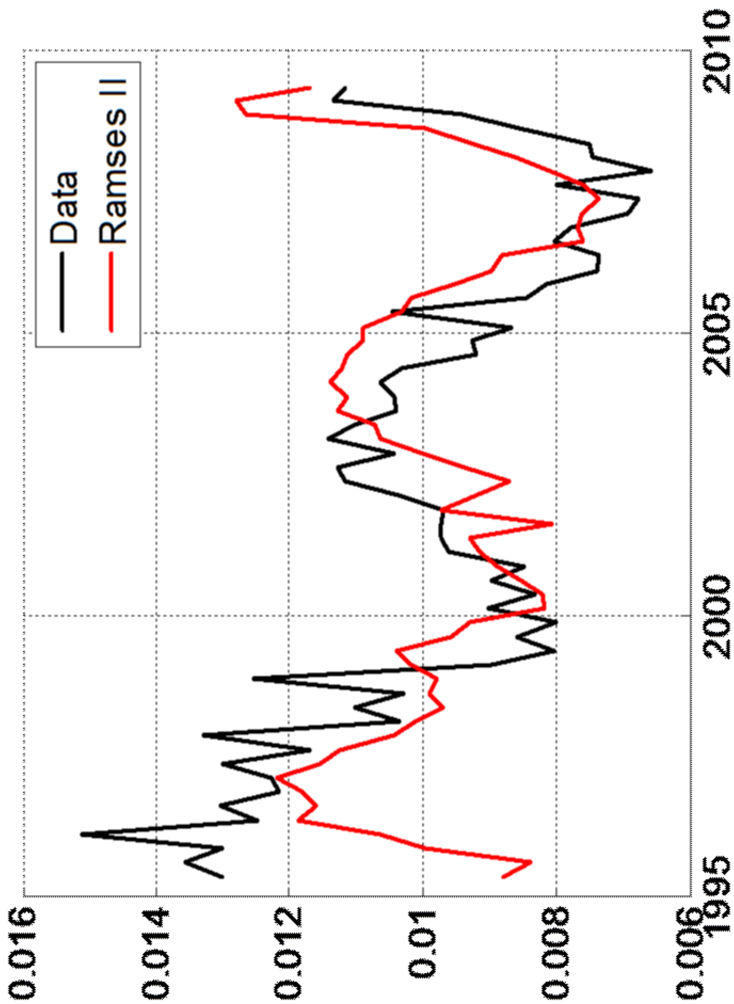


Figure H. Smoothed (two-sided Kalman filtered) estimate of the bankruptcy rate in Ramses II and actual bankruptcy data (black line) from UC AB - Sweden's largest business and credit information agency.

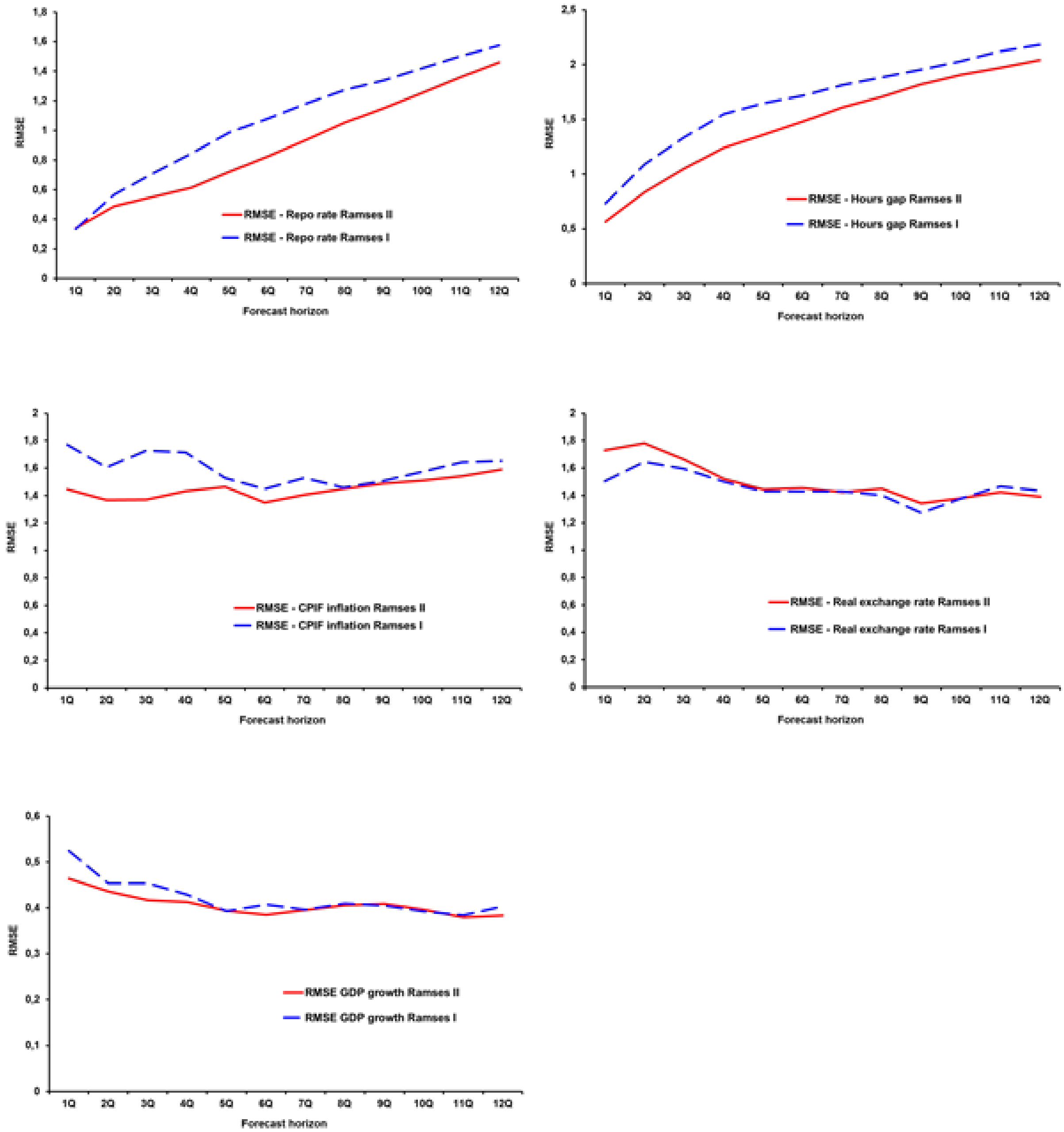


Figure I. Root Mean Squared Error

B. Appendix

B.1. Scaling of Variables

We adopt the following scaling of variables. The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{B.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+\Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods and C_t are final consumption goods) are scaled by z_t^+. Government consumption, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by z_t^+ to induce stationarity. $\tilde{P}_t$ is the within-sector relative price of a good. w_t denotes the ratio between the (Nash) wage paid to workers $\tilde{W}_t$ and the "rental rate of homogenous labor" W_t in the labor market model. Finally, the expected discounted future surplus of a match to an employment agency, D_t^j is scaled like most other nominal variables. Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ i_t^m = \frac{I_t^m}{z_t^+} \\ c_t^m &= \frac{C_t^m}{z_t^+}, c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ a_t \equiv \frac{S_t A_{t+1}^*}{P_t z_t^+}, \\ x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ (y_t =)\, \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t},\ D_{z^+,t}^j \equiv \frac{D_t^j}{P_t z_t^+}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the domestic homogeneous good. We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},$$
$$\pi_t^i = \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'CPI'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$\begin{aligned} p_t^{m,x} &= \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t}, \\ p_t^x &= \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t}. \end{aligned} \tag{B.2}$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$, and in the production of final investment goods in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

We denote the real exchange rate by q_t :

$$q_t = \frac{S_t P_t^*}{P_t^c}. \tag{B.3}$$

B.2. Functional forms

We adopt the following functional form for a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b (1 - \sigma_a) u + \sigma_b ((\sigma_a/2) - 1), \tag{B.4}$$

where σ_a and σ_b are the parameters of this function.

The functional form for investment adjustment costs, as well as its derivatives are:

$$\begin{aligned} \tilde{S}(x) &= \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}\,(x - \mu_{z+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}\,(x - \mu_{z+}\mu_\Psi)\right] - 2\right\} \\ &= 0,\ x = \mu_{z+}\mu_\Psi. \end{aligned} \tag{B.5}$$

$$
\begin{aligned}
\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-\mu_{z+}\mu_{\Psi}\right)\right]-\exp\left[-\sqrt{\tilde{S}''}\left(x-\mu_{z+}\mu_{\Psi}\right)\right]\right\} \\
&= 0,\ x=\mu_{z+}\mu_{\Psi}.
\end{aligned} \tag{B.6}
$$

$$
\begin{aligned}
\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-\mu_{z+}\mu_{\Psi}\right)\right]+\exp\left[-\sqrt{\tilde{S}''}\left(x-\mu_{z+}\mu_{\Psi}\right)\right]\right\} \\
&= \tilde{S}'',\ x=\mu_{z+}\mu_{\Psi}.
\end{aligned}
$$

In the employment friction model we assume a log-normal distribution for idiosyncratic productivities of workers. This implies the following:

$$
\mathcal{E}\left(\bar{a}_t^j;\sigma_{a,t}\right)=\int_{\bar{a}_t^j}^{\infty} ad\mathcal{F}\left(a;\sigma_{a,t}\right)=1-prob\left[v<\frac{\log\left(\bar{a}_t^j\right)+\frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}}-\sigma_{a,t}\right], \tag{B.7}
$$

where *prob* refers to the standard normal distribution and eq. (B.7) simply is eq. (2.58) spelled out under this distributional assumption. We similarly spell out eq. (2.59):

$$
\begin{aligned}
\mathcal{F}\left(\bar{a}^j;\sigma_a\right) &= \int_0^{\bar{a}^j} d\mathcal{F}(a;\sigma_a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma}} \exp^{\frac{-v^2}{2}}dv \\
&= prob\left[v<\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma_a}\right].
\end{aligned} \tag{B.8}
$$

B.3. Baseline Model

B.3.1. First order conditions for domestic homogenous good price setting

Substituting eq. (2.7) into eq. (2.6) to obtain, after rearranging,

$$
E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\{\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d-1}}-mc_{t+j}\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d-1}}\},
$$

or,

$$
E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\{\left(X_{t,j}\tilde{p}_t\right)^{1-\frac{\lambda_d}{\lambda_d-1}}-mc_{t+j}\left(X_{t,j}\tilde{p}_t\right)^{\frac{-\lambda_d}{\lambda_d-1}}\},
$$

where

$$
\frac{P_{i,t+j}}{P_{t+j}}=X_{t,j}\tilde{p}_t,\ X_{t,j}\equiv\left\{\begin{array}{c}\frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}},\ j>0\\ 1,\ j=0.\end{array}\right. .
$$

The i^{th} firm maximizes profits by choice of the within-sector relative price $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling we obtain:

$$
E_t\sum_{j=0}^{\infty}\left(\beta\xi_d\right)^j A_{t+j}\left[\tilde{p}_tX_{t,j}-\lambda_d mc_{t+j}\right]=0,
$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j}\tilde{y}_{t+j}X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} \left(\beta\xi_d\right)^j A_{t+j}\lambda_d mc_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta\xi_d\right)^j A_{t+j}X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$\begin{aligned} K_t^d &\equiv E_t \sum_{j=0}^{\infty} \left(\beta\xi_d\right)^j A_{t+j}\lambda_d mc_{t+j} \\ F_t^d &= E_t \sum_{j=0}^{\infty} \left(\beta\xi_d\right)^j A_{t+j}X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$\begin{aligned} E_t \left[\psi_{z^+,t}\tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d\right] &= 0 \\ E_t \left[\lambda_d\psi_{z^+,t}\tilde{y}_t mc_t + \beta\xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d\right] &= 0. \end{aligned}$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di\right]^{(1-\lambda_d)} \\ &= \left[\left(1-\xi_p\right)\tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p \left(\tilde{\pi}_{d,t}P_{t-1}\right)^{\frac{1}{1-\lambda_d}}\right]^{(1-\lambda_d)} \end{aligned} \tag{B.9}$$

After dividing by P_t and rearranging:

$$\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} = \left(\tilde{p}_t^d\right)^{\frac{1}{1-\lambda_d}}. \tag{B.10}$$

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogenous good are:[22]

$$E_t\left[\psi_{z^+,t}y_t+\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F_{t+1}^d-F_t^d\right] = 0 \tag{B.11}$$

$$E_t\left[\lambda_d\psi_{z^+,t}y_t mc_t+\beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}}K_{t+1}^d-K_t^d\right] = 0, \tag{B.12}$$

$$\mathring{p}_t=\left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d}+\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{B.13}$$

$$\left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{(1-\lambda_d)} = \frac{K_t^d}{F_t^d} \tag{B.14}$$

$$\tilde{\pi}_{d,t}\equiv(\pi_{t-1})^{\kappa_d}\left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d}(\breve{\pi})^{\varkappa_d} \tag{B.15}$$

B.3.2. First order conditions for export good price setting

$$E_t\left[\psi_{z^+,t}q_t p_t^c p_t^x x_t+\left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}}\beta\xi_x F_{x,t+1}-F_{x,t}\right] = 0 \tag{B.16}$$

$$E_t\left[\lambda_x\psi_{z^+,t}q_t p_t^c p_t^x x_t mc_t^x+\beta\xi_x\left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}}K_{x,t+1}-K_{x,t}\right] = 0, \tag{B.17}$$

$$\mathring{p}_t^x=\left[(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right)^{\lambda_x}+\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_x}{1-\lambda_x}}\right]^{\frac{1-\lambda_x}{\lambda_x}} \tag{B.18}$$

$$\left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{(1-\lambda_x)} = \frac{K_{x,t}}{F_{x,t}} \tag{B.19}$$

When we linearize around steady state and $\varkappa_{m,j}=0$, equations (B.16)-(B.19) reduce to:

[22] When we linearize about steady state and set $\varkappa_d=0$, we obtain,

$$\begin{aligned}\hat{\pi}_t-\widehat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1}-\widehat{\bar{\pi}}_{t+1}^c\right)+\frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1}-\widehat{\bar{\pi}}_t^c\right)\\ &\quad -\frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\widehat{\bar{\pi}}_t^c\\ &\quad +\frac{1}{1+\kappa_d\beta}\frac{\left(1-\beta\xi_d\right)\left(1-\xi_d\right)}{\xi_d}\widehat{mc}_t,\end{aligned}$$

where a hat indicates log-deviation from steady state.

$$\hat{\pi}_t^x = \frac{\beta}{1+\kappa_x\beta} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\kappa_x\beta} \hat{\pi}_{t-1}^x + \frac{1}{1+\kappa_x\beta} \frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x} \widehat{mc}_t^x,$$

where a hat over a variable indicates log deviation from steady state.

B.3.3. Demand for domestic inputs in export production

Integrating eq. (2.27):

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) \int_0^1 X_{i,t} di \tag{B.20}$$

$$= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t \frac{\int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di}{\left(P_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}}.$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$:

$$\mathring{P}_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di\right]^{\frac{\lambda_{x,t}-1}{-\lambda_{x,t}}}.$$

Then,

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} = \int_0^1 \left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di,$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} (1-\omega_x) X_t \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}, \tag{B.21}$$

where

$$\mathring{p}_t^x \equiv \frac{\mathring{P}_t^x}{P_t^x},$$

and the law of motion of $\mathring{p}_t^x$ is given in (B.18).

We now simplify (B.21). Rewriting the second equality in (2.24), we obtain:

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^c p_t^x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*}} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (B.21), we obtain:

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} Y_t^*$$

B.3.4. Demand for Imported Inputs in Export Production

After scaling expression (2.29), we obtain:

$$x_t^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x\right)^{-\eta_f} y_t^*. \tag{B.22}$$

B.3.5. First order conditions for import good price setting

$$E_t \left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t} \right] = 0 \tag{B.23}$$

$$E_t \left[\lambda_{m,j} \psi_{z^+,t} p_t^{m,j} mc_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t} \right] = 0, \tag{B.24}$$

$$\mathring{p}_t^{m,j} = \left[(1-\xi_{m,j}) \left(\frac{1-\xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}} \right)^{\lambda_{m,j}} + \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \mathring{p}_{t-1}^{m,j} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} \right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \tag{B.25}$$

$$\left[\frac{1-\xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}} \right]^{(1-\lambda_{m,j})} = \frac{K_{m,j,t}}{F_{m,j,t}}, \tag{B.26}$$

for $j = c, i, x$.[23] Here,

$$\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \\ i_t^m & j = i \end{cases}.$$

B.3.6. Household Consumption and Investment Decisions

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c \left(1+\tau_t^c\right) = 0. \tag{B.27}$$

[23] When we linearize around steady state and $\varkappa_{m,j} = 0$,

$$\begin{aligned} \hat{\pi}_t^{m,j} - \widehat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_{m,j}\beta} E_t \left(\hat{\pi}_{t+1}^{m,j} - \widehat{\bar{\pi}}_{t+1}^c \right) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta} \left(\hat{\pi}_{t-1}^{m,j} - \widehat{\bar{\pi}}_t^c \right) \\ &\quad - \frac{\kappa_{m,j}\beta\left(1-\rho_\pi\right)}{1+\kappa_{m,j}\beta} \widehat{\bar{\pi}}_t^c \\ &\quad + \frac{1}{1+\kappa_{m,j}\beta} \frac{\left(1-\beta\xi_{m,j}\right)\left(1-\xi_{m,j}\right)}{\xi_{m,j}} \widehat{mc}_t^{m,j}, \end{aligned}$$

To define the intertemporal Euler equation associated with the household's capital accumulation decision, we need to define the rate of return on a period t investment in a unit of physical capital, R^k_{t+1} :

$$R^k_{t+1} = \frac{\left(1-\tau^k_t\right)\left[u_{t+1}\bar{r}^k_{t+1} - \frac{p^i_{t+1}}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau^k_t \delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{B.28}$$

where it is convenient to recall

$$\frac{p^i_t}{\Psi_t}P_t = P^i_t,$$

the date t price of the homogeneous investment good. Here, $P_{k',t}$ denotes the price of a unit of newly installed physical capital, which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R^k_{t+1} represents the period $t+1$ payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets in (B.28) captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R^k_t in terms of scaled variables:

$$\begin{aligned} R^k_{t+1} &= \frac{P_{t+1}\Psi_{t+1}}{P_t\Psi_{t+1}} \frac{\left(1-\tau^k_t\right)\left[u_{t+1}\bar{r}^k_{t+1} - \frac{p^i_{t+1}}{\Psi_{t+1}}a(u_{t+1})\right] + (1-\delta)P_{k',t+1} + \tau^k_t \delta \frac{P_t}{P_{t+1}} P_{k',t}}{P_{k',t}} \\ &= \pi_{t+1} \frac{\left(1-\tau^k_t\right)\left[u_{t+1}\bar{r}^k_{t+1} - p^i_{t+1}a(u_{t+1})\right] + (1-\delta)\Psi_{t+1}P_{k',t+1} + \tau^k_t \delta \frac{P_t}{P_{t+1}} \Psi_{t+1} P_{k',t}}{\Psi_{t+1}P_{k',t}}. \end{aligned}$$

so that

$$R^k_{t+1} = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{\left(1-\tau^k_t\right)\left[u_{t+1}\bar{r}^k_{t+1} - p^i_{t+1}a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau^k_t \delta \frac{\mu_{\Psi,t+1}}{\pi_{t+1}} p_{k',t}}{p_{k',t}}. \tag{B.29}$$

Capital is a good hedge against inflation, except for the way depreciation is treated. A rise in inflation effectively raises the tax rate on capital because of the practice of valuing depreciation at historical cost. The first order condition for capital implies:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R^k_{t+1}}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{B.30}$$

We differentiate the Lagrangian representation of the household's problem as displayed in ALLV, with respect to I_t :

$$-\upsilon_t P^i_t + \omega_t \Upsilon_t F_1\left(I_t, I_{t-1}\right) + \beta\omega_{t+1}\Upsilon_{t+1}F_2\left(I_{t+1}, I_t\right) = 0,$$

where υ_t denotes the multiplier on the household's nominal budget constraint and ω_t denotes the multiplier on the capital accumulation technology. In addition, the price of capital is the ratio of these multipliers:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}.$$

Expressing the investment first order condition in terms of scaled variables,

$$
\begin{aligned}
-\frac{\psi_{z^+,t}}{z_t^+}\frac{p_t^i}{\Psi_t}+\upsilon_t P_t P_{k',t}\Upsilon_t\left[1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right]& \\
+\beta\upsilon_{t+1}P_{t+1}P_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)^2 &= 0.
\end{aligned}
$$

Now multiply by $z_t^+\Psi_t$

$$
\begin{aligned}
-\psi_{z^+,t}p_t^i+\psi_{z^+,t}p_{k',t}\Upsilon_t\left[1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right]& \qquad \text{(B.31)}\\
+\beta\psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{\Psi,t+1}\mu_{z^+,t+1} &= 0.
\end{aligned}
$$

Our first order condition for I_t appears to differ slightly from the first order condition in ALLV, equation (2.55), but the two actually coincide when we take into account the definition of f.

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = p_t^i a'(u_t),$$

or, in scaled terms,

$$\bar{r}_t^k = p_t^i a'(u_t). \qquad \text{(B.32)}$$

The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

B.3.7. Wage setting conditions in the baseline model

We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t=\left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}}dj\right]^{\lambda_w},\ 1\le\lambda_w<\infty,$$

where h_j denotes the j^{th} household supply of labor services. Households are, in the baseline version of the model, subject to Calvo wage setting frictions as in Erceg, Henderson and Levin (2000) (EHL). With probability $1-\xi_w$ the j^{th} household is able to reoptimize its wage and with probability ξ_w it sets its wage according to:

$$
\begin{aligned}
W_{j,t+1} &= \tilde{\pi}_{w,t+1}W_{j,t} && \text{(B.33)}\\
\tilde{\pi}_{w,t+1} &= (\pi_t^c)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)}(\breve{\pi})^{\varkappa_w}(\mu_{z^+})^{\vartheta_w}, && \text{(B.34)}
\end{aligned}
$$

where $\kappa_w,\varkappa_w,\vartheta_w,\kappa_w+\varkappa_w\in(0,1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{(h_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \right], \tag{B.35}$$

where τ_t^y is a tax on labor income and τ_t^w is a payroll tax. Also, υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}. \tag{B.36}$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$.

Substituting eq. (B.36) into the objective function eq. (B.35),

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L}$$
$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

It is convenient to recall the scaling of variables:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \ \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \ \tilde{y}_t = \frac{Y_t}{z_t^+}, \ w_t = \tilde{W}_t / W_t, \ z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Then,

$$\begin{aligned} \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\ &= \frac{W_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}, \end{aligned}$$

where

$$\begin{aligned} X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}, \ i > 0 \\ &= 1, \ i = 0. \end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi_t^c \cdots \pi_{t+i-1}^c \right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \cdots \bar{\pi}_{t+i}^c \right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i \right)^{\varkappa_w} \left(\mu_{z^+}^i \right)^{\vartheta_w}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}$$

In steady state,

$$
\begin{aligned}
X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w}\left(\bar{\pi}^i\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}^i\right)^{\varkappa_w}\left(\mu_{z^+}^i\right)^{\vartheta_w}}{\bar{\pi}^i\mu_{z^+}^i} \\
&= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w}\left(\mu_{z^+}^i\right)^{\vartheta_w-1} \\
&\to 0,
\end{aligned}
$$

in the no-indexing case, when $\breve{\pi}=1$, $\varkappa_w=1$ and $\vartheta_w=0$.

Simplifying using the scaling notation,

$$
\begin{aligned}
&E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\
&+\upsilon_{t+i}W_{t+i}\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],
\end{aligned}
$$

or,

$$
\begin{aligned}
&E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\
&+\psi_{z^+,t+i}w_t\bar{w}_tX_{t,i}\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],
\end{aligned}
$$

or,

$$
\begin{aligned}
&E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \\
&+\psi_{z^+,t+i}w_t^{1+\frac{\lambda_w}{1-\lambda_w}}\bar{w}_tX_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],
\end{aligned}
$$

Differentiating with respect to w_t,

$$
\begin{aligned}
&E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}\lambda_w\left(1+\sigma_L\right)w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)-1} \\
&+\psi_{z^+,t+i}w_t^{\frac{\lambda_w}{1-\lambda_w}}\bar{w}_tX_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}]=0
\end{aligned}
$$

Dividing and rearranging,

$$
\begin{aligned}
&E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L} \\
&+\frac{\psi_{z^+,t+i}}{\lambda_w}w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}}\bar{w}_tX_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}]=0
\end{aligned}
$$

Solving for the wage rate:

$$\begin{aligned}
w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}} \\
&= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}
\end{aligned}$$

where

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L} \\
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}.
\end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1}\right)^{1+\sigma_L} \\
&\quad + (\beta\xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\left(\pi_t^c\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2}\right)^{1+\sigma_L} \\
&\quad + ...
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t\beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \{\zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2}\right)^{1+\sigma_L} \zeta_{t+2}^h + ...\} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1},
\end{aligned}$$

using,

$$\pi_{w,t+1}=\frac{W_{t+1}}{W_t}=\frac{\bar{w}_{t+1}z_{t+1}^{+}P_{t+1}}{\bar{w}_t z_t^{+}P_t}=\frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \tag{B.37}$$

Also,

$$\begin{aligned}
F_{w,t} &= E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i\frac{\psi_{z^+,t+i}}{\lambda_w}X_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}\\
&= \frac{\psi_{z^+,t}}{\lambda_w}H_t\frac{1-\tau_t^y}{1+\tau_t^w}\\
&\quad+\beta\xi_w\frac{\psi_{z^+,t+1}}{\lambda_w}\left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left(\frac{\left(\pi_t^c\right)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}H_{t+1}\frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w}\\
&\quad+\left(\beta\xi_w\right)^2\frac{\psi_{z^+,t+2}}{\lambda_w}\left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\\
&\quad\times\left(\frac{\left(\pi_t^c\pi_{t+1}^c\right)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}^2\right)^{\varkappa_w}\left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}H_{t+2}\frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w}\\
&\quad+...
\end{aligned}$$

or,

$$\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w}H_t\frac{1-\tau_t^y}{1+\tau_t^w}\\
&\quad+\beta\xi_w\left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left(\frac{\left(\pi_t^c\right)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}\{\frac{\psi_{z^+,t+1}}{\lambda_w}H_{t+1}\frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w}\\
&\quad+\beta\xi_w\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left(\frac{\left(\pi_{t+1}^c\right)^{\kappa_w}\left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}\frac{\psi_{z^+,t+2}}{\lambda_w}H_{t+2}\frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w}\\
&\quad+...\}\\
&= \frac{\psi_{z^+,t}}{\lambda_w}H_t\frac{1-\tau_t^y}{1+\tau_t^w}+\beta\xi_w\left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}F_{w,t+1},
\end{aligned}$$

so that

$$F_{w,t}=\frac{\psi_{z^+,t}}{\lambda_w}H_t\frac{1-\tau_t^y}{1+\tau_t^w}+\beta\xi_w E_t\left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}F_{w,t+1},$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t=\left[\left(1-\xi_w\right)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}}+\xi_w\left(\tilde{\pi}_{w,t}W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t=\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{B.38}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{B.39}$$

In addition to (B.39), we have following equilibrium conditions associated with sticky wages[24]:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{B.41}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{B.42}$$

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{B.43}$$

B.3.8. Output and aggregate factors of production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production.

[24] Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w = 0$, we obtain

$$E_t \begin{bmatrix} \eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \left(\hat{\pi}_t - \widehat{\bar{\pi}}_t^c \right) + \eta_4 \left(\hat{\pi}_{t+1} - \rho_{\widehat{\bar{\pi}}^c} \widehat{\bar{\pi}}_t^c \right) \\ +\eta_5 \left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c \right) + \eta_6 \left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c} \widehat{\bar{\pi}}_t^c \right) \\ +\eta_7 \hat{\psi}_{z^+,t} + \eta_8 \hat{H}_t + \eta_9 \hat{\tau}_t^y + \eta_{10} \hat{\tau}_t^w + \eta_{11} \hat{\zeta}_t^h \\ +\eta_{12} \hat{\mu}_{z^+,t} + \eta_{13} \hat{\mu}_{z^+,t+1} \end{bmatrix} = 0, \tag{B.40}$$

where

$$b_w = \frac{[\lambda_w \sigma_L - (1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w \xi_w \\ \left(\sigma_L \lambda_w - b_w \left(1 + \beta \xi_w^2 \right) \right) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ (1-\lambda_w) \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w) \frac{\tau^y}{(1-\tau^y)} \\ -(1-\lambda_w) \frac{\tau^w}{(1+\tau^w)} \\ -(1-\lambda_w) \\ -b_w \xi_w \\ b_w \beta \xi_w \end{pmatrix}.$$

Consider the unweighted average of the intermediate goods:

$$
\begin{aligned}
Y_t^{sum} &= \int_0^1 Y_{i,t} di \\
&= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+} \phi \right] di \\
&= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{it}} \right)^{\alpha} H_{it} - z_t^{+} \phi \right] di \\
&= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{it} di - z_t^{+} \phi
\end{aligned}
$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$
Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^{+} \phi.
$$

Recall that the demand for $Y_{j,t}$ is

$$
\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},
$$

so that

$$
\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1-\lambda_d}},
$$

say, where

$$
\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{B.44}
$$

Dividing by P_t,

$$
\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},
$$

or,

$$
\mathring{p}_t = \left[(1-\xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1-\xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{B.45}
$$

The preceding discussion implies:

$$
Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^{+} \phi \right],
$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{B.46}$$

We need to replace aggregate homogeneous labor, H_t, with aggregate household labor, h_t. From eq. (B.38) we have $H_t = \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t$. Plugging this is we obtain:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right].$$

which completes the derivation.

B.3.9. Restrictions across inflation rates

We now consider the restrictions across inflation rates implied by our relative price formulas. In terms of the expressions in (B.2) there are the restrictions implied by $p_t^{m,j}/p_{t-1}^{m,j}$, $j = x, c, i$, and p_t^x. The restrictions implied by the other two relative prices in (B.2), p_t^i and p_t^c, have already been exploited in (2.20) and (2.38), respectively. Finally, we also exploit the restriction across inflation rates implied by q_t/q_{t-1} and (B.3). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \tag{B.47}$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{B.48}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{B.49}$$

$$\frac{p_t^x}{p_{t-1}^x} = \frac{\pi_t^x}{\pi_t^*} \tag{B.50}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^*}{\pi_t^c}. \tag{B.51}$$

B.3.10. Endogenous Variables of the Baseline Model

In the above sections we derived following 71 equations,

$$2.3, 2.4, 2.5, 2.10, 2.11, 2.12, 2.14, 2.15, 2.16, 2.18, 2.19, 2.20, 2.21, 2.24, 2.25, 2.30,$$
$$B.27, 2.33, 2.35, 2.36, 2.38, 2.39, 2.39, 2.41, B.29, 2.70, 2.72, 2.73, 2.74, B.4, B.5,$$
$$B.6, B.11 - B.15, B.16 - B.19, B.22, B.23 - B.26, B.34, B.37, B.38, B.39, B.41, B.42, B.43,$$
$$B.46, B.47 - B.51,$$

which can be used to solve for the following 71 unknowns:

$$\begin{aligned}\bar{r}_t^k, \bar{w}_t, R_t^{\nu,*}, R_t^f, R_t^x, R_t, mc_t, mc_t^x, mc_t^{m,c}, mc_t^{m,i}, mc_t^{m,x}, \pi_t, \pi_t^x, \pi_t^c, \pi_t^i, \pi_t^{m,c}, \pi_t^{m,i}, \pi_t^{m,x},\\ p_t^c, p_t^x, p_t^i, p_t^{m,x}, p_t^{m,c}, p_t^{m,i}, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, q_t, i_t, c_t, x_t, a_t, s_t, \psi_{z^+,t}, y_t\\ K_t^d, F_t^d, \tilde{\pi}_{d,t}, \mathring{p}_t, K_{x,t}, F_{x,t}, \tilde{\pi}_t^x, \mathring{p}_t^x, \{K_{m,j,t}, F_{m,j,t}, \tilde{\pi}_t^{m,j}, \mathring{p}_t^{m,j}; j = c, i, x\}, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k\\ \Phi_t, \tilde{S}_t, \tilde{S}_t', a(u_t), \mathring{w}_t, c_t^m, i_t^m, x_t^m, \pi_w.\end{aligned}$$

B.4. Equilibrium Conditions for the Financial Frictions Model

B.4.1. Derivation of Aggregation Across Entrepreneurs

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth, N_{t+1}. Then, aggregate average net worth, $\bar{N}_{t+1}$, is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1})\, dN_{t+1}.$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N, where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \tag{B.52}$$

where $\bar{K}_t^N$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t)\, dN_t.$$

Multiplying (B.52) by $f(N_t)$ and integrating over all entrepreneurs,

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.$$

Writing this out more fully:

$$\begin{aligned} V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})]\, \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &\quad - \left\{ [1 - F(\bar{\omega}_t; \sigma_{t-1})]\, \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t. \end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$. Substituting:

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t),$$

which implies eq. (2.50) in the main text

B.4.2. Equilibrium Conditions

In this subsection we indicate how the equilibrium conditions of the baseline model must be modified to accommodate financial frictions.

Consider the households. Households no longer accumulate physical capital, and the first order condition, (2.39), must be dropped. No other changes need to be made to the household first order conditions. Equation (2.33) can be interpreted as applying to the household's decision to make bank deposits. The household equations, (2.38) and (2.39), pertaining to investment can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the first order condition of many identical, competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables, noting that W_t^e is set so that its scaled counterpart is constant:

$$n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+},\ w^e = \frac{W_t^e}{P_t z_t^+}.$$

Dividing both sides of (2.50) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} \left[R_t^k p_{k',t-1} \bar{k}_t - R_{t-1} \left(p_{k',t-1} \bar{k}_t - n_t \right) - \mu G\left(\bar{\omega}_t ; \sigma_{t-1} \right) R_t^k p_{k',t-1} \bar{k}_t \right] + w^e, \tag{B.53}$$

for $t = 0, 1, 2, \ldots$. Equation (B.53) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payments to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, eq. (2.47), can be expressed in terms of the scaled variables as:

$$\Gamma(\bar{\omega}_{t+1}; \sigma_t) - \mu G(\bar{\omega}_{t+1}; \sigma_t) = \frac{R_t}{R_{t+1}^k} \left(1 - \frac{n_{t+1}}{p_{k',t} \bar{k}_{t+1}} \right), \tag{B.54}$$

for $t = -1, 0, 1, 2, \ldots$. The optimality condition for bank loans is (2.46).

The output equation, (2.72), does not have to be modified. Instead, the resource constraint for domestic homogenous goods (2.73) needs to be adjusted for the monitoring costs:

$$\begin{aligned} y_t - d_t \quad &= \quad g_t + (1 - \omega_c) \left(p_t^c \right)^{\eta_c} c_t + \left(p_t^i \right)^{\eta_i} \left(i_t + a\left(u_t \right) \frac{\bar{k}_t}{\mu_{\psi,t} \mu_{z^+,t}} \right) (1 - \omega_i) \\ &+ \left[\omega_x \left(p_t^{m,x} \right)^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) \left(\mathring{p}_t^x \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p_t^x \right)^{-\eta_f} y_t^*, \end{aligned} \tag{B.55}$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t;\sigma_{t-1})R_t^k p_{k',t-1}\bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

When we bring the model to the data measured GDP is y_t adjusted for both monitoring costs and, as in the baseline model, capital utilization costs:

$$gdp_t = y_t - d_t - \left(p_t^i\right)^{\eta_i}\left(a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i).$$

Account has to be taken of the consumption by exiting entrepreneurs. The net worth of these entrepreneurs is $(1-\gamma_t)\,V_t$ and we assume a fraction, $1-\Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ, is consumed by the exiting entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta\frac{1-\gamma_t}{\gamma_t}(n_{t+1}-w^e)z_t^+ P_t$$

from the right side of (2.13). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

The financial frictions brings a net increase of 2 equations (we add (2.46), (B.53) and (B.54), and delete (2.39)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72. The financial frictions also introduce the additional shocks, σ_t and γ_t.

B.5. Equilibrium Conditions from the Employment Frictions Model

B.5.1. Labor Hours

Scaling (2.56) by $P_t z_t^+$ yields:

$$\bar{w}_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L}\frac{1}{\psi_{z^+,t}\frac{1-\tau_t^y}{1+\tau_t^w}} \tag{B.56}$$

Note, that the ratio

$$\frac{\mathcal{G}_t^i}{\varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since no other variables in (B.56) are indexed by cohort.

B.5.2. Vacancies and the Employment Agency Problem

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $W_{i,t}$, as in (2.51):

$$W_{i,t} = \tilde{\pi}_{w,t}W_{i-1,t-1},\ \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}\right)^{\kappa_w}\left(\bar{\pi}_t\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}, \tag{B.57}$$

for $i = 1,...,N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w+\varkappa_w \in (0,1)$.

After wages are set, employment agencies in cohort i decide on endogenous separation, post vacancies to attract new workers in the next period and supply labor services, $l_t^i \varsigma_{i,t}$, into competitive labor markets. Simplifying,

$$
\begin{aligned}
F\left(l_t^0, \omega_t\right) \quad = \quad & \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j} [\Big(W_{t+j} \mathcal{E}_{t+j}^j - \Gamma_{t,j} \omega_t \left[1 - \mathcal{F}_{t+j}^j\right]\Big) \varsigma_{j,t+j} \\
& - P_{t+j} \frac{\kappa z_{t+j}^+}{\varphi} \left(\tilde{v}_t^j\right)^{\varphi} \left(1 - \mathcal{F}_{t+j}^j\right)] l_{t+j}^j \\
& + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right),
\end{aligned} \tag{B.58}
$$

For convenience, we omit the expectation operator E_t below. Let

Writing out (B.58):

$$
\begin{aligned}
F\left(l_t^0, \omega_t\right) \quad = \quad & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left[\left(W_t \mathcal{E}_t^0 - \omega_t \left(1 - \mathcal{F}_t^0\right)\right) \varsigma_t - P_t \frac{\kappa z_t^+}{\varphi} \left(\tilde{v}_t^0\right)^{\varphi} \left(1 - \mathcal{F}_t^0\right)\right] l_t^0 \\
& + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t \left(1 - \mathcal{F}_{t+1}^1\right)\right) \varsigma_{t+1} - P_{t+1} \frac{\kappa z_{t+1}^+}{\varphi} \left(\tilde{v}_{t+1}^1\right)^{\varphi} \left(1 - \mathcal{F}_{t+1}^1\right)\right] \\
& \times \left(\chi_t^0 + \rho\right) \left[1 - \mathcal{F}_t^0\right] l_t^0 \\
& + \beta^2 E_t \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2} \mathcal{E}_{t+2}^2 - \Gamma_{t,2} \omega_t \left(1 - \mathcal{F}_{t+2}^2\right)\right) \varsigma_{t+2} - P_{t+2} \frac{\kappa z_{t+2}^+}{\varphi} \left(\tilde{v}_{t+2}^2\right)^{\varphi} \left(1 - \mathcal{F}_{t+2}^2\right)\right] \\
& \times \left(\chi_{t+1}^1 + \rho\right) \left(\chi_t^0 + \rho\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left(1 - \mathcal{F}_t^0\right) l_t^0 \\
& + \ldots + \\
& + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right)\}.
\end{aligned}
$$

$$
\begin{aligned}
J\left(\omega_t\right) \quad = \quad & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left(W_t \mathcal{E}_t^0 - \omega_t \left(1 - \mathcal{F}_t^0\right)\right) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{\varphi} \left(\tilde{v}_t^0\right)^{\varphi} \left[1 - \mathcal{F}_t^0\right] \\
& + \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t \left(1 - \mathcal{F}_{t+1}^1\right)\right) \varsigma_{1,t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{\varphi} \left(\tilde{v}_{t+1}^1\right)^{\varphi} \left(1 - \mathcal{F}_{t+1}^1\right)\right] \times \\
& \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(1 - \mathcal{F}_t^0\right) \\
& + \beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2} \mathcal{E}_{t+2}^2 - \Gamma_{t,2} \omega_t \left(1 - \mathcal{F}_{t+2}^2\right)\right) \varsigma_{2,t+2} - P_{t+2} z_{t+2}^+ \frac{\kappa}{\varphi} \left(\tilde{v}_{t+2}^2\right)^{\varphi} \left(1 - \mathcal{F}_{t+2}^2\right)\right] \times \\
& \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \left(1 - \mathcal{F}_{t+1}^1\right) \left[1 - \mathcal{F}_t^0\right] \\
& + \ldots + \\
& + \beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \times \\
& \left(1 - \mathcal{F}_{t+N-1}^{N-1}\right) \cdots \left(1 - \mathcal{F}_t^0\right)\}.
\end{aligned} \tag{B.59}
$$

We derive optimal vacancy posting decisions of employment agencies by differentiating (B.59) with respect to $\tilde{v}_t^0$ and multiply the result by $\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}$, to obtain:

$$
\begin{aligned}
0 &= -P_t z_t^+ \kappa \left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&\quad +\beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left[1-\mathcal{F}_{t+1}^1\right]\right)\varsigma_{1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left[1-\mathcal{F}_{t+2}^2\right]\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right]\} \\
&= J\left(\omega_t\right)-\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \\
&\quad -P_t z_t^+\kappa\left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}
\end{aligned}
$$

Since the latter expression must be zero, we conclude:

$$
\begin{aligned}
J\left(\omega_t\right) &= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \\
&\quad +P_t z_t^+\kappa\left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\kappa\left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_t^0\right)^{\varphi}+\left(\tilde{v}_t^0\right)^{\varphi-1}\frac{\rho}{Q_t^{1-\iota}}\right]\left[1-\mathcal{F}_t^0\right].
\end{aligned}
$$

Next, we obtain simple expressions for the vacancy decisions from their first order necessary conditions for optimality. Multiplying the first order condition for $\tilde{v}_{t+1}^1$ by

$$
\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}},
$$

we obtain:

$$
\begin{aligned}
0 &= -\beta\frac{\upsilon_{t+1}}{\upsilon_t}P_{t+1}z_{t+1}^+\kappa\left(\tilde{v}_{t+1}^1\right)^{\varphi-1}\left[1-\mathcal{F}_{t+1}^1\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}}\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left[1-\mathcal{F}_{t+2}^2\right]\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right].
\end{aligned}
$$

Substitute out the period $t+2$ and higher terms in this expression using the first order condition for $\tilde{v}_t^0$. After rearranging, we obtain,

$$\frac{P_t z_t^+ \kappa \left(\tilde{v}_t^0\right)^{\varphi-1}}{Q_t^{1-\iota}} = \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\begin{array}{c} \left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t \left[1-\mathcal{F}_{t+1}^1\right]\right) \varsigma_{1,t+1} \\ +P_{t+1} z_{t+1}^+ \kappa \left(1-\mathcal{F}_{t+1}^1\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^1\right)^{\varphi} + \left(\tilde{v}_{t+1}^1\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}}\right] \end{array} \right].$$

Following the pattern set with $\tilde{v}_{t+1}^1$, multiply the first order condition for $\tilde{v}_{t+2}^2$ by

$$\left(\tilde{v}_{t+2}^2 Q_{t+2}^{1-\iota} + \rho\right) \frac{1}{Q_{t+2}^{1-\iota}}.$$

Substitute the period $t+3$ and higher terms in the first order condition for $\tilde{v}_{t+2}^2$ using the first order condition for $\tilde{v}_{t+1}^1$ to obtain, after rearranging,

$$\frac{P_{t+1} z_{t+1}^+ \kappa \left(\tilde{v}_{t+1}^1\right)^{\varphi-1}}{Q_{t+1}^{1-\iota}} = \beta \frac{\upsilon_{t+2}}{\upsilon_{t+1}} \left[\begin{array}{c} \left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t \left[1-\mathcal{F}_{t+2}^2\right]\right) \varsigma_{2,t+2} \\ +P_{t+2} z_{t+2}^+ \kappa \left(1-\mathcal{F}_{t+2}^2\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+2}^2\right)^{\varphi} + \left(\tilde{v}_{t+1}^2\right)^{\varphi-1} \frac{\rho}{Q_{t+2}^{1-\iota}}\right] \end{array} \right].$$

Continuing in this way, we obtain,

$$\frac{P_{t+j} z_{t+j}^+ \kappa \left(\tilde{v}_{t+j}^j\right)^{\varphi-1}}{Q_{t+j}^{1-\iota}} = \beta \frac{\upsilon_{t+j+1}}{\upsilon_{t+j}} \left[\begin{array}{c} \left(W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1} - \Gamma_{t,j+1}\omega_t \left[1-\mathcal{F}_{t+j+1}^{j+1}\right]\right) \varsigma_{j+1,t+j+1} \\ +P_{t+j+1} z_{t+j+1}^+ \kappa \left(1-\mathcal{F}_{t+j+1}^{j+1}\right) \left[\begin{array}{c} \left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi} \\ +\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi-1} \frac{\rho}{Q_{t+j+1}^{1-\iota}} \end{array} \right] \end{array} \right],$$

for $j = 0, 1, ..., N-2$. Now consider the first order necessary condition for the optimality of $\tilde{v}_{t+N-1}^{N-1}$. After multiplying this first order condition by

$$\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}},$$

we obtain,

$$\begin{aligned} 0 \;\; = \;\; & -\beta^{N-1} \frac{\upsilon_{t+N-1}}{\upsilon_t} P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \cdots \\ & \cdots \left(\tilde{v}_{t+N-2}^{N-2} Q_{t+N-2}^{1-\iota} + \rho\right) \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}} \left[1-\mathcal{F}_{t+N-2}^{N-2}\right] \cdots \left[1-\mathcal{F}_t^0\right] \\ & +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \times \\ & \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \cdots \left[1-\mathcal{F}_t^0\right] \} \end{aligned}$$

or,

$$P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \frac{1}{Q_{t+N-1}^{1-\iota}} = \beta \frac{\upsilon_{t+N}}{\upsilon_{t+N-1}} J\left(\tilde{W}_{t+N}\right).$$

Making use of our expression for J, we obtain:

$$P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \frac{1}{Q_{t+N-1}^{1-\iota}} = \beta \frac{\upsilon_{t+N}}{\upsilon_{t+N-1}} \left[\begin{array}{c} \left(W_{t+N}\mathcal{E}_{t+N}^0 - \tilde{W}_{t+N}\left(1-\mathcal{F}_{t+N}^0\right)\right) \varsigma_{0,t+N} \\ +P_{t+N} z_{t+N}^+ \kappa \left[\begin{array}{c} \left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+N}^0\right)^{\varphi} \\ +\left(\tilde{v}_{t+N}^0\right)^{\varphi-1} \frac{\rho}{Q_{t+N}^{1-\iota}} \end{array} \right] \left[1-\mathcal{F}_{t+N}^0\right] \end{array} \right].$$

The above first order conditions apply over time to a group of agencies that bargain at date t. We now express the first order conditions for a fixed date and different cohorts:

$$
\begin{aligned}
P_t z_t^+ \kappa \left(\tilde{v}_t^j\right)^{\varphi-1} \frac{1}{Q_t^{1-\iota}} &= \beta \frac{\upsilon_{t+1}}{\upsilon_t} [\left(W_{t+1}\mathcal{E}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\tilde{W}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1} \\
&\quad + P_{t+1}z_{t+1}^+\kappa\left(1-\mathcal{F}_{t+1}^{j+1}\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi} + \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)], \\
&\quad \text{for } j = 0, ..., N-2.
\end{aligned}
$$

Scaling by $P_t z_t^+$ yields the following scaled first order optimality conditions:

$$
\begin{aligned}
\kappa \left(\tilde{v}_t^j\right)^{\varphi-1} \frac{1}{Q_t^{1-\iota}} &= \beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [\left(\bar{w}_{t+1}\mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\left(1-\mathcal{F}_{t+1}^{j+1}\right)\right)\varsigma_{j+1,t+1} \\
&\quad + \kappa\left(1-\mathcal{F}_{t+1}^{j+1}\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi} + \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)], \\
&\quad \text{for } j = 0, ..., N-2,
\end{aligned}
\tag{B.60}
$$

where

$$
\begin{aligned}
G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}}\left(\frac{1}{\mu_{z^+,t-i+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+1}}\right),\ i \geq 0, \\
w_t &= \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t}.
\end{aligned}
\tag{B.61}
$$

Also,

$$
G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}\left(\frac{1}{\mu_{z^+,t+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+j}}\right) & j > 0 \\ 1 & j = 0 \end{cases}. \tag{B.62}
$$

The scaled vacancy first order condition of agencies that are in the last period of their contract is:

$$
\begin{aligned}
\kappa\left(\tilde{v}_t^{N-1}\right)^{\varphi-1}\frac{1}{Q_t^{1-\iota}} &= \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}[\left(\bar{w}_{t+1}\mathcal{E}_{t+1}^0 - w_{t+1}\bar{w}_{t+1}\left(1-\mathcal{F}_{t+1}^0\right)\right)\varsigma_{0,t+1} \\
&\quad + \kappa\left(1-\mathcal{F}_{t+1}^0\right)\left(\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^0\right)^{\varphi} + \left(\tilde{v}_{t+1}^0\right)^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)].
\end{aligned}
\tag{B.63}
$$

B.5.3. Agency Separation Decisions

This section presents details of the employment agency separation decision. We start by considering the separation decision of a representative agency in the $j = 0$ cohort which renegotiates the wage in the current period. After that, we consider $j > 0$.

The Separation Decision of Agencies that Renegotiate the Wage in the Current Period

We start by considering the impact of $\bar{a}_t^0$ on agency and worker surplus, respectively. The aggregate surplus across all the l_t^0 workers in the representative agency is given by (**??**). The object, $\mathcal{F}_t^0$, is a function of $\bar{a}_t^0$ as indicated in (2.59). We denote its derivative by

$$\mathcal{F}_t^{j\prime} \equiv \frac{d\mathcal{F}_t^j}{d\bar{a}_t^j}, \tag{B.64}$$

for $j = 0...N-1$. Where convenient, in this subsection we include expressions that apply to the representative agency in cohort $j > 0$ as well as to those in cohort, $j = 0$. According to (2.56), $\bar{a}_t^0$ affects V_t^0 via its impact on hours worked, $\varsigma_{0,t}$. Hours worked is a function of $\bar{a}_t^0$ because $\mathcal{G}_t^0$ is (see (2.57), (2.56) and (2.65)). These observations about V_t^0 also apply to V_t^j, for $j > 0$. Thus, differentiating (2.65), we obtain:

$$V_t^{j\prime} \equiv \frac{d}{d\bar{a}_t^j} V_t^j = \left[\Gamma_{t-j,j}\tilde{W}_{t-j}\frac{1-\tau_t^y}{1+\tau_t^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\upsilon_t}\right]\varsigma_{j,t}^{\prime}, \tag{B.65}$$

where

$$\varsigma_{j,t}^{\prime} \equiv \frac{d\varsigma_{j,t}}{d\bar{a}_t^j} = \frac{1}{\sigma_L}\left(\varsigma_{j,t}\right)^{1-\sigma_L}\frac{W_t\upsilon_t}{\zeta_t A_L}\frac{1-\tau_t^y}{1+\tau_t^w}\mathcal{G}_t^{j\prime}, \tag{B.66}$$

and

$$\mathcal{G}_t^{j\prime} \equiv \frac{d\mathcal{G}_t^j}{d\bar{a}_t^j}. \tag{B.67}$$

The counterpart to (B.66) in terms of scaled variables is:

$$\varsigma_{j,t}^{\prime} \equiv \frac{1}{\sigma_L}\left(\varsigma_{j,t}\right)^{1-\sigma_L}\frac{\bar{w}_t w_t\psi_{z^+,t}}{\zeta_t A_L}\frac{1-\tau_t^y}{1+\tau_t^w}\mathcal{G}_t^{j\prime} \tag{B.68}$$

The value of being unemployed, U_t, is not a function of the $\bar{a}_t^0$ chosen by the representative agency because U_t is determined by economy-wide aggregate variables such as the job finding rate (see (2.66)).

According to (2.64) agency surplus per worker in l_t^0 is given by $J(\omega_t)$ and this has the following representation:

$$J(\omega_t) = \max_{\bar{a}_t^0}\tilde{J}\left(\omega_t;\bar{a}_t^0\right)\left(1-\mathcal{F}_t^0\right).$$

where

$$\tilde{J}\left(\omega_t;\bar{a}_t^0\right) = \max_{\tilde{v}_t^0}\left\{\left(W_t\mathcal{G}_t^0-\omega_t\right)\varsigma_{0,t} - P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(\chi_t^0+\rho\right)J_{t+1}^1(\omega_t)\right\}, \tag{B.69}$$

denotes the value to an agency in cohort 0 of an employee after endogenous separations has taken place and

$$J_{t+1}^{j+1}(\omega_t) = \max_{\left\{\bar{a}_{t+i}^i, \tilde{v}_{t+i}^i\right\}_{i=j}^{N-1}} \{\left[\left(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^{+}\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right] \tag{B.70}$$
$$\times\left(1-\mathcal{F}_{t+1}^{j+1}\right)$$
$$+\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}\mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2}\omega_{t-j}\right)\varsigma_{j+2,t+2} - P_{t+2}z_{t+2}^{+}\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\right]$$
$$\times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)$$
$$+...+$$
$$+\beta^{N-j}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)\left(\chi_{t+N-j-1}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\},$$

for $j=0$.

In (B.69) and (B.70), it is understood that χ_{t+j}^{j}, $\tilde{v}_{t+j}^{j}$ are connected by (2.60). Thus, the surplus of the representative agency with workforce, l_t^0, is given by (2.64). Differentiation of $\tilde{J}$ with respect to $\bar{a}_t^j$ need only be concerned with the impact of $\bar{a}_t^j$ on $\mathcal{G}_t^j$ and $\varsigma_{j,t}$. Generalizing (B.69) to cohort j:

$$\tilde{J}\left(\omega_{t-j};\bar{a}_t^j\right) = \max_{\tilde{v}_t^j}\left\{\left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_t\right)\varsigma_{j,t} - P_t z_t^{+}\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(\chi_t^j+\rho\right)J_{t+1}^{j+1}(\omega_t)\right\}.$$

Then,

$$\tilde{J}_{\bar{a}^j}\left(\omega_{t-j};\bar{a}_t^j\right) \equiv \frac{d\tilde{J}\left(\omega_{t-j};\bar{a}_t^j\right)}{d\bar{a}_t^j} = \left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t}' + W_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}, \tag{B.71}$$

where $\varsigma_{j,t}'$ and $\mathcal{G}_t^{j\prime}$ are defined in (B.66) and (B.67), respectively.

We now evaluate $\mathcal{F}_t^{j\prime}$, $\mathcal{G}_t^{j\prime}$ and $\varsigma_{j,t}'$, for $j\geq 0$. We assume that productivity, a, is drawn from a log-normal distribution having the properties, $Ea=1$ and $Var(\log a)=\sigma_a^2$. This assumption simplifies the analysis because analytic expressions are available for objects such as $\mathcal{F}_t^{j\prime}$, $\mathcal{G}_t^{j\prime}$. Although these expressions are readily available in the literature (see, for example, BGG), we derive them here for completeness. It is easily verified that $\mathcal{F}$ has the following representation:[25]

$$\mathcal{F}\left(\bar{a}^j;\sigma_a\right) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log\bar{a}^j} e^x e^{\frac{-\left(x+\frac{1}{2}\sigma_a^2\right)^2}{2\sigma_a^2}}dx,$$

where $x=\log a$. Combining the exponential terms,

$$\mathcal{F}\left(\bar{a}^j;\sigma_a\right) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log\bar{a}^j} \exp\frac{-\left(x-\frac{1}{2}\sigma_a^2\right)^2}{2\sigma_a^2}dx.$$

Now, make the change of variable,

$$v\equiv\frac{x-\frac{1}{2}\sigma_a^2}{\sigma_a}$$

[25] Note that $Ea=1$ is imposed by specifying $E\log a=-\sigma_a^2/2$.

so that

$$dv = \frac{1}{\sigma_a} dx.$$

Substituting into the expression for $\mathcal{F}$:

$$\mathcal{F}\left(\bar{a}^j; \sigma_a\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma_a}} \exp^{\frac{-v^2}{2}} dv.$$

This is just the standard normal cumulative distribution, evaluated at $\left(\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma_a^2\right)/\sigma_a$. Differentiating $\mathcal{F}$, we obtain an expression for (B.64):

$$\mathcal{F}_t^{j\prime} = \frac{1}{\bar{a}^j \sigma_a \sqrt{2\pi}} \exp^{-\frac{\left(\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2\right)^2}{2\sigma_a^2}}. \tag{B.72}$$

The object on the right of the equality is just the normal density with variance σ_a^2 and mean $-\sigma_a^2/2$, evaluated at $\log\left(\bar{a}^j\right)$ and divided by $\bar{a}^j$. From (2.58) we obtain:

$$\mathcal{E}_t^{j\prime} = -\bar{a}_t^j \mathcal{F}_t^{j\prime}. \tag{B.73}$$

Differentiating (B.67),

$$\mathcal{G}_t^{j\prime} = \frac{\mathcal{E}_t^{j\prime}\left(1-\mathcal{F}_t^j\right) + \mathcal{E}_t^j \mathcal{F}_t^{j\prime}}{\left[1-\mathcal{F}_t^j\right]^2} \tag{B.74}$$

The surplus criterion governing the choice of $\bar{a}_t^0$ is (**??**). The first order necessary condition for an interior optimum is given by (**??**), which we reproduce here for convenience:

$$s_w V_t^{0\prime} + s_e \tilde{J}_{\bar{a}^0}\left(\tilde{W}_t; \bar{a}_t^0\right) = \left[s_w\left(V_t^0 - U_t\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0},$$

where we have made use of the fact that the wage paid to workers in the bargaining period is denoted $\tilde{W}_t$. After substituting from (B.65) and (B.71):

$$s_w\left(\tilde{W}_t \frac{1-\tau_t^y}{1+\tau_t^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\upsilon_t}\right)\varsigma_{0,t}' + s_e\left[\left(W_t \mathcal{G}_t^0 - \tilde{W}_t\right)\varsigma_{0,t}' + W_t \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] = \tag{B.75}$$
$$\left[s_w\left(V_t^0 - U_t\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}.$$

In scaled terms this is

$$s_w\left(w_t W_t \frac{1-\tau_t^y}{1+\tau_t^w} - A_L P_t z_t^+ \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}' + s_e\left[\left(W_t \mathcal{G}_t^0 - w_t W_t\right)\varsigma_{0,t}' + W_t \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] =$$
$$\left[s_w\left(P_t z_t^+ V_{z^+,t}^0 - U_{z^+,t} P_t z_t^+\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}$$
$$P_t z_t^+ s_w\left(w_t \bar{w}_t \frac{1-\tau_t^y}{1+\tau_t^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}' + P_t z_t^+ \bar{w}_t s_e\left[\left(\mathcal{G}_t^0 - w_t\right)\varsigma_{0,t}' + \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] =$$
$$P_t z_t^+\left[s_w\left(V_{z^+,t}^0 - U_{z^+,t}\right) + s_e \tilde{J}_{z^+,t}^0\right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}$$

Dividing through by $P_t z_t^+$ yields:

$$s_w \left(w_t \bar{w}_t \frac{1-\tau_t^y}{1+\tau_t^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}} \right) \varsigma_{0,t}' + s_e \bar{w}_t \left[\left(\mathcal{G}_t^0 - w_t \right) \varsigma_{0,t}' + \mathcal{G}_t^{0\prime} \varsigma_{0,t} \right] = \left[\begin{array}{c} s_w \left(V_{z^+,t}^0 - U_{z^+,t} \right) \\ + s_e \tilde{J}_{z^+,t}^0 \end{array} \right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0} \tag{B.76}$$

The Separation Decision of Agencies that Renegotiated in Previous Periods We now turn to the $\bar{a}_t^j$ decision, for $j = 1, ..., N-1$. The representative agency that selects $\bar{a}_t^j$ is a member of the cohort of agencies that bargained j periods in the past. We denote the present discounted value of profits of the representative agency in cohort j by $F_t^j \left(\omega_{t-j} \right)$:

$$\begin{aligned}
\frac{F_t^j \left(l_t^j, \omega_{t-j} \right)}{l_t^j} \equiv\ & J_t^j \left(\omega_{t-j} \right) = \max_{\left\{ \bar{a}_{t+i}^{j+i}, \tilde{v}_{t+i}^{j+i} \right\}_{i=0}^{N-j-1}} \{ \left[\left(W_t \mathcal{G}_t^j - \Gamma_{t-j,j} \omega_{t-j} \right) \varsigma_{j,t} - P_t z_t^+ \frac{\kappa}{\varphi} \left(\tilde{v}_t^j \right)^{\varphi} \right] \\
& \times \left(1 - \mathcal{F}_t^j \right) \\
& + \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} \mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1} \omega_{t-j} \right) \varsigma_{j+1,t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{\varphi} \left(\tilde{v}_{t+1}^{j+1} \right)^{\varphi} \right] \\
& \times \left(1 - \mathcal{F}_{t+1}^{j+1} \right) \left(\chi_t^j + \rho \right) \left(1 - \mathcal{F}_t^j \right) \\
& + ... + \\
& + \beta^{N-j} \frac{\upsilon_{t+N-j}}{\upsilon_t} J \left(\tilde{W}_{t+N-j} \right) \left(\chi_{t+N-1-j}^{N-1} + \rho \right) \left(1 - \mathcal{F}_{t+N-j-1}^{N-1} \right) \cdots \\
& \left(\chi_t^j + \rho \right) \left(1 - \mathcal{F}_t^j \right) \}.
\end{aligned}$$

Here, we exploit that $F_t^j \left(l_t^j, \omega_{t-j} \right)$ is proportional to l_t^j, as in the case $j = 0$ considered in (2.64). In particular, $J_t^j \left(\omega_{t-j} \right)$ is not a function of l_t^j and corresponds to the object in (B.70) with the time index, t, replaced by $t - j$. We can write $J_t^j \left(\omega_{t-j} \right)$ in the following form:

$$J_t^j \left(\omega_{t-j} \right) = \tilde{J}_t^j \left(\omega_{t-j}; \bar{a}_t^j \right) \left(1 - \mathcal{F}_t^j \right),$$

where

$$\tilde{J}_t^j \left(\omega_{t-j}; \bar{a}_t^j \right) = \left(W_t \mathcal{G}_t^j - \Gamma_{t-j,j} \omega_{t-j} \right) \varsigma_{j,t} - P_t z_t^+ \frac{\kappa}{\varphi} \left(\tilde{v}_t^j \right)^{\varphi} + \beta \frac{\upsilon_{t+1}}{\upsilon_t} J_{t+1}^{j+1} \left(\omega_{t-j} \right) \left(\chi_t^j + \rho \right).$$

from a generalization of (B.69) to $j = 1...N-1$.

In this way, we obtain an expression for agency surplus for agencies that have not negotiated for j periods which is symmetric to (**??**):

$$F_t^j \left(\omega_{t-j} \right) = \tilde{J}_t^j \left(\omega_{t-j}; \bar{a}_t^j \right) \left(1 - \mathcal{F}_t^j \right) l_t^j. \tag{B.77}$$

Our expression for total surplus is the analog of (**??**):

$$\left[s_w \left(V_t^j - U_t \right) + s_e \tilde{J}_t^j \left(\omega_{t-j}; \bar{a}_t^j \right) \right] \left(1 - \mathcal{F}_t^j \right) l_t^j. \tag{B.78}$$

Differentiating,

$$s_w V_t^{j\prime} + s_e \tilde{J}_{\bar{a}^j}\left(\omega_{t-j};\bar{a}_t^j\right) = \left[s_w\left(V_t^j - U_t\right) + s_e \tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}, \tag{B.79}$$

which corresponds to (2.69). Here, $\tilde{J}_{\bar{a}^j}\left(\omega_{t-1};\bar{a}_t^j\right)$ is the analog of (B.71) with index 0 replaced by j. After substituting from the analogs for cohort j of (B.65), (B.71):

$$s_w\left(\Gamma_{t-j,j}\tilde{W}_{t-j}\frac{1-\tau_t^y}{1+\tau_t^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\upsilon_t}\right)\varsigma_{j,t}' + s_e\left[\left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\tilde{W}_{t-j}\right)\varsigma_{j,t}' + W_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}\right] =$$
$$\left[s_w\left(V_t^j - U_t\right) + s_e\tilde{J}\left(\tilde{W}_{t-j};\bar{a}_t^j\right)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}.$$

Scaling analogously to (B.76) and plugging in $\tilde{W}_{t-j} = w_{t-j}\bar{w}_{t-j}P_{t-j}z_{t-j}^+$ and $\bar{w}_t z_t^+ P_t = W_t$ we obtain:

$$s_w\left(G_{t-j,j}w_{t-j}\bar{w}_{t-j}\frac{1-\tau_t^y}{1+\tau_t^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{j,t}' + s_e\left[\left(\bar{w}_t\mathcal{G}_t^j - G_{t-j,j}\bar{w}_{t-j}w_{t-j}\right)\varsigma_{j,t}' + \bar{w}_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}\right] = \tag{B.80}$$
$$\left[s_w\left(V_{z^+,t}^j - U_{z^+,t}\right) + s_e\tilde{J}_{z^+,t}^j\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}$$

Finally, we need an explicit expression for $\tilde{J}\left(\tilde{W}_t;\bar{a}_t^j\right)$, or rather its scaled equivalent $\tilde{J}_{z^+,t}^j$. For this we use (B.70) to write out $J_{t+1}^{j+1}\left(\omega_t\right)$ for $j = 1...N$ and plug into (B.69):

$$\tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right) = \left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}J_{t+1}^{j+1}\left(\omega_{t-1}\right)\left(\chi_t^j + \rho\right)$$

Redisplaying (B.70) for convenience:

$$\begin{aligned}
J_{t+1}^{j+1}\left(\omega_t\right) = & \max_{\left\{\bar{a}_{t+i}^i,\tilde{v}_{t+i}^i\right\}_{i=j}^{N-1}}\{\left[\left(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right] \\
& \times\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}\mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2}\omega_{t-j}\right)\varsigma_{j+2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\right] \\
& \times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +... + \\
& +\beta^{N-j}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)\left(\chi_{t+N-j-1}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots \\
& \left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\},
\end{aligned}$$

Accordingly:

$$
\begin{aligned}
\tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right) &= \left(W_t\mathcal{G}_t^j-\Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t}-P_tz_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}+\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(\chi_t^j+\rho\right)\{ \\
&\left[\left(W_{t+1}\mathcal{G}_{t+1}^{j+1}-\Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right]\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
&+\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}\mathcal{G}_{t+2}^{j+2}-\Gamma_{t-j,j+2}\omega_{t-j}\right)\varsigma_{j+2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\right] \\
&\times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
&+...+ \\
&+\beta^{N-j}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)\left(\chi_{t+N-j-1}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots \\
&\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\}
\end{aligned}
$$

for $j=0,...,N-1$. Plugging in for $\omega_{t-j}=\tilde{W}_{t-j}=w_{t-j}\bar{w}_{t-j}P_{t-j}z_{t-j}^+$ and scaling obtains:

$$
\begin{aligned}
\tilde{J}_{z^+,t}^j\left(\tilde{W}_{t-j};\bar{a}_t^j\right) &\equiv \frac{\tilde{J}^j\left(\tilde{W}_t;\bar{a}_t^j\right)}{P_tz_t^+}=\left(\bar{w}_t\mathcal{G}_t^j-G_{t-j,j}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j,t}-\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}+ \\
&\beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\frac{P_tz_t^+}{P_{t+1}z_{t+1}^+}\left(\chi_t^j+\rho\right) \\
&\times\{\frac{P_{t+1}z_{t+1}^+}{P_tz_t^+}\left[\left(\bar{w}_{t+1}\mathcal{G}_{t+1}^{j+1}-G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+1,t+1}-\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right]\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
&+\beta\frac{\psi_{z^+,t+2}}{\psi_{z^+,t+1}}\frac{P_{t+1}z_{t+1}^+}{P_{t+2}z_{t+2}^+}\frac{P_{t+2}z_{t+2}^+}{P_tz_t^+}\left[\begin{array}{c}\left(\bar{w}_{t+2}\mathcal{G}_{t+2}^{j+2}-G_{t-j,j+2}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+2,t+2} \\ -\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\end{array}\right] \\
&\times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
&+...+ \\
&+\beta^{N-j}\frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t+1}}\frac{P_{t+1}z_{t+1}^+}{P_{t+N-j}z_{t+N-j}^+}\frac{P_{t+N-j}z_{t+N-j}^+}{P_tz_t^+}J_{z^+,t+N-j} \\
&\times\left(\chi_{t+N-j-1}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\}
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\tilde{J}^j_{z^+,t}\left(\tilde{W}_{t-j};\bar{a}^j_t\right) &= \left(\bar{w}_t\mathcal{G}^j_t - G_{t-j,j}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j,t} - \frac{\kappa}{\varphi}\left(\tilde{v}^j_t\right)^{\varphi} + \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left(\chi^j_t+\rho\right) \qquad \text{(B.81)}\\
&\times\{\left[\left(\bar{w}_{t+1}\mathcal{G}^{j+1}_{t+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+1,t+1} - \frac{\kappa}{\varphi}\left(\tilde{v}^{j+1}_{t+1}\right)^{\varphi}\right]\left(1-\mathcal{F}^{j+1}_{t+1}\right)\\
&+\beta\frac{\psi_{z^+,t+2}}{\psi_{z^+,t+1}}\left[\left(\bar{w}_{t+2}\mathcal{G}^{j+2}_{t+2} - G_{t-j,j+2}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+2,t+2} - \frac{\kappa}{\varphi}\left(\tilde{v}^{j+2}_{t+2}\right)^{\varphi}\right]\\
&\times\left(1-\mathcal{F}^{j+2}_{t+2}\right)\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right)\\
&+\ldots+\\
&+\beta^{N-j}\frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t+1}}J_{z^+,t+N-j}\left(\chi^{N-1}_{t+N-j-1}+\rho\right)\\
&\times\left(1-\mathcal{F}^{N-1}_{t+N-j-1}\right)\cdots\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right)\}
\end{aligned}
$$

Re-writing this in a way that makes use of Ω^i_t defined in (B.85) below:

$$
\begin{aligned}
\tilde{J}^j_{z^+,t}\left(\tilde{W}_{t-j};\bar{a}^j_t\right) &= \left(\bar{w}_t\mathcal{G}^j_t - G_{t-j,j}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j,t} - \frac{\kappa}{\varphi}\left(\tilde{v}^j_t\right)^{\varphi} + \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\frac{1}{\left(1-\mathcal{F}^j_t\right)} \qquad \text{(B.82)}\\
&\times\{\left[\left(\bar{w}_{t+1}\mathcal{G}^{j+1}_{t+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+1,t+1} - \frac{\kappa}{\varphi}\left(\tilde{v}^{j+1}_{t+1}\right)^{\varphi}\right]\Omega^{j+1}_{t+1}\\
&+\beta\frac{\psi_{z^+,t+2}}{\psi_{z^+,t+1}}\left[\left(\bar{w}_{t+2}\mathcal{G}^{j+2}_{t+2} - G_{t-j,j+2}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+2,t+2} - \frac{\kappa}{\varphi}\left(\tilde{v}^{j+2}_{t+2}\right)^{\varphi}\right]\Omega^{j+2}_{t+2}\\
&+\ldots+\\
&+\beta^{N-j-1}\frac{\psi_{z^+,t+N-j}}{\psi_{z^+,t+1}}J_{z^+,t+N-j}\Omega^{N-j}_{t+N-j}\}
\end{aligned}
$$

for $j=0,...,N-1$.

B.5.4. Bargaining Problem

The first order condition associated with the Nash bargaining problem is:

$$
\eta_t V_{w,t}J_{z^+,t} + (1-\eta_t)\left[V^0_{z^+,t} - U_{z^+,t}\right]J_{w,t} = 0, \qquad \text{(B.83)}
$$

after division by $z^+_t P_t$.

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables:

$$
\begin{aligned}
J_{z^+,t} &= \sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}\left[\left(\bar{w}_{t+j}\frac{\mathcal{E}^j_{t+j}}{1-\mathcal{F}^j_{t+j}} - G_{t,j}w_t\bar{w}_t\right)\varsigma_{j,t+j} - \frac{\kappa}{\varphi}\left(\tilde{v}^j_{t+j}\right)^{\varphi}\right]\Omega^j_{t+j}\\
&+\beta^N\frac{\psi_{z^+,t+N}}{\psi_{z^+,t}}J_{z^+,t+N}\frac{\Omega^N_{t+N}}{1-\mathcal{F}^0_{t+N}}. \qquad \text{(B.84)}
\end{aligned}
$$

We also require the derivative of J with respect to ω_t, i.e. the marginal surplus of the employment agency with respect to the negotiated wage. By the envelope condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and only be concerned with the direct impact of ω_t on J. Taking the derivative of (B.59):

$$\begin{aligned}
J_{w,t} \;=\;& -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\
& -\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1}\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\
& -\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+2}^2\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
& -...-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+1}^{N-2}+\rho\right)\times \\
& \left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left[1-\mathcal{F}_t^0\right].
\end{aligned}$$

Let,

$$\Omega_{t+j}^j=\begin{cases}\left(1-\mathcal{F}_{t+j}^j\right)\prod_{l=0}^{j-1}\left(\chi_{t+l}^l+\rho\right)\left(1-\mathcal{F}_{t+l}^l\right) & j>0 \\ 1-\mathcal{F}_t^0 & j=0\end{cases}. \tag{B.85}$$

It is convenient to express this in recursive form:

$$\begin{aligned}
\Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_{t+1}^1=\left(1-\mathcal{F}_{t+1}^1\right)\left(\chi_t^0+\rho\right)\overbrace{\left(1-\mathcal{F}_t^0\right)}^{\Omega_t^0}, \\
\Omega_{t+2}^2 &= \left(1-\mathcal{F}_{t+2}^2\right)\left(\chi_{t+1}^1+\rho\right)\overbrace{\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_t^0\right)\left(1-\mathcal{F}_{t+1}^1\right)}^{\Omega_{t+1}^1},...
\end{aligned}$$

so that

$$\Omega_{t+j}^j=\left(1-\mathcal{F}_{t+j}^j\right)\left(\chi_{t+j-1}^{j-1}+\rho\right)\Omega_{t+j-1}^{j-1},$$

for $j=1,2,....$. It is convenient to define these objects at date t as a function of variables dated t and earlier for the purposes of implementing these equations in Dynare:

$$\begin{aligned}
\Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_t^1=\left(1-\mathcal{F}_t^1\right)\left(\chi_{t-1}^0+\rho\right)\overbrace{\left(1-\mathcal{F}_{t-1}^0\right)}^{\Omega_{t-1}^0}, \\
\Omega_t^2 &= \left(1-\mathcal{F}_t^2\right)\left(\chi_{t-1}^1+\rho\right)\overbrace{\left(\chi_{t-2}^0+\rho\right)\left(1-\mathcal{F}_{t-2}^0\right)\left(1-\mathcal{F}_{t-1}^1\right)}^{\Omega_{t-1}^1}
\end{aligned}$$

so that

$$\Omega_t^j=\left(1-\mathcal{F}_t^j\right)\left(\chi_{t-1}^{j-1}+\rho\right)\Omega_{t-1}^{j-1}.$$

Then, in terms of scaled variables we obtain:

$$J_{w,t}=-\sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}G_{t,j}\Omega_{t+j}^j\varsigma_{j,t+j}. \tag{B.86}$$

Scaling V_t^i by $P_t z_t^+$, we obtain:

$$\begin{aligned} V_{z^+,t}^i &= G_{t-i,i} w_{t-i} \bar{w}_{t-i} \varsigma_{i,t} \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\,\psi_{z^+,t}} \\ &+\beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right) U_{z^+,t+1}\right], \end{aligned} \tag{B.87}$$

for $i = 0, 1, ..., N-1$, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i,\ U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.$$

In our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have:

$$\mathcal{M}_{t+j} = \left(1-\mathcal{F}_t^0\right)\cdots\left(1-\mathcal{F}_{t+j}^j\right), \tag{B.88}$$

for $j = 0, ..., N-1$. Then, the derivative of V^0, which we denote by $V_w^0(\omega_t)$, is:

$$\begin{aligned} V_w^0(\omega_t) &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}. \end{aligned} \tag{B.89}$$

Note that ω_t has no impact on the intensity of labor effort. This is determined by (B.56), independent of the wage rate paid to workers.

Scaling (2.66),

$$U_{z^+,t} = b^u\left(1-\tau_t^y\right) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [f_t V_{z^+,t+1}^x + (1-f_t)\, U_{z^+,t+1}]. \tag{B.90}$$

This value function applies to any unemployed worker, whether they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of an exogenous separation, or because they arrived because of an endogenous separation.

B.5.5. Final equilibrium conditions

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m (1-L_t)^\sigma v_t^{1-\sigma}, \tag{B.91}$$

where

$$L_t = \sum_{j=0}^{N-1} \left(1-\mathcal{F}_t^j\right) l_t^j. \tag{B.92}$$

and σ_m is the productivity of the matching technology.

In our environment, there is a distinction between effective hours and measured hours. Effective hours is the hours of each person, adjusted by their productivity, a. Recall that the average productivity of a worker in working in cohort j (i.e., who has survived the endogenous productivity cut) is $\mathcal{E}_t^j / \left(1 - \mathcal{F}_t^j\right)$. The number of workers who survive the productivity cut in cohort j is $\left(1 - \mathcal{F}_t^j\right) l_t^j$, so that our measure of total effective hours is:

$$H_t = \sum_{j=0}^{N-1} \varsigma_{j,t} \mathcal{E}_t^j l_t^j, \tag{B.93}$$

In contrast, total measured hours is:

$$H_t^{meas} = \sum_{j=0}^{N-1} \varsigma_{j,t} \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{B.94}$$

The job finding rate is:

$$f_t = \frac{m_t}{1 - L_t}. \tag{B.95}$$

The probability of filling a vacancy is:

$$Q_t = \frac{m_t}{v_t}. \tag{B.96}$$

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$v_t = \frac{1}{Q_t^l} \sum_{j=0}^{N-1} \tilde{v}_t^j \left(1 - \mathcal{F}_t^j\right) l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (B.96), (2.68) and (2.60).

B.5.6. Characterization of the Bargaining Set

Implicitly, we assumed that the scaled wage,

$$w_t^i = \frac{W_t^i}{z_t^+ P_t},$$

paid by an employment agency which has renegotiated most recently i periods in the past is always inside the bargaining set, $[\underline{\mathrm{w}}_t^i, \bar{w}_t^i]$, $i = 0, 1, ..., N-1$. Here, $\bar{\mathrm{w}}_t^i$ has the property that if $w_t^i > \bar{w}_t^i$ then the agency prefers not to employ the worker and $\underline{\mathrm{w}}_t^i$ has the property that if $w_t^i < \underline{\mathrm{w}}_t^i$ then the worker prefers to be unemployed. We now describe our strategy for computing $\underline{\mathrm{w}}_t^i$ and $\bar{\mathrm{w}}_t^i$.

The lower bound, $\underline{\mathrm{w}}_t^i$, sets the surplus of a worker, $\left(1 - \mathcal{F}_t^i\right)\left(V_{z^+,t}^i - U_{z^+,t}\right)$, in an agency in cohort i to zero. By (B.87):

$$\begin{aligned} U_{z^+,t} \quad &= \quad \underline{\mathrm{w}}_t^i \varsigma_{i,t} \frac{1 - \tau_t^y}{1 + \tau_t^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1 + \sigma_L)\,\psi_{z^+,t}} \\ &\quad + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho\left(1 - \mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left(1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}\right) U_{z^+,t+1}\right], \end{aligned}$$

for i = 0, ..., N - 1. In steady state, this is

$$\underline{\mathrm{w}}^i = \frac{U_{z^+} + \zeta^h A_L \frac{\varsigma_i^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+}} - \beta\left[\rho\left(1-\mathcal{F}^{i+1}\right)V_{z^+}^{i+1} + \left(1-\rho+\rho\mathcal{F}^{i+1}\right)U_{z^+}\right]}{\varsigma_i \frac{1-\tau^y}{1+\tau^w}}$$

where a variable without time subscript denotes its steady state value. We now consider the upper bound, $\bar{w}_t^i$, which sets the surplus $J_{z^+,t}$ of an agency in cohort i to zero, i = 0, ..., N - 1. From (B.84)

$$\begin{aligned} 0 \quad = \quad & \sum_{j=0}^{N-1-i} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+j} \frac{\mathcal{E}_{t+j}^j}{1-\mathcal{F}_{t+j}^j} - G_{t,j}\bar{w}_t^i\right)\varsigma_{j,t+j} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+j}^j\right)^{\varphi}\right]\Omega_{t+j}^j \\ & + \beta^{N-i}\frac{\psi_{z^+,t+N-i}}{\psi_{z^+,t}} J_{z^+,t+N-i}\frac{\Omega_{t+N-i}^{N-i}}{1-\mathcal{F}_{t+N-i}^0} \end{aligned}$$

for i = 0, ..., N - 1. In steady state:

$$\begin{aligned} 0 \quad = \quad & \sum_{j=0}^{N-1-i} \beta^j \left[\left(\bar{w}\frac{\mathcal{E}^j}{1-\mathcal{F}^j} - G_j\bar{w}^i\right)\varsigma_j - \frac{\kappa}{\varphi}\left(\tilde{v}^j\right)^{\varphi}\right]\Omega^j \\ & + \beta^{N-i} J_{z^+}\frac{\Omega^{N-i}}{1-\mathcal{F}^0}. \end{aligned}$$

For the dynamic economy, the additional unknowns are the $2N$ variables composed of $\underline{\mathrm{w}}_t^i$ and $\bar{w}_t^i$ for i = 0, 1, ..., N - 1. We have an equal number of equations to solve for them.

B.6. Summary of equilibrium conditions for Employment Frictions in the Baseline Model

This subsection summarizes the equations of the labor market that define the equilibrium and how they are integrated with the baseline model. The equations include the N efficiency conditions that determines hours worked, (B.56); the law of motion of the workforce in each cohort, (2.61); the first order conditions associated with the vacancy decision, (B.60), (B.63), $j = 0, ..., N-1$; the derivative of the employment agency surplus with respect to the wage rate, (B.86); scaled agency surplus, (B.84); the value function of a worker, $V_{z^+,t}^i$, (B.87); the derivative of the worker value function with respect to the wage rate, (B.89); the growth adjustment term, $G_{t,j}$ (B.62); the scaled value function for unemployed workers, (B.90); first order condition associated with the Nash bargaining problem, (B.83); the (suitably modified) resource constraint, (**??**); the equations that characterize the productivity cutoff for job separations, (B.76) and (B.80); the equations that characterize $\tilde{J}_{z^+,t}^j$ (B.82); the value of finding a job, (2.67); the job finding rate, (B.95); the probability of filling a vacancy, (B.96); the matching function, (2.68); the wage updating equation for cohorts that do not optimize, (B.57); the equation determining total employment, (B.92); the equation determining

Ω^j_{t+j}, (B.85); the equation determining the hiring rate, χ^i_t (2.60); the equation determining the number of matches (the matching function), (B.91); the definition of total effective hours (B.93); the equations defining $\mathcal{M}^j_t$, (B.88); the equations defining $\mathcal{F}^j_t$, (B.8); the equations defining $\mathcal{E}^i_t$, (B.7); the equations defining $\mathcal{G}^{j\prime}_t$ (B.74); the equations defining $\mathcal{F}^{j\prime}_t$ (B.72)

The following additional endogenous variables are added to the list of endogenous variables in the baseline model:

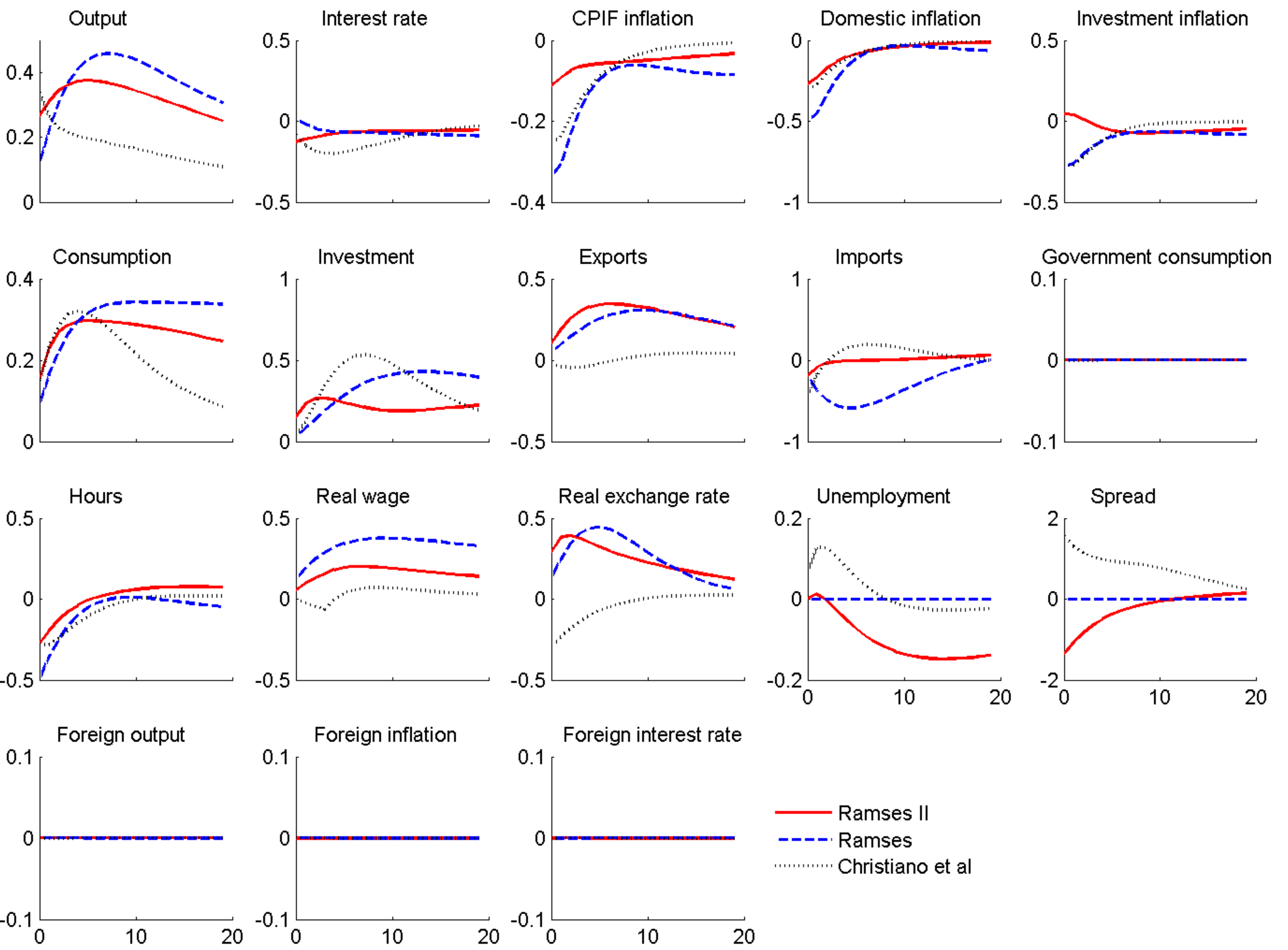
$$l^j_t, \mathcal{E}^j_t, \mathcal{F}^j_t, \varsigma_{j,t}, \mathcal{M}^j_t, \bar{a}^j_t, \tilde{v}^j_t, G_{t,j}, Q_t, \Omega^j_{t+j}, J_{w,t}, w_t, J_{z^+,t}, V^j_{z^+,t}, U_{z^+,t}, V^0_{w,t},$$
$$V^x_{z^+,t}, f_t, m_t, v_t, \chi^j_t, \tilde{\pi}_{w,t}, L_t, \mathcal{G}^{j\prime}_t, \mathcal{F}^{j\prime}_t \text{ and } \tilde{J}^j_{z^+,t}$$

We drop the equations from the baseline model that determines wages, eq. (B.41), (B.42), (B.43),(B.39) and (2.52).

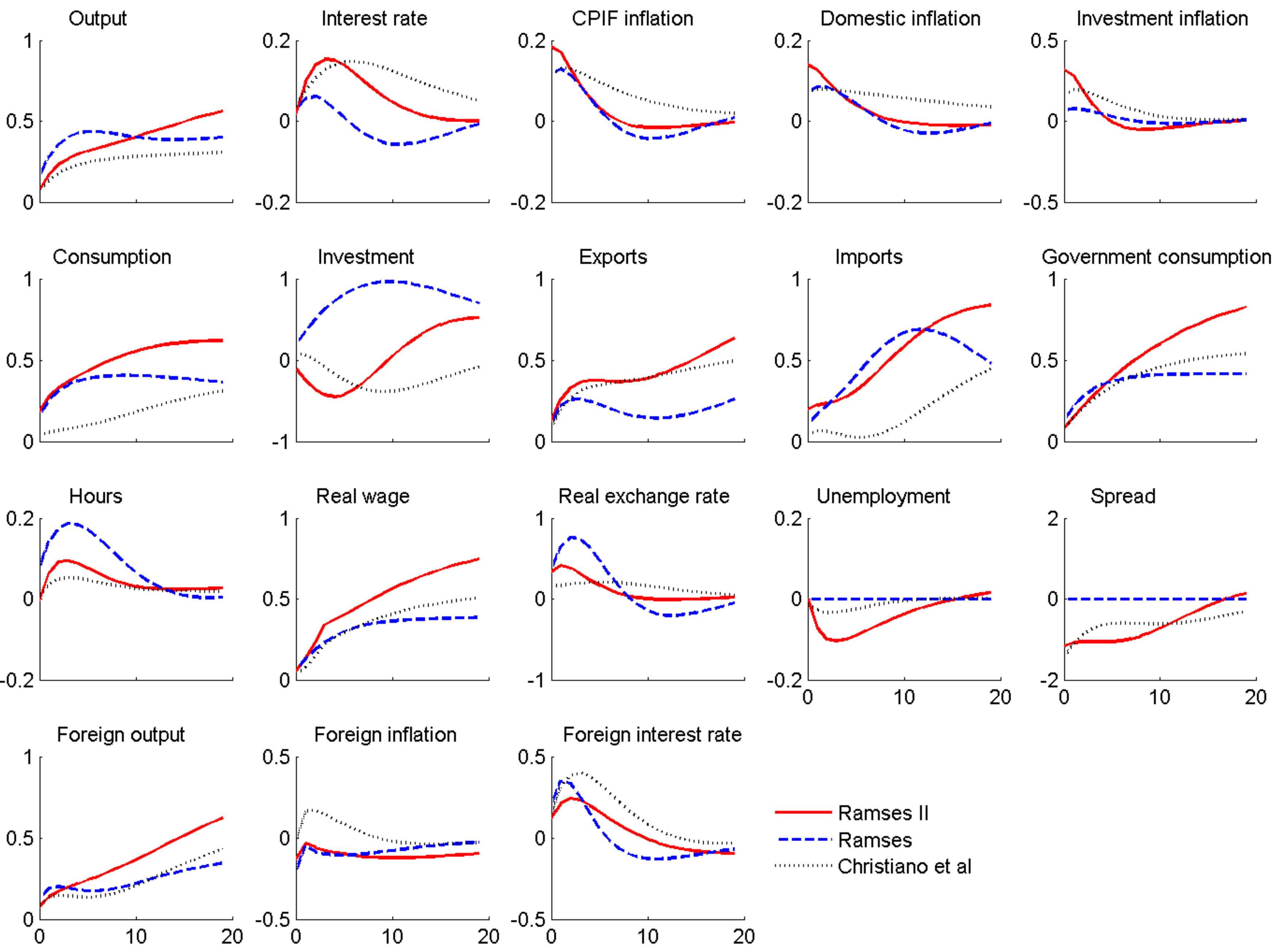
B.7. Summary of equilibrium conditions of the Full Model

In this subsection, we integrate financial frictions and labor market frictions together into what we call the full model.

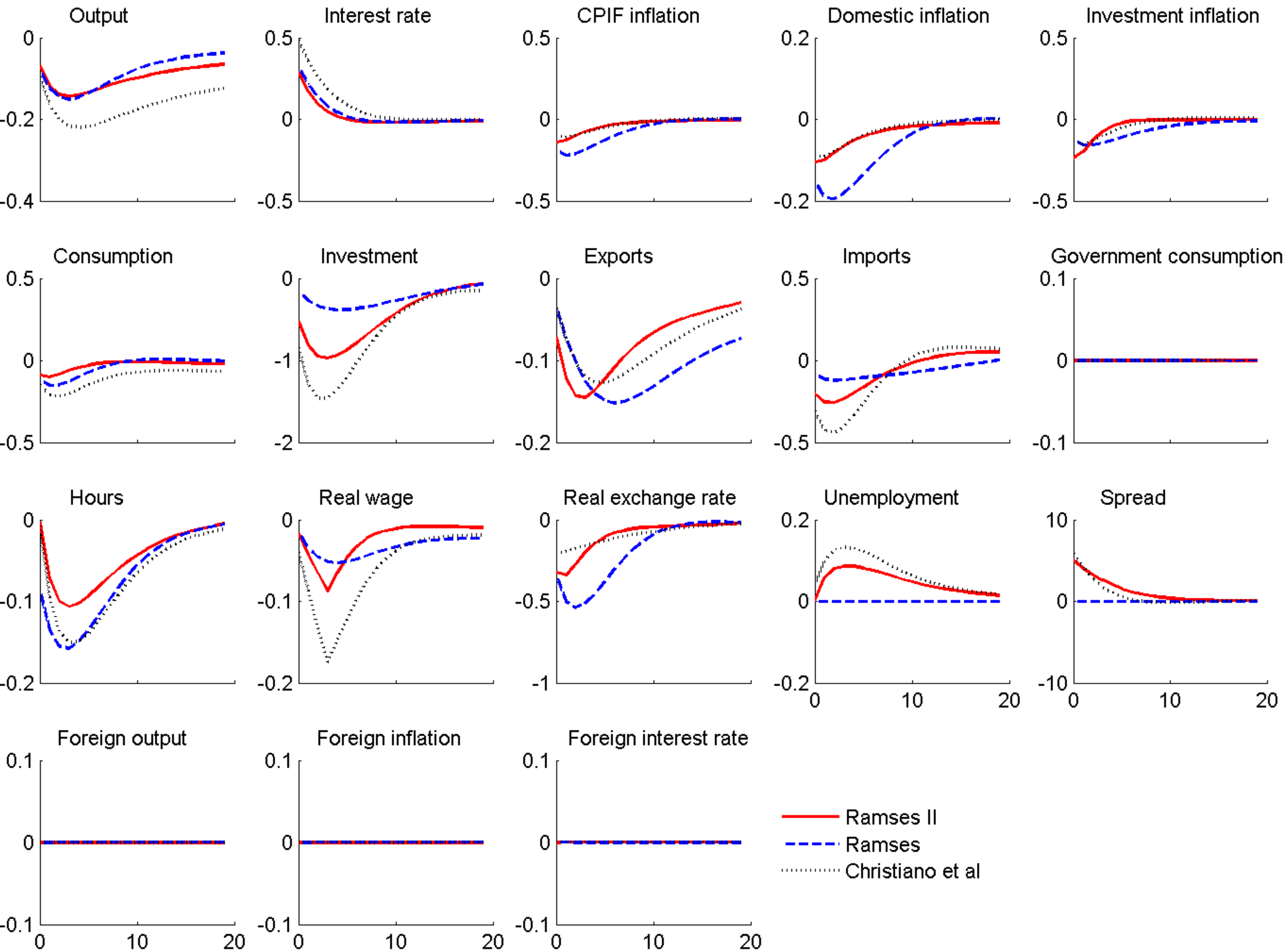
The equations which describe the dynamic behavior of the model are those of the baseline model discussed in section B.3.10 and section B.3 plus those discussed in the financial frictions model specified in section B.4.2 plus those discussed in the employment friction model presented in section B.6. Finally, the resource constraint needs to be adjusted to include monitoring as well as recruitment costs. Similarly measured GDP is adjusted to exclude both monitoring costs and recruitment costs (and, as in the baseline model, capital utilization costs).



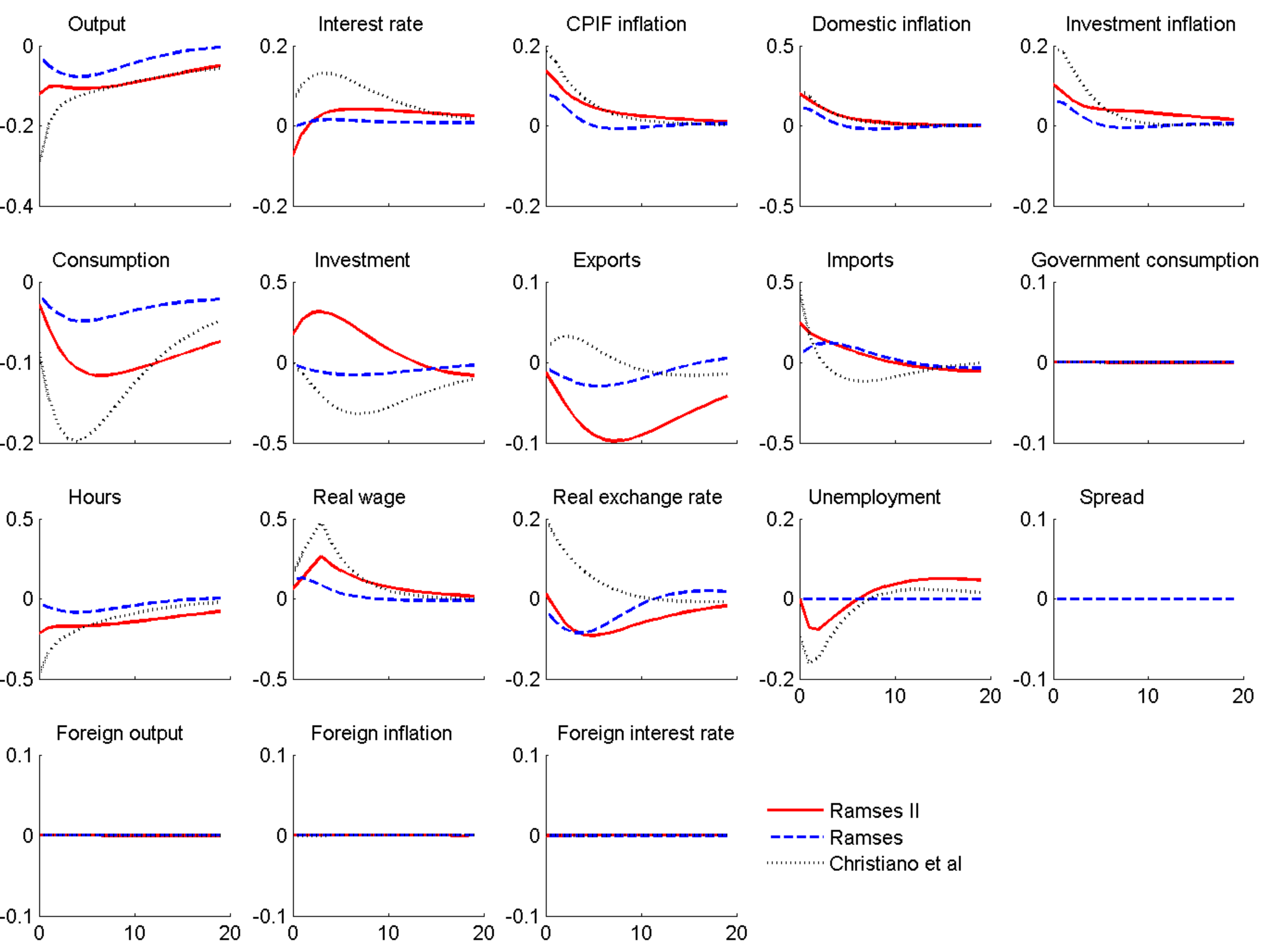
Stationary Neutral Technology Shock



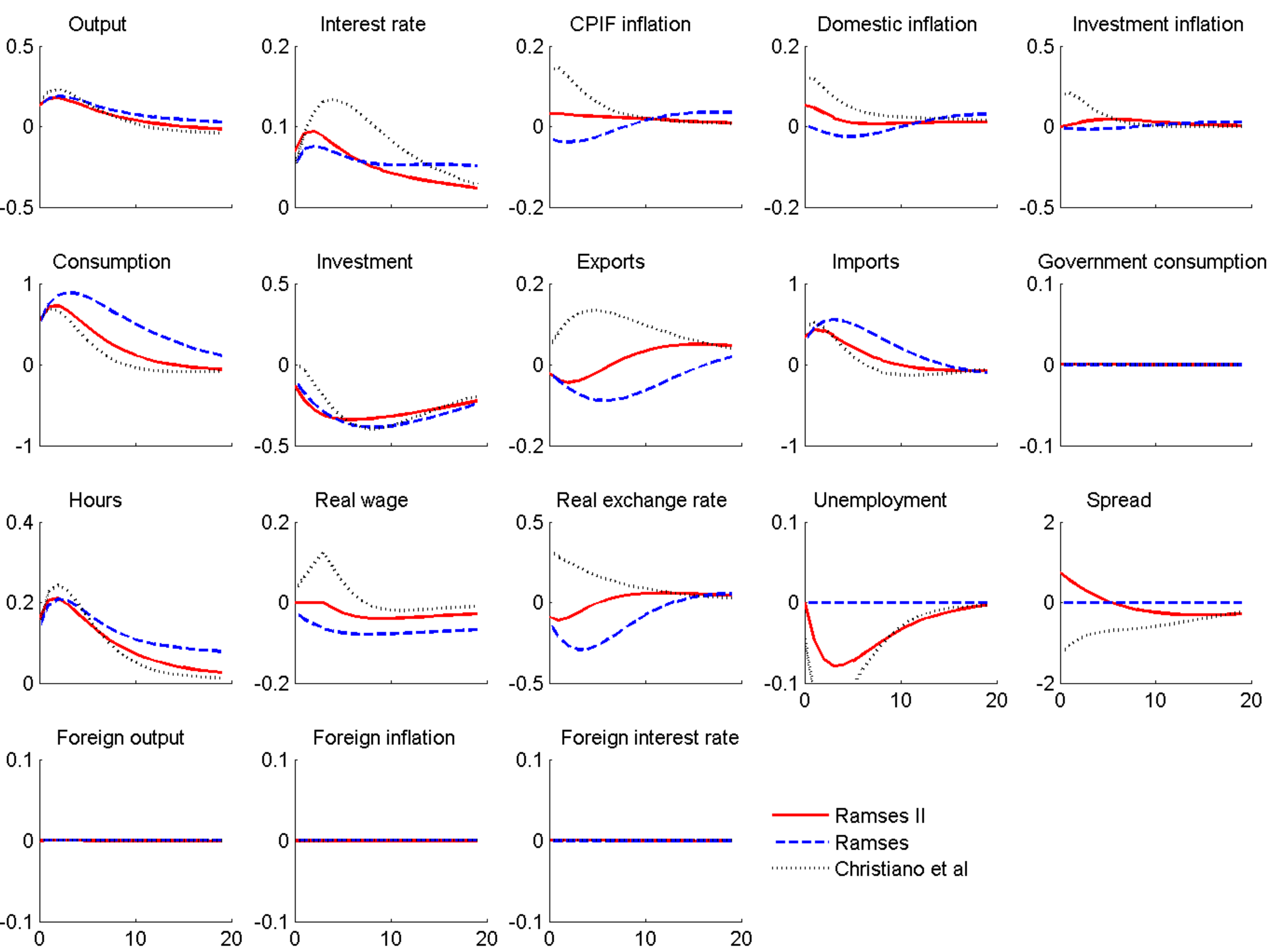
Unit-root Neutral Technology Shock



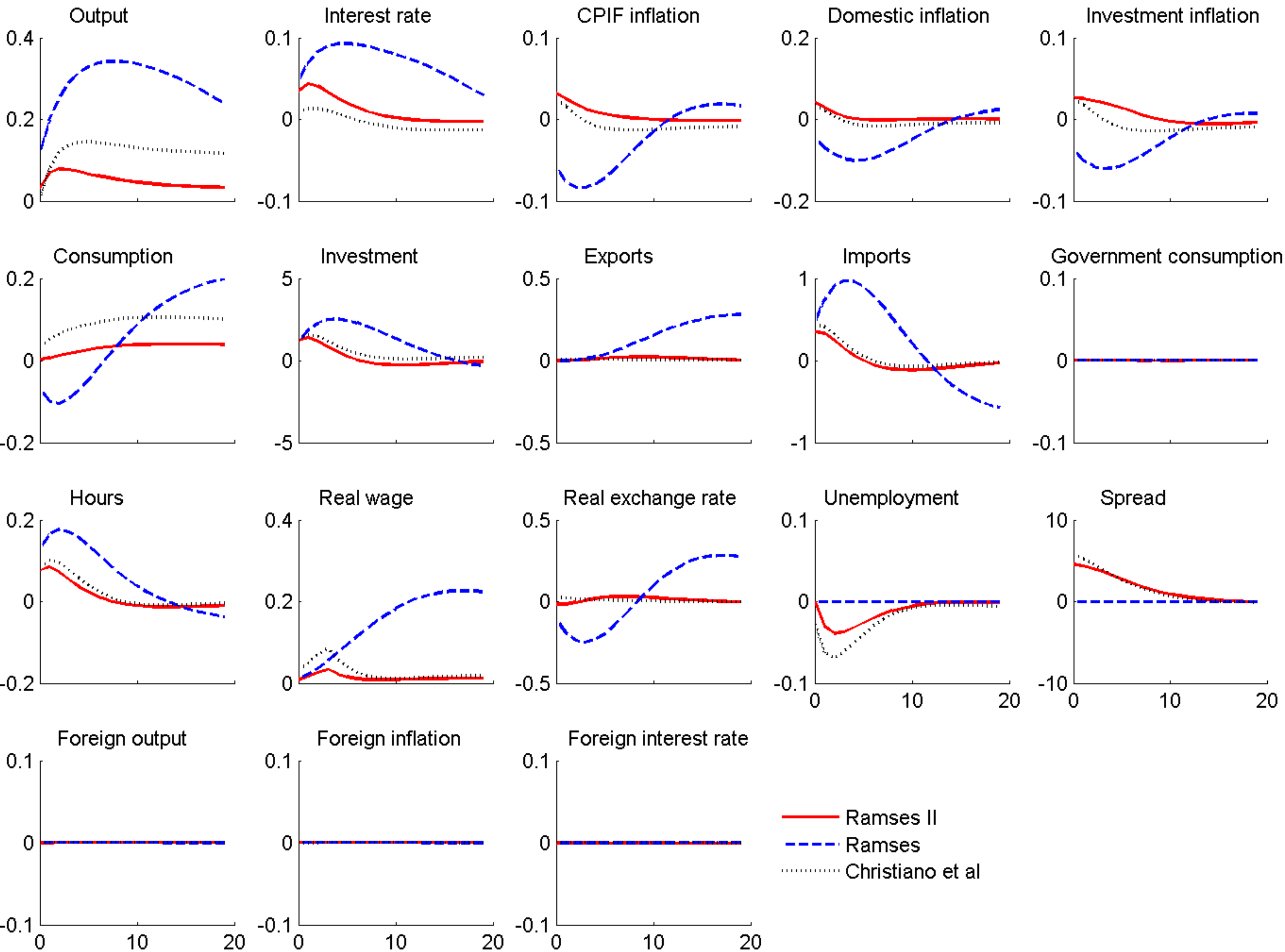
Monetary Policy Shock



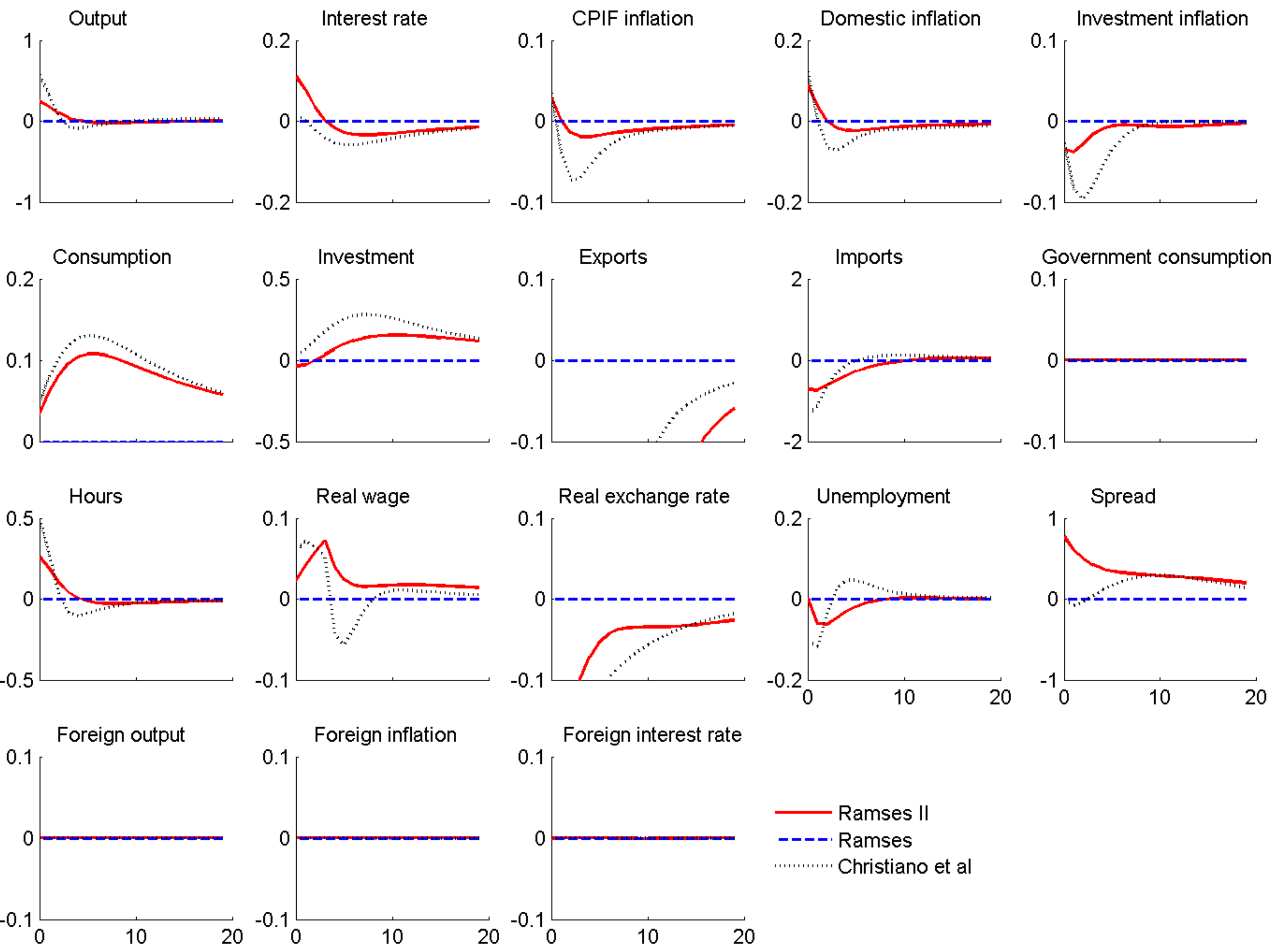
Labor Preference Shock



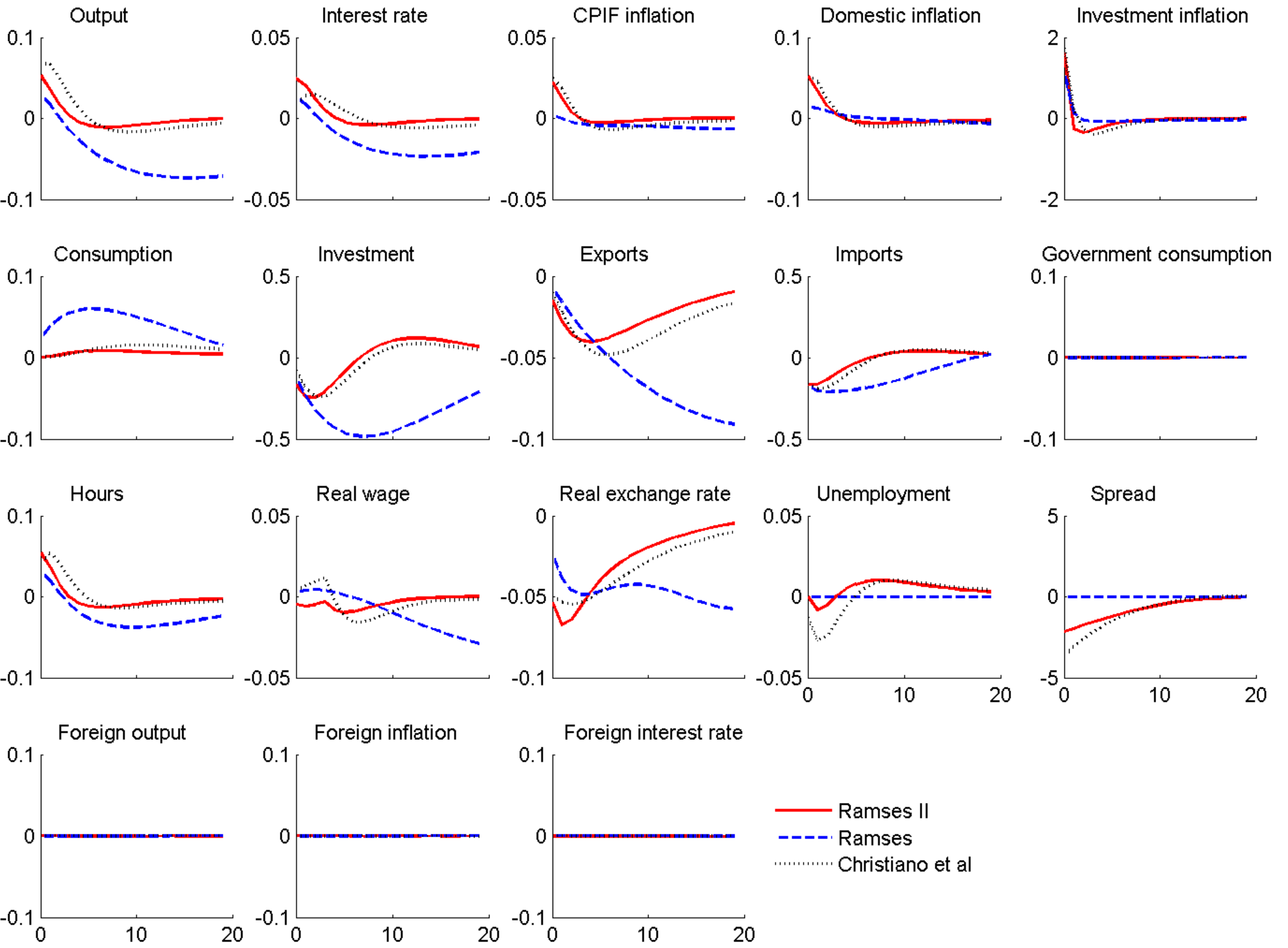
Consumption Preference Shock



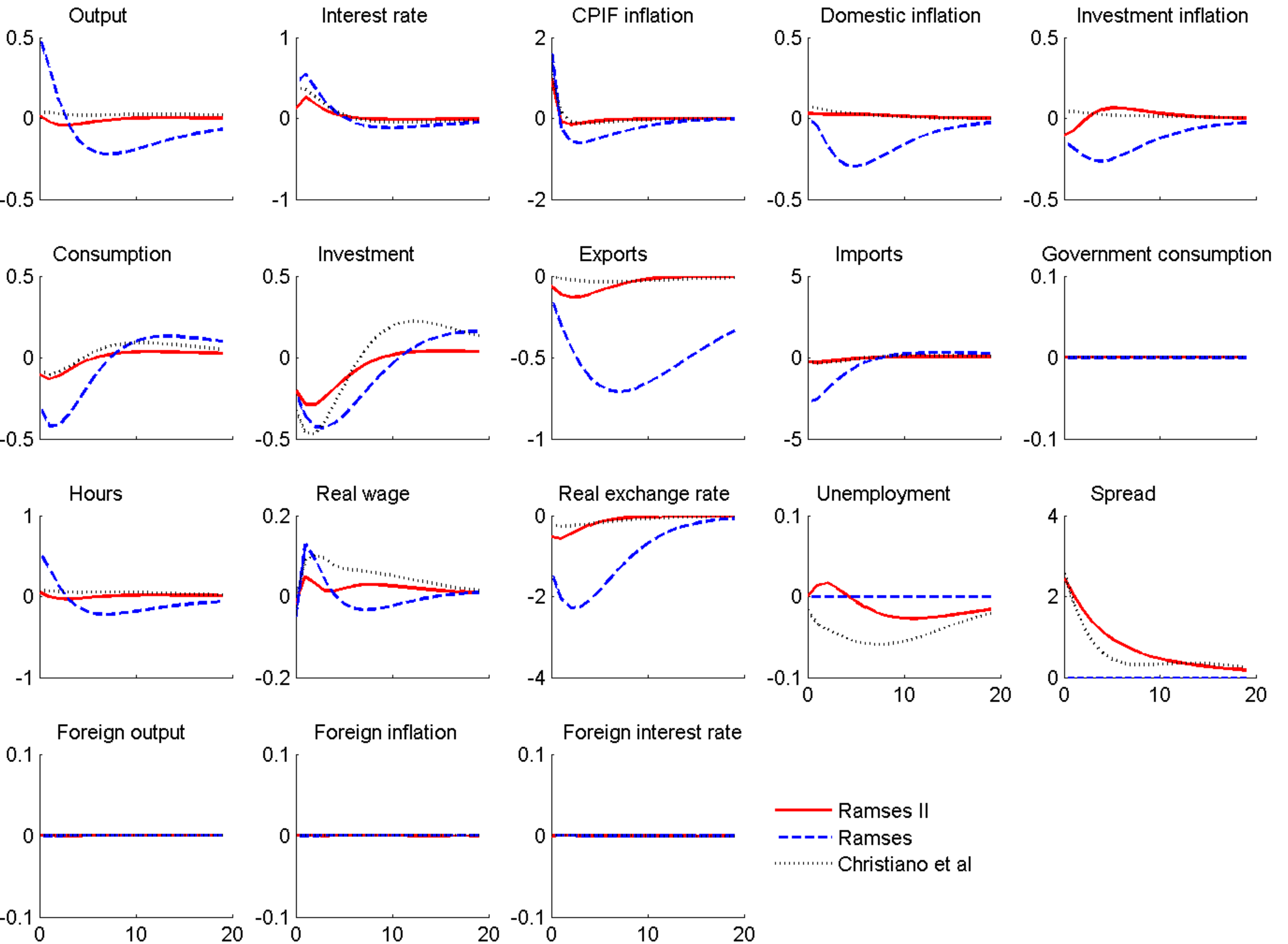
Stationary Investment Technology Shock



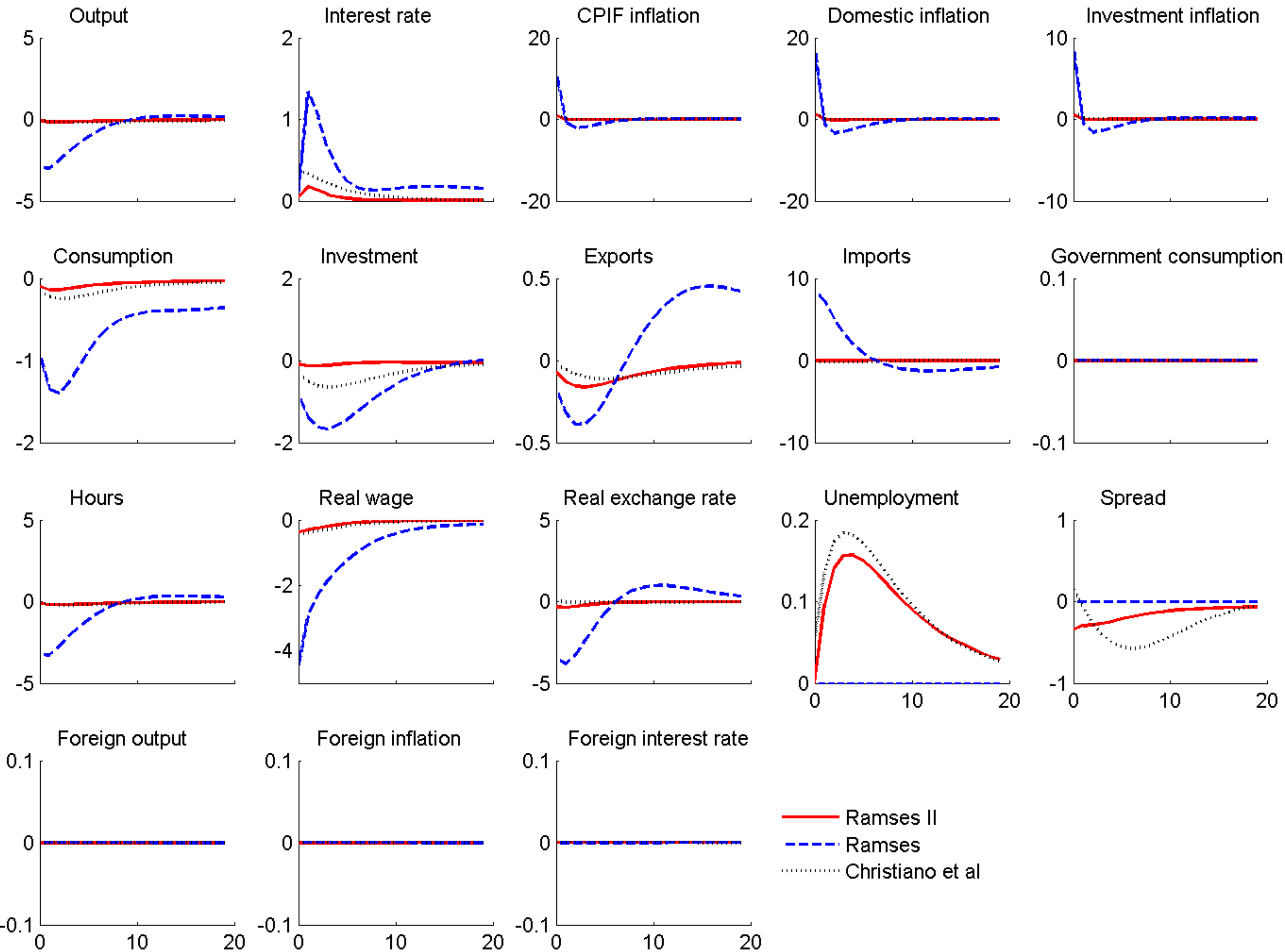
Import Export Markup Shock



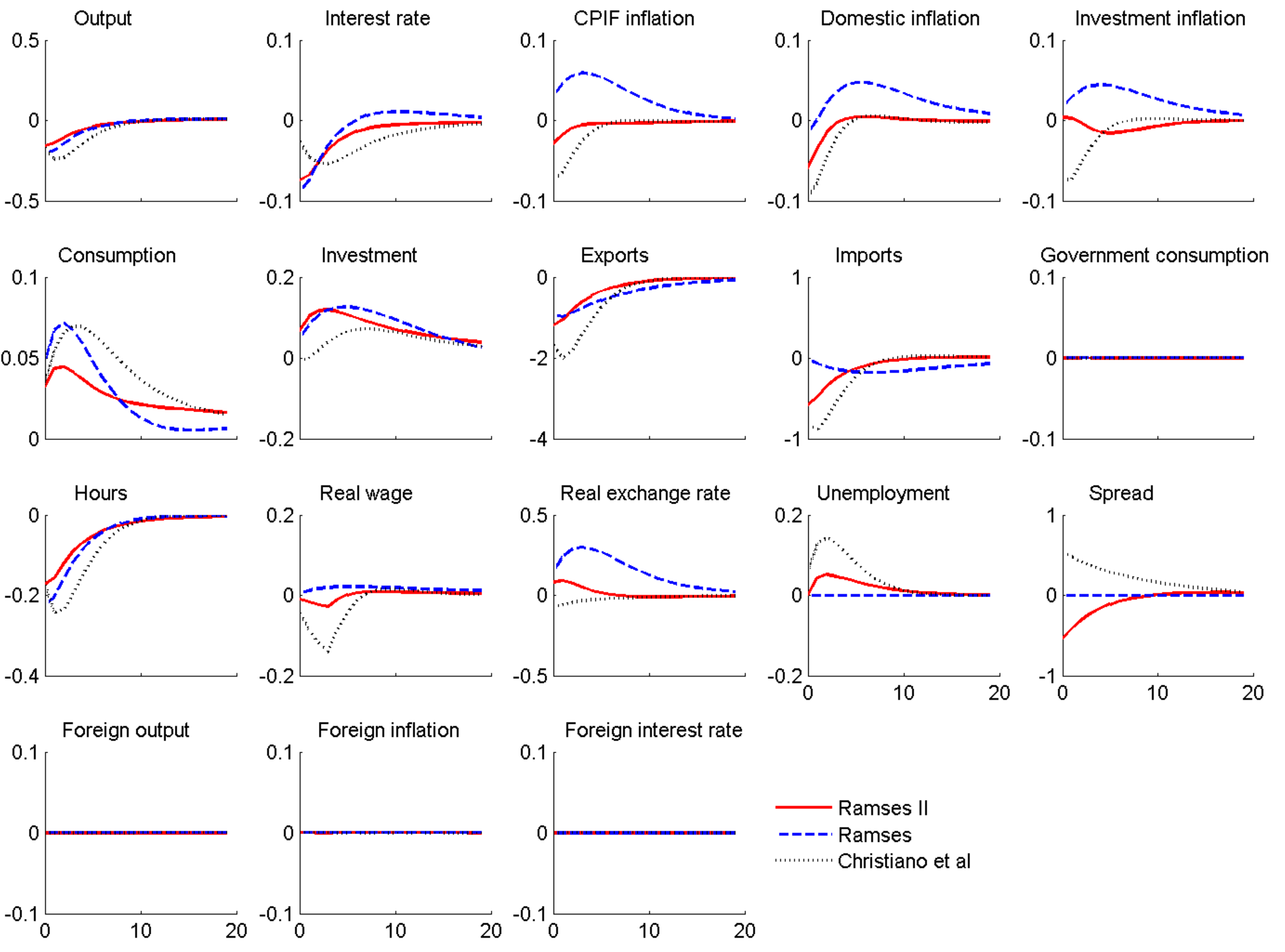
Import Investment Markup Shock



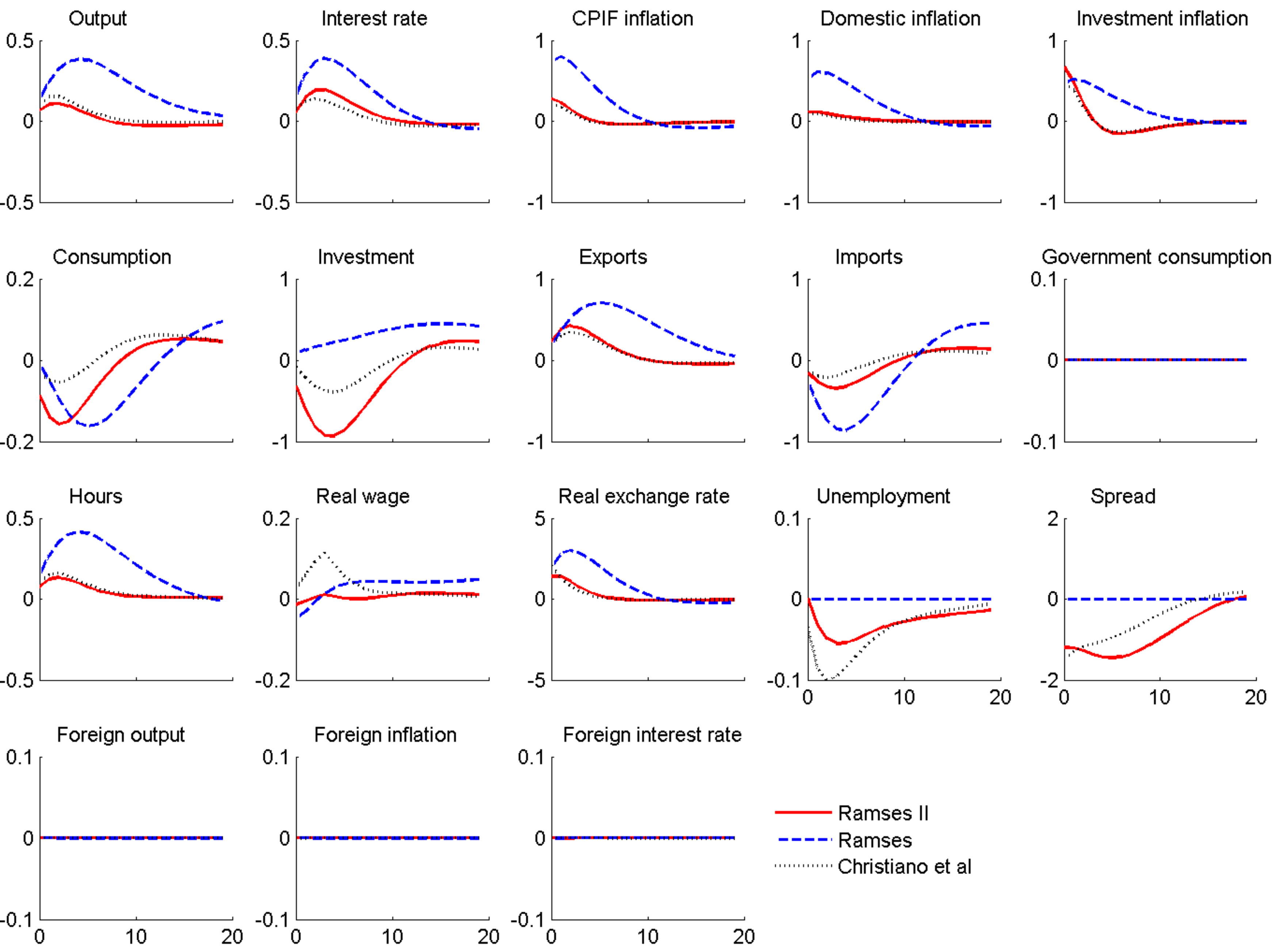
Import Consumption Markup Shock



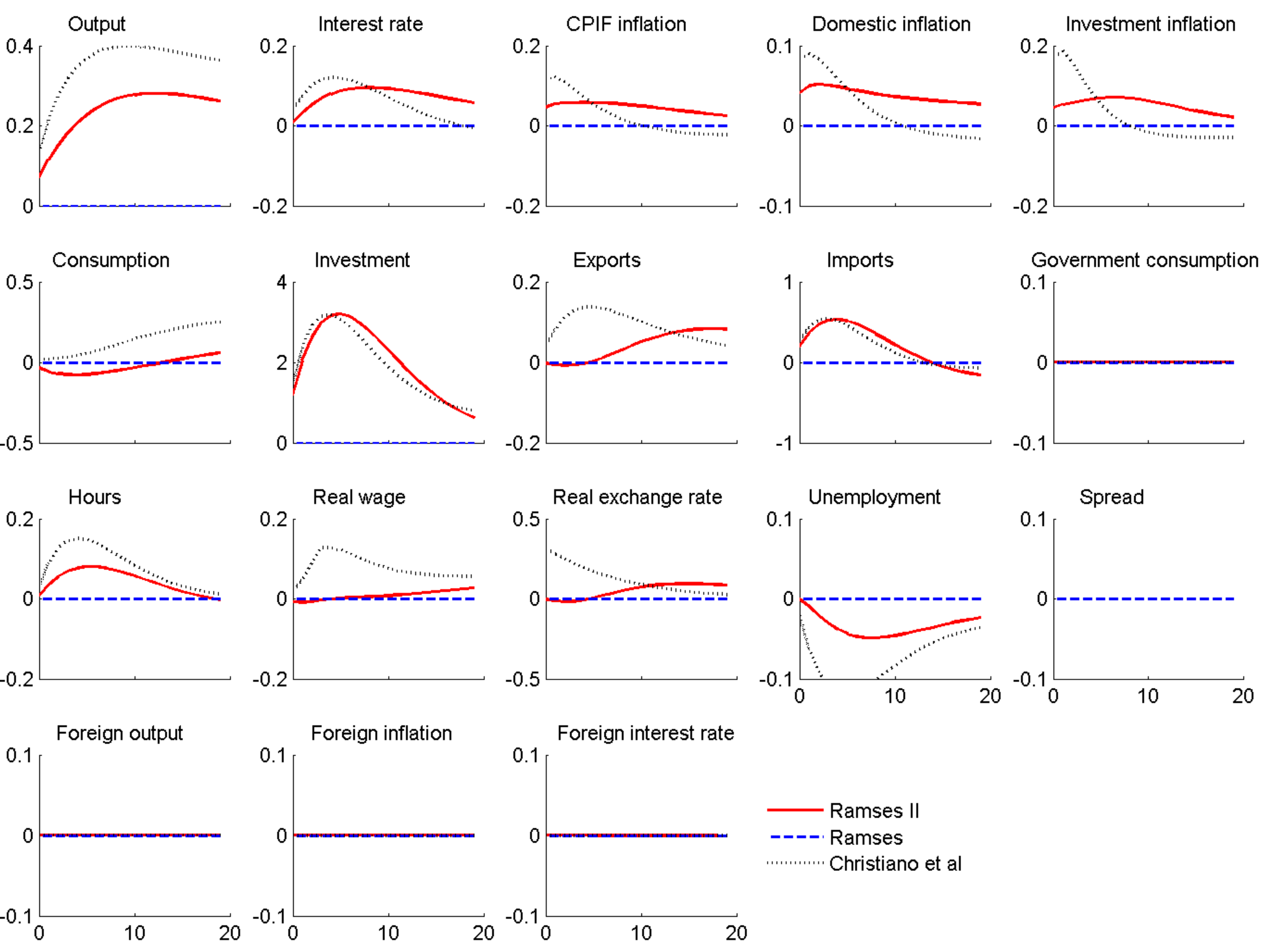
Domestic Markup Shock



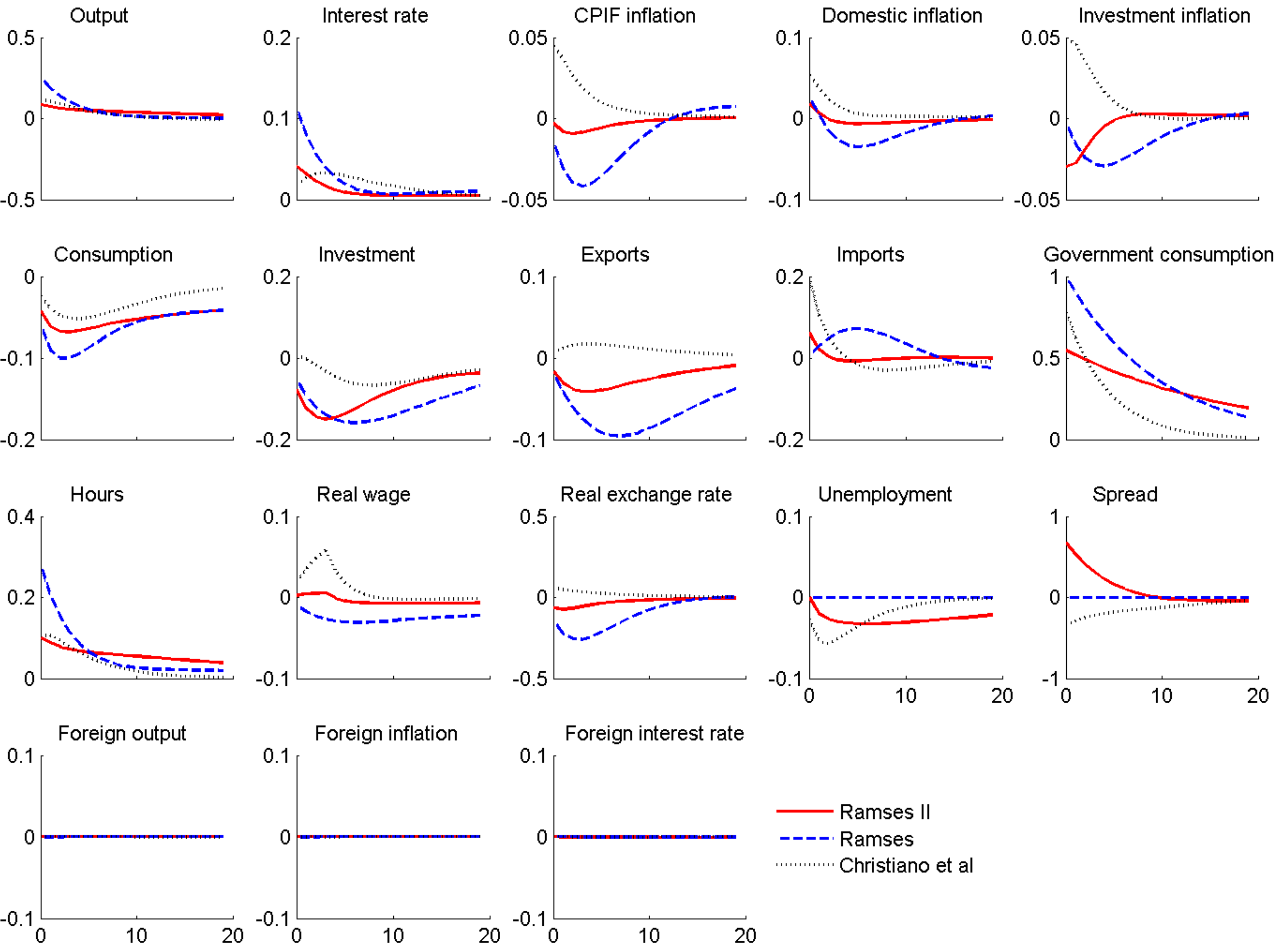
Export Markup Shock



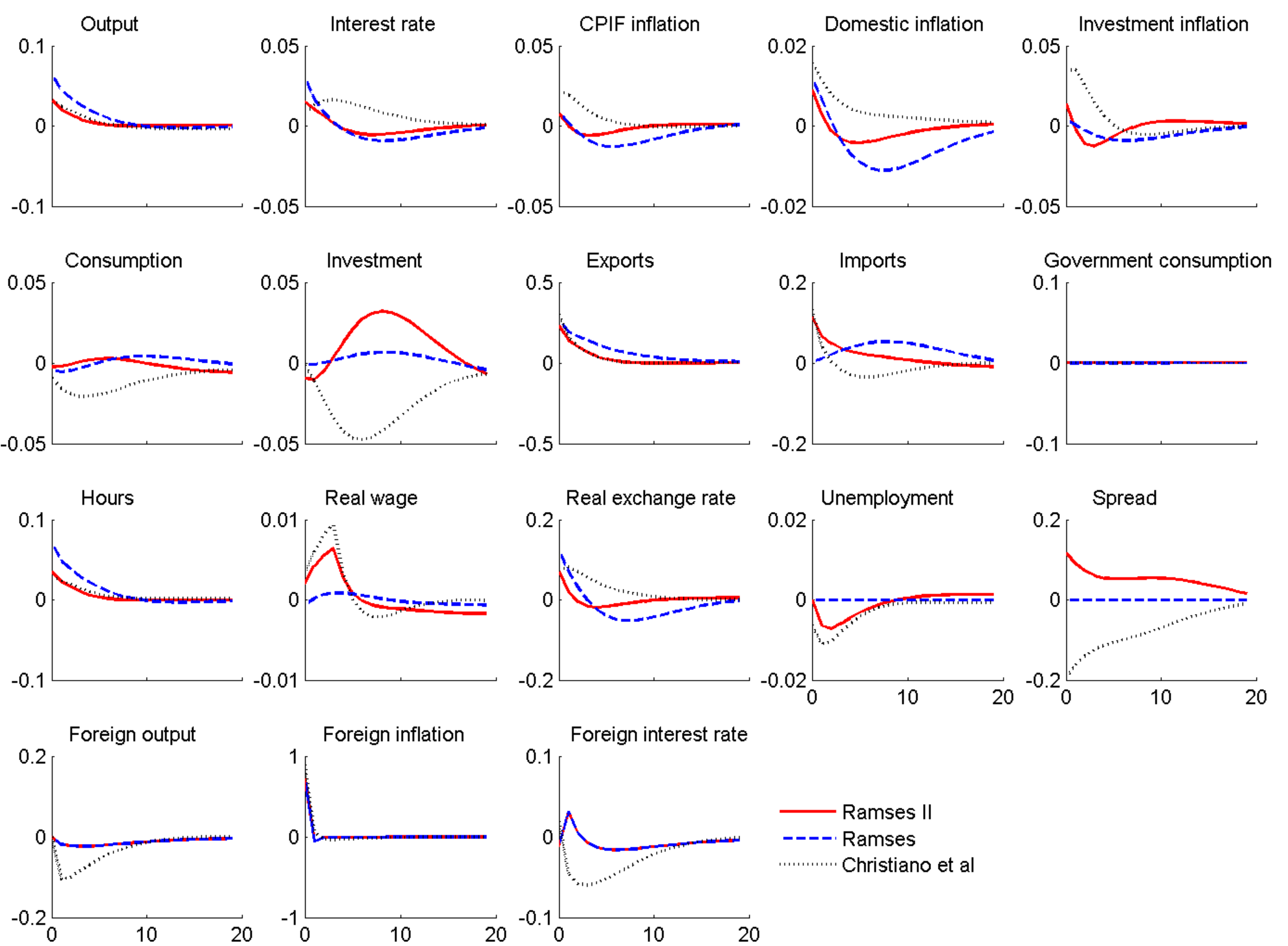
UIP Risk Premium Shock



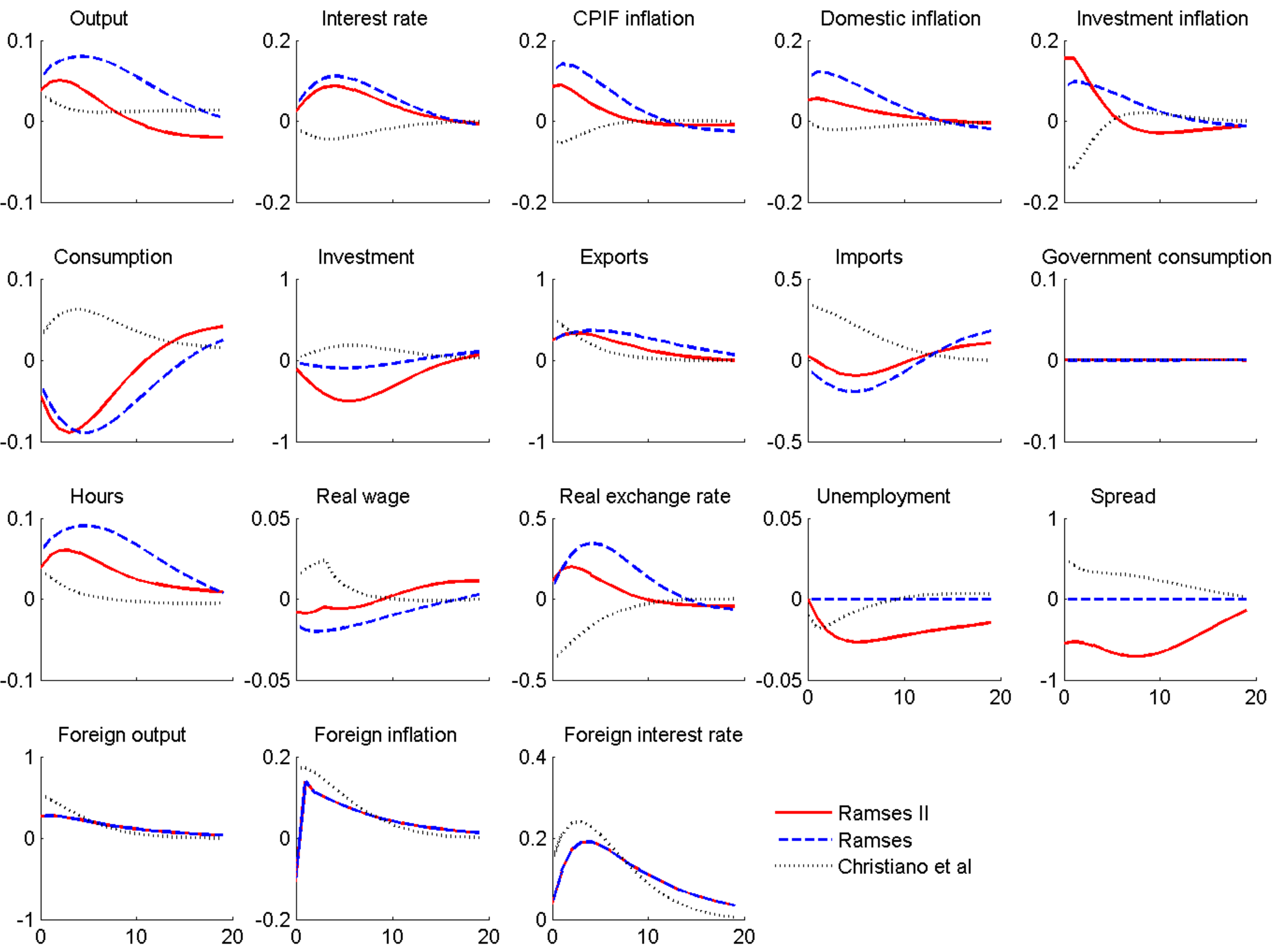
Entrepreneur Survival Prob. Shock



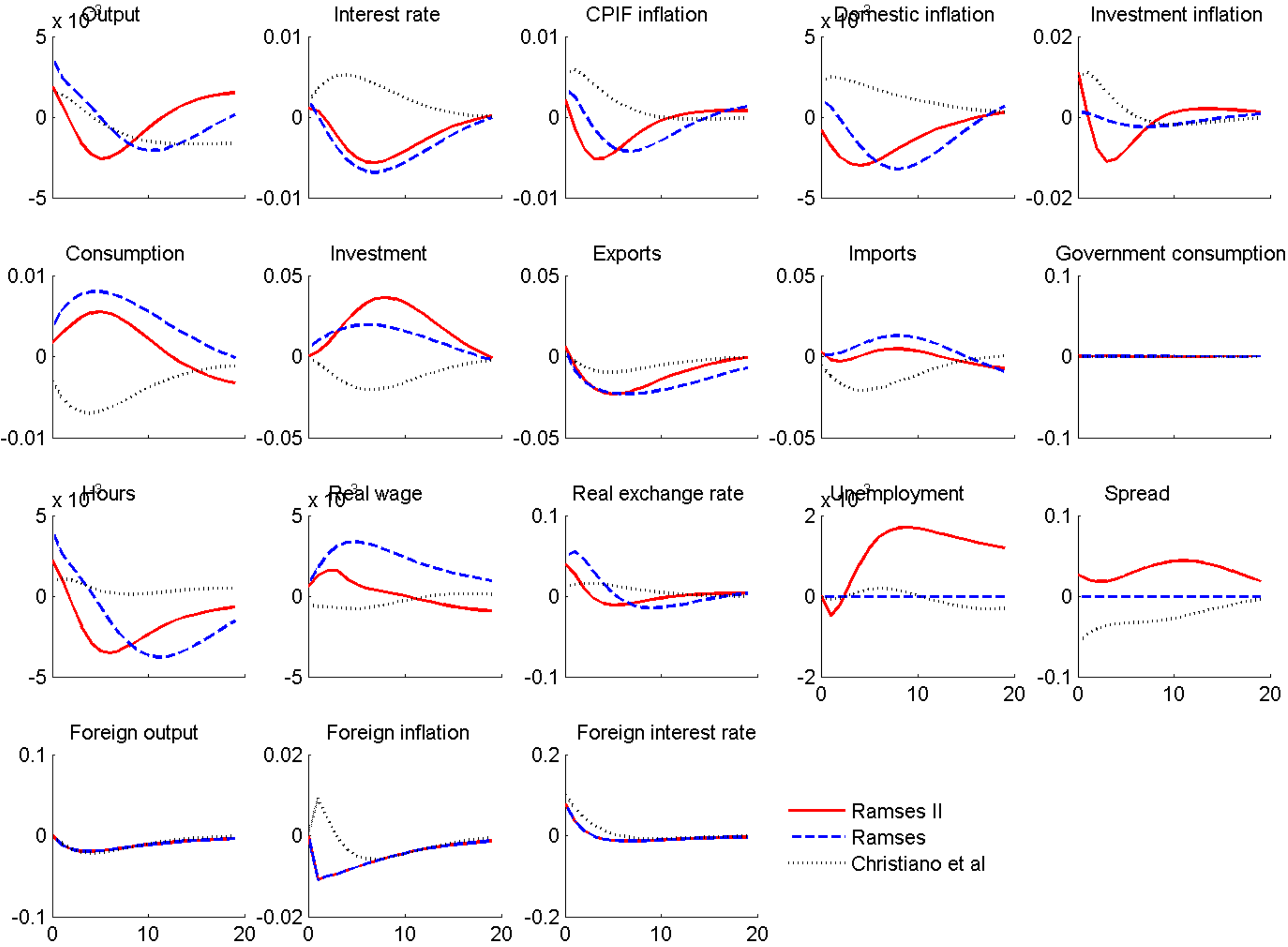
Government Consumption Shock



Foreign Inflation Shock



Foreign Output Shock



Foreign Nominal Interest Rate Shock

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